

## CPM Educational Program

A California, Non-Profit Corporation

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### CPM Courses and Their Core Threads

Each course is built around a few core ideas as recommended by the TIMSS Report, which in turn are developed and deepened over the four year program. Specific skills and concepts are tied to these threads and presented in context--both conceptually and in realistic problems--so that students will see the connections among the ideas and make sense of mathematics.

#### Math 1 (Algebra 1) (detailed below)

Problem Solving  
Graphing  
Writing Equations  
Solving Equations  
Ratios  
Symbol Manipulation

#### Math 2 (Geometry)

Problem Solving  
Graphing  
Ratios  
Geometric Properties  
Algebra  
Spatial Visualization  
Conjecture & Explanation (Proof)

#### Math 3 (Algebra 2)

Problem Solving  
Representation & Modeling  
Functions & Graphing  
Intersections and Systems  
Algorithms  
Reasoning and Communication

#### Math 4 (Mathematical Analysis)

Problem Solving  
Concepts of Calculus  
Analysis of Models  
Trigonometry  
Advanced Functions  
Algebraic Fluency & Accuracy

### CPM Snapshot: Developing the Factoring Thread in Algebra 1 (Math 1)

Students begin preparing for trinomial factoring with the third problem in the text (GS-3). The "diamond problems" introduce "sum and product" as a patterning problem in a puzzle format. These problems recur in homework throughout units 0-7. In Unit 1 the tiles are used to introduce combining like terms, followed by the Distributive property in Unit 2. In Unit 2 and subsequent units, the tiles are used to create area models for multiplying algebraic terms. Notice how units 4, 6 and 7 continue to practice the skills students will need to factor trinomials.

Toward the end of the first semester tiles and generic rectangles are used to introduce multiplying binomials. After a unit and a half of practicing this multiplication, the course turns to undoing the multiplication, that is, trinomial factoring with  $a = 1$  (factoring  $ax^2 + bx + c$  comes in Unit 10, after students have had at least a month to consolidate basic trinomial factoring). Once again tiles, area models, and patterning are used to help concretize and visualize factoring, and the process is tied back to the diamond problems. The basic factoring thread culminates with an investigation in the theme problem, AP-79, in Unit 8. Factoring is revisited in each of the subsequent CPM courses.

Answers appear in **bold** print, as they do in the teacher version of the text.

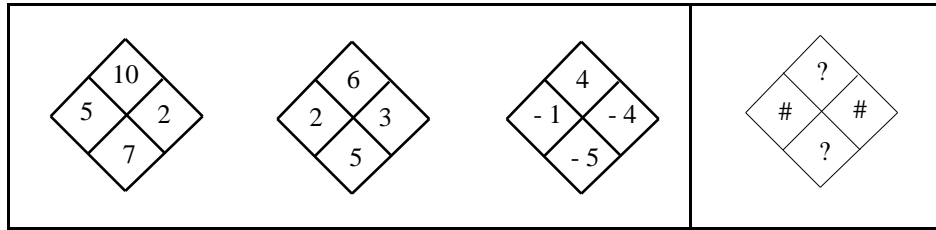
The references to student "tool kits" is CPM's way of teaching and requiring students to have study notes for each unit. The tool kit icon means "put this in your tool kit--NOW!"

## Unit 0 Getting Started

(Pre-problem in puzzle form for trinomial factoring)

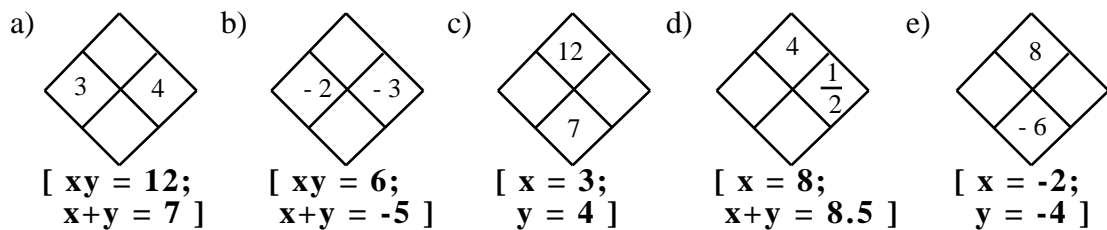
### GS-3. DIAMOND PROBLEMS

With your study team, see if you can discover a pattern in the three diamonds below. In the fourth diamond, if you know the numbers (#), can you find the unknowns (?) ? Explain how you would do this. Note that "#" is a standard symbol for the word "number".



Patterns are an important problem solving skill we use in algebra. The patterns in Diamond Problems will be used later in the course to solve algebraic problems.

Copy the Diamond Problems below and use the pattern you discovered to complete each of them.



