



Algebra Connections

A Framework-based description of the course

General Introduction

College Preparatory Mathematics

CPM Algebra Connections looks different from most other algebra courses. It is, in fact, these differences that make it an effective way for students of various abilities and interests to learn algebra. The California Mathematics Framework calls for a complete mathematics program that connects computational and procedural skills, conceptual understanding, and problem solving. In the Algebra Connection course, problem solving, understanding, and skills development are not treated as separate activities, but are carefully woven together so that students and teachers see a balanced treatment of each of them. Approximately 20 studies have examined statewide test results, high- and low-performing students, and SAT and ACT scores. These studies, as well as detailed investigations of individual schools, show that CPM students learn from 5% to 39% more than their counterparts who use other programs.

How does the CPM Algebra Connections course work? This course is designed to enable teachers to use multiple approaches for teaching the material to the diverse student population in California. Teachers are encouraged to use a variety of modes to accommodate different needs of students, such as lecture, class discussions, small student study teams, interaction with individual students, questioning, and summary. (See any lesson plan provided in the teacher materials for an example.) The regular interaction with the students in a problem-based setting allows teachers to monitor the individual progress of each student and the class as a whole by moving among the student study teams as each lesson progresses. In this way, they can intervene with one student or a study team to help overcome a moment of confusion and correct errors. When necessary, the teachers can pull the class together and address a challenging concept in a prescriptive lecture or make notes about what needs to be addressed during the lesson's closure.

The problem-based nature of each lesson provides guided, purposeful work that supports the learning of the lesson's targeted standards. The problem set structures the students' work so that they see how an idea develops, how it is related to other ideas, and why an algorithm works. Lessons are developed to build on prior skills and concepts so that as the course progresses, students see increasingly complex problems in each topic area. Problems are designed to accommodate different learning modalities, especially visual and kinesthetic learners. For example, in addition to the written text and teacher instruction, geometric representations help students visualize algebraic abstractions.

In addition to accommodating California's diverse learning population, lessons are designed to flexibly accommodate different classroom structures. Consequently, a learning team may consist of the entire class, four students, or pairs. During class, teachers may instruct students to brainstorm collectively or in teams to devise a solution strategy for the core problem in the lesson. The text regularly contains several questions in the lesson introduction or offers "Discussion Points" to help the class get started. For challenging problems and activities, a "Further Guidance" section provides support for students who need extra help. The use of small learning teams gives students the opportunity to discuss mathematics and to ask questions of their peers and the teacher.

After a concept or skill is developed, the text presents formal vocabulary, emphasizes key ideas, and provides explicit examples of algorithms in a “Math Notes” box for individual study and reference. There is online homework help (www.hotmath.com) as well as additional assistance using the Extra Practice resource booklet for use in the classroom and at home. There is also a Parent Guide booklet that explains concepts in a straightforward manner so that parents can assist their child. Both booklets offer explanations of concepts and skills, contain step-by-step examples, practice sets, and answers. They are available in book form for classroom use and posted for free download for students and parents at www.cpm.org.

Standards Alignment

All of the California Algebra Content Standards are addressed in the textbook. A few standards addressed in Chapters 1 - 12 are further developed and practiced in the “Additional Topics for California” section that follows Chapter 12. The text develops the content standards by building from basic ideas to deeper understanding and more complex situations by the last three chapters. For example, students learn the fundamentals of linear and quadratic relationships over several chapters, and then formalize them as functions in Chapter 11. When there are connections between ideas that will enhance student understanding, retention, and the ability to apply the mathematics they are learning, students are challenged to make them. For example, students spend three chapters studying graphing and equations, then tie together patterns, tables, rules, and graphs as multiple representations of given situations in Chapter 4. These multiple representations then become a way to analyze other non-linear functions in Chapters 8 and 11.

The text introduces new ideas in “problem-based” lessons, a research-based approach to learning, that are structured to develop one main idea. (The details of the CPM research base are in the front of the Teacher Edition under the “Research and Results” tab.) Teachers alternate between questioning students, leading discussions, and lecturing as needed during the class period, so that students meet the objectives of the lesson. The teacher is instructed to bring closure to each lesson and is provided suggestions for how to do so in the teacher materials. Using “Math Notes” entries, the text formalizes topics shortly after they have been introduced and students have had an opportunity to practice them.