## Unit 1 Organizing Data

(Introducing manipulatives for symbol manipulation.)

SQ-67.

Summarize the idea of Combining Like Terms in your tool kit. Then represent the following situations with an algebraic expression.



Combining tiles that have the same area to write a simpler expression is called **COMBINING LIKE TERMS**.

**Example:**

$x^2$

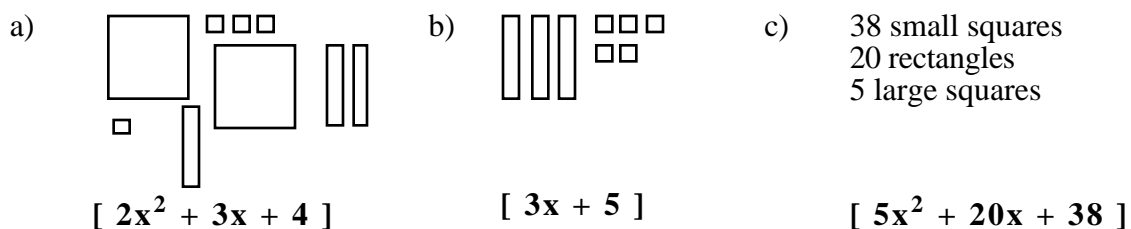
$x$

$x$

$x^2 + 2x + 2$

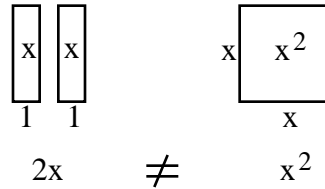
We write  $2x$  to show  $2(x)$  or  $2 \cdot x$ .

Represent each of the following situations with an algebraic expression.



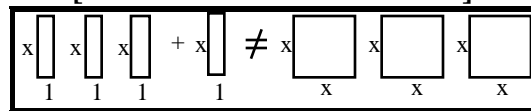
SQ-71.

**Example:** To show that  $2x$  does not usually equal  $x^2$ , you need two rectangles and one big square.

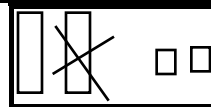


[ Solutions shown below ]

a) Show that  $3x + x \neq 3x^2$ .



b) Show that  $2x - x \neq 2$ .

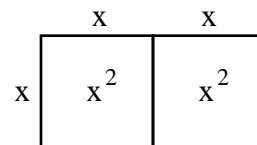


**Unit 2 Area and Subproblems** (Area and perimeter visual basis for multiplying.)

**MULTIPLYING WITH ALGEBRA TILES**

Example: The dimensions of this rectangle are

$x$  by  $2x$



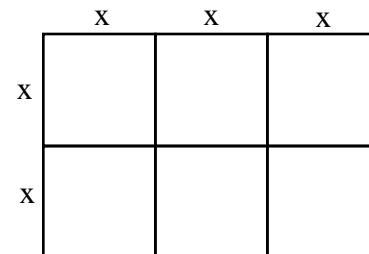
Since two large squares cover the area, the area is  $2x^2$ .

We can write the area as a multiplication problem using its dimensions:

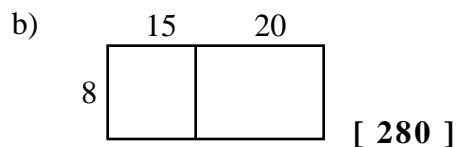
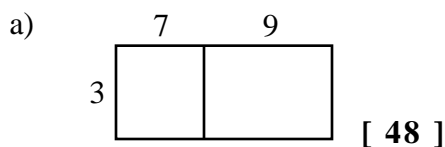
$$x(2x) = 2x^2$$

KF-53. Use the figure at right to answer these questions.

- a) What are the dimensions of the rectangle? [  $2x$  by  $3x$  ]
- b) What is the area of this rectangle? [  $6x^2$  ]
- c) Write the area as a multiplication problem.  
[  $(2x)(3x) = 6x^2$  ]



KF-74. Show two methods of finding the area of the rectangles below. Be sure to write down and label your subproblems.



KF-78. In your own words, add the following information to your tool kit.



### THE DISTRIBUTIVE PROPERTY

With two rectangles ( $2x$ ) and eight small squares ( $8$ ), we can represent the product  $2(x + 4)$  by grouping the tiles together. Below right, a **GENERIC RECTANGLE** represents the same product as if its parts were the dimensions (length and width) of a rectangle. Study these representations and find their similarities and differences. Then record both examples in your tool kit.