Often the development of some of the larger ideas in the course takes several weeks and months. Consequently, the primary standards citations show where a topic is developed, while the supporting citations list where it is practiced over time. Each standard is developed beyond a minimal level of competence. For example, for Standard 3.0, students start with solving basic linear equations and inequalities in Chapters 2 and 3, but continue to develop skills to solve increasingly challenging equations so that later they can solve equations such as $(\sqrt{|x+5|} - 6)^2 + 4 = 20$ in Chapter 10 (problem 10-92).

Field testing of the course has been extensive, with two years of reviews by teachers and several rewritings in response to their suggestions. As a result, we know that students learn the material by completing the work in the textbook, including homework. However, the design of the book recognizes that not all students learn at the same time and at the same pace. Research-based “spaced practice” addresses this reality by giving students the opportunity to practice, revisit, and eventually master and build on ideas that may have been difficult at first. Students, teachers, and parents have additional practice available in the Extra Practice resource booklet and the Parent Guide. Both of these resources, available at www.cpm.org, use an explicit format to provide succinct summaries of ideas coupled with solved examples and additional practice. Tutorial solutions to the homework problems are available at www.hotmath.com to provide additional support for students and parents.

Program Organization

The first nine chapters each contain two or three topics that develop the fundamentals of algebra. The remainder of the course extends this core work with more formal and advanced study. Concepts are explored and practiced as they are introduced, but practice to mastery extends over days, weeks, and sometimes months, depending on the complexity of the topic. For example, after factoring is taught in the first section of Chapter 8, there are 25 practice problems in the remainder of the chapter, then more than 70 additional problems that require students to factor in the rest of the book.

Most lessons are designed for a 50-minute class, while multiple day lessons are divided into 50-minute segments. They are constructed to foster conceptual understanding concurrent with the development of rules and algorithms. Each homework section includes work with the concept or skill itself as well as practice in situations where it is used in the context of a larger problem. For example, factoring is practiced in the work with rational expressions. Students have time to develop experience with ideas, work with them at a fundamental level, and grow in their understanding of them through direct practice as well as use them in other contexts to reach mastery. Since practice with the material is spaced over time, and because the classroom structure allows considerable opportunities for teachers to work individually with students who need assistance, teachers are encouraged to follow the pacing suggestions in the Teacher Edition. (See the “Universal Access” section in the Teacher Edition for additional information about pacing.)

As part of a lesson’s closure activities, students are frequently asked to explain what they have learned by making an entry in their “Learning Log.” This activity gives them a chance to individually consolidate an idea. Teachers then review the concept so that students can verify that their Learning Log entry is complete and accurate. Shortly thereafter, usually within a lesson or two, the idea is given formally in a “Math Notes” entry along with an example. For example, in Lesson 5.1.2, students make an entry that describes the pattern they see in problems that develop the Distributive Property. They do additional work with the Distributive Property in Lesson 5.1.3, which concludes with the formal statement of the property and a multi-dimensional example in the “Math Notes” box on page 198.

Each chapter begins with an overview of what will be learned. In addition, the Teacher Edition has a lesson map of each chapter, a detailed discussion of how the chapter develops, and notes about any special challenges and study team issues. There is also a brief description of how and where the ideas will be used later in the course. The teacher introduction to each lesson follows a standard structure that addresses the objectives, pacing, core problems, materials needed, team roles, and homework. In the section titled “Suggested Lesson Activity,” there is a discussion of the lesson that conveys the authors’ intent for the developmental problems and contains various suggestions for teachers to differentiate instruction. In particular, this section outlines the pacing of each part of the lesson, suggests questions to ask students during their study team work, notes where there might be difficulties and how to address them, and provides explanations of any special elements of the lesson such as the use of manipulatives or the construction of a model. The lesson introduction concludes with closure suggestions. There is additional assistance with closure in the front of the Teacher Edition (“Closure” tab).

Since students usually work on the core problems in study teams, a research-based approach to learning, the teacher materials offer regular support for managing the teams. The use of structured study teams helps students to focus on the lesson and participate effectively in the class. The structure of the problems fosters discourse about the mathematics in the lesson. All students have the opportunity to contribute to the team in several ways: raising and answering questions, facilitating the team’s work (getting started, seeking consensus, summarizing progress), recording and organizing the team’s work, keeping the team on task, and interacting with the teacher with questions and the acquisition of necessary resources for the lesson.

The student text has also been carefully designed to serve as an effective study aid. The index at the end helps students not only find problems that introduce a topic, but also directs them to corresponding “Math Notes” boxes. The glossary provides formal mathematical definitions for each vocabulary term with illustrations whenever appropriate. At the end of Chapters 1 - 11, a closure section provides a set of review problems for both current and past work (titled “What Have I Learned?”). Each of these chapter reviews is followed by answers to the problems along with a list of where to go for help and additional practice with each type of problem. Chapter 12 ends the course with five challenging activities that require students to review the main skills and concepts of the course.

Assessment

The assessment resource that comes with the Teacher Edition has three components: an assessment handbook, a plan for assessment in each chapter, and a problem bank that serves as the resource for the construction of various types of tests. The handbook discusses the nature of assessment, then offers several formats for assessment itself and suggestions for how to use each of them: individual tests, team tests, participation quizzes, student presentations, class observations, and portfolios. There is also advice about how to use rubric scoring and evaluate student presentations. The next component is a plan for how, when, and where to use each of the five assessment strategies in each chapter, as discussed in the “Assessment Handbook” tabbed section. The plan also specifies the content for the individual student test. Finally, the test bank has problems and sample tests for each chapter. For example, the Chapter 3 problem bank has study team test items, questions that use topics from previous chapters, a sample study team test, individual test items, including material from previous chapters, and a sample individual test. These pages are printed four to a page to use for reference, since all of the assessment materials are on a CD in both Word and PDF formats so that teachers can create their assessments electronically.

In the broadest sense, assessment takes place on a daily basis, since teachers are able to interact with each student during the study team portion of the lesson. The discussion of each lesson in the Teacher Edition frequently suggests strategies to help teachers address student difficulties. By observing and interacting with individual students and study teams, teachers can readily assess how well students understand the topics in the lesson, check on the understanding of previous topics, and review or re-teach topics tailored to the specific concerns of the student or study team. The use of more formal assessment methods discussed in the previous paragraph compliment the teacher’s daily observations of the students.

Universal Access

The Teacher Edition has a section that discusses how to work with students of varied mathematical backgrounds and needs (“Universal Access” tab). It reinforces the “spaced practice” structure of the text that gives students multiple opportunities to learn, practice, and master an idea. There are specific suggestions for addressing benchmark, strategic, and intensive students as well as those with special needs, English learners, and advanced students.

The design of the course discussed in detail elsewhere in this essay makes it possible to tailor learning to the needs of individual students. Lessons give teachers options for conducting them. The frequent and varied assessment methods, coupled with built-in regular observations of student work during class, give teachers the information they need to individualize their instruction for the needs of their students. In particular, each chapter has several options to use for closure and assessment. Included with each chapter’s resource pages are vocabulary cards that teachers can use at any time to help students identify the terms they need to know and to build their mathematics vocabulary.

There is a resource section in the front of the Teacher Edition that describes various learning strategies for teachers to use with student study teams (“Study Team Strategies” tab). The instructions for using each strategy found there are complimented with practice using them during the Algebra Connections workshops provided by CPM for first-year teachers of the course. Teachers can then incorporate these strategies into their lesson plans. In addition, the “Lesson Overview” and “Suggested Lesson Activity” for each lesson offer teachers suggestions and options for conducting the lessons and responding to individual student needs.

Instructional Planning and Support

The Algebra Connections course is a self-contained program that teaches the requisite California standards for Algebra 1 in a single school year. There are about 130 lessons, which leave ample time for chapter closure activities and assessment. The student and teacher texts are the core resources for the program. The Teacher Edition contains the lesson plans, resource pages, materials list, and assessment resources (both print and electronic versions). Homework help (www.hotmath.com), an Extra Practice resource, and a Parent Guide (the latter items at www.cpm.org or available in soft cover) provide tutorial help, alternate explanations of concepts, examples, and additional practice (with answers) for students who need this assistance. Teachers may use these resources for additional practice for individual students as well as with their entire class if needed.

The lesson plans provide multi-dimensional support for the teachers. The detailed discussion in the “lesson activities” section explains how the mathematical content is developed through the students’ work on the problems. Teachers who need additional help with the mathematics may refer to the Extra Practice and Parent Guide resources for straightforward explanations of topics with examples. The expected outcomes from the problems and connections between and among the ideas in the lesson and other topics in the course are noted for teachers. The lesson notes address what the authors expect students to learn from the problems and guide teachers as to what to do if students have difficulty; there are also cautions about how much understanding to expect at various points of the development of major ideas. Each plan specifies the core problems that must be covered to meet the objectives of the lesson and those that are extensions, lead to deeper understanding, or provide practice with an idea. Technology-based options have their own lesson discussion sections as well as a general section in the front of the Teacher Edition (“Technology Notes” tab). These lessons utilize graphing calculators and offer alternative strategies for how to teach the lesson without them. The necessary programs are available at the CPM web site.

The “spaced practice” structure of the homework assures that students practice ideas and skills over time to consolidate their knowledge for long-term retention. Each homework set is designed to contain work with the topics just introduced, as well as problems to practice, extend, and apply previously learned skills and concepts. The “Research and Results” tabbed section has both a summary and the complete text of the paper that documents the research base for problem-based lessons, student study teams, and spaced practice.

CPM provides teachers who are using a CPM course for the first time with four days of professional development workshops during the summer and three days during the school year. CPM mentor teachers, who use the course in their own classrooms, conduct the sessions using carefully developed workshop plans after attending training at annual CPM Leadership Institutes. The workshops emphasize how to use the teaching strategies described in this document. These workshops strengthen teaching methodologies, review the mathematics of the course, and discuss suggestions for the use of the assessment resources and classroom management. It is expected that teachers, schools, and districts that use Algebra Connections for the first time will attend the workshop series.

Conclusion

The Algebra Connections course is the product of classroom teachers who created lessons that work with the diverse student population of California. The author team based the structure of the book on the collective experience of hundreds of teachers who have offered suggestions for more than a decade. Comments and suggestions by students and parents also helped shape this course. Care was taken to pilot and field test the lessons with thousands of students, mostly in the authors' and other CPM mentor teachers' classrooms, to assure their effectiveness. Likewise, the professional development support for the course is a synthesis of ideas from several hundred CPM mentor teachers based on their experiences with conducting workshops for more than 7,500 teachers since 1992.

Every assessment of CPM students has shown that they learn basic mathematical skills and procedures at least as well as students who use other programs. Most of the studies show that they do better. Studies that measure the other elements of a complete curriculum—conceptual understanding and problem solving ability—show that they do considerably better in these areas. These studies are available at www.cpm.org/teachers/info.htm, along with the research base of the CPM program.

Student Edition Highlights

General

- Note to Students
- Illustrated glossary
- List of symbols
- Index
- Online homework help (www.hotmath.com)
- Extra practice resource (www.cpm.com)

Each Chapter

- Overview, outline, and guiding questions
- Lesson introductions
- Homework with review
- Math notes (definitions, examples)
- Closure activities
- Self-check review with answers

Parent Guide & Extra Practice Highlights

Parent Guide

- Suggestions for working with a student
- Explanations of concepts
- Solved examples
- Practice problems with answers

Extra Practice

- Explanations of concepts
- Solved examples
- Practice problems with answers

Teacher Edition Highlights

Tabbed Resources

- Table of contents
- California standards correlations
- Research and results
- Teaching notes
 - ※ Technology
 - ※ Student study teams
 - ※ Closure
- Universal access
- Timeline, materials, index, & glossary
- Assessment handbook
- Assessment resources
- Answers

Elements of Each Chapter

Opening pages:

- Objectives
- Chapter pacing matrix
- Chapter overview
- Where are the ideas going?

Each lesson plan:

- Lesson objectives
- Length of activity
- Core problems
- Ways of thinking
- Materials needed
- Suggested lesson activity
- Closure with review
- Team strategies
- Homework

End of chapter:

- Lesson resource pages