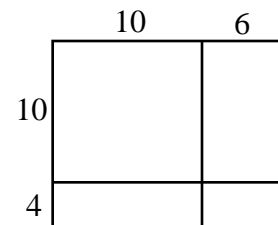


$$2(x + 4) = 2 \cdot x + 2 \cdot 4 = 2x + 8$$

In general, when you use tiles and rectangles to multiply two or more terms by the same number, variable, or mixed expression, you can see the multiplying pattern we have been using. The pattern is called the **DISTRIBUTIVE PROPERTY** and can be written as a rule:

$$a(b + c) = ab + ac.$$

KF-87. Find the total area of the rectangle at right in as many different ways as you can. Explain your methods. Are there more than two ways to solve this problem?  
 [  $100 + 60 + 40 + 24 = 224$ ;  $(10 + 6)(10 + 4) = 224$  ]



## Unit 4 Writing & Solving Equations

*(Practice continues.)*

CP-52. Use Distributive Property to rewrite each expression. Multiply in parts (a) and (b). Factor in parts (c) and (d).

- |                 |                  |               |                |
|-----------------|------------------|---------------|----------------|
| a) $2(3x - 2)$  | [ $6x - 4$ ]     | c) $5x + 15$  | [ $5(x + 3)$ ] |
| b) $2x(5x + 3)$ | [ $10x^2 + 6x$ ] | d) $x^2 + 4x$ | [ $x(x + 4)$ ] |

# Unit 6 Graphing and Systems of Linear Equations

(Extend model to binomials.)

WR-66. Write the area as a sum and as a product.

a)

[  $(13)(16) = 48 + 30 + 80 + 50 = 208$  ]

c)

[  $3x(x + 5) = 3x^2 + 15x$  ]

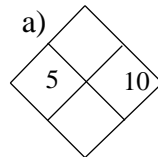
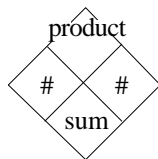
b)

[  $2(x + 4) = 2x + 8$  ]

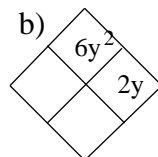
d) Find the perimeters of the three rectangles above.

[  $58, 2x + 12, 8x + 10$  ]

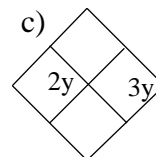
WR-67. Copy and solve these Diamond Problems:



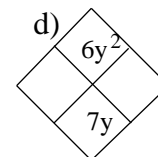
[  $xy = 50;$   
 $x + y = 15$  ]



[  $x = 3y;$   
 $sum = 5y$  ]



[  $prod = 6y^2;$   
 $sum = 5y$  ]



[  $y$  and  $6y$  ]

WR-70.

In this semester we have developed several big ideas. One of these ideas has been the Distributive Property and multiplication of variables. In the next two days we will expand our understanding of multiplication to include pairs of sums.

Use your Algebra Tiles to construct each of the following rectangles with the specified dimensions. Draw the complete picture of each rectangle on your paper, then write its area as a product and as a sum. Label everything as shown in the example below.

Example:

[  $(x + 1)(x + 4) = x^2 + 5x + 4$  ]

**product**                      **sum**

a)

[  $(x + 5)(x + 3) = x^2 + 8x + 15$  ]

b)

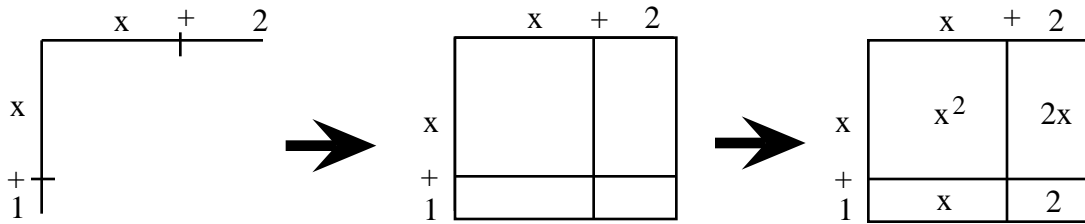
[  $(x + 2)(x + 4) = x^2 + 6x + 8$  ]

c)

[  $x(x + 6) = x^2 + 6x$  ]

WR-71.

We can make our work drawing tiled rectangles easier by not filling in the whole picture. That is, we can show a **generic rectangle** by using an outline instead of drawing in all the dividing lines for the rectangular tiles and unit squares. For example, we can represent the rectangle whose dimensions are  $x + 1$  by  $x + 2$  with the generic rectangle shown below:



**area as a product**

$$(x + 1)(x + 2) = x^2 + 2x + 1x + 2 = x^2 + 3x + 2$$

**area as a sum**

In the last step we found the area of each of the parts of the generic rectangle (the four interior rectangles) by multiplying each length and width, and then recorded the area of each part as a sum.

*(The complete problem continues with exercises like those in WR-70.)*

WR-72.

Carefully read this information about binomials. Then add a description of binomials and the example of multiplying binomials to your tool kit.



These are examples of **BINOMIALS**:

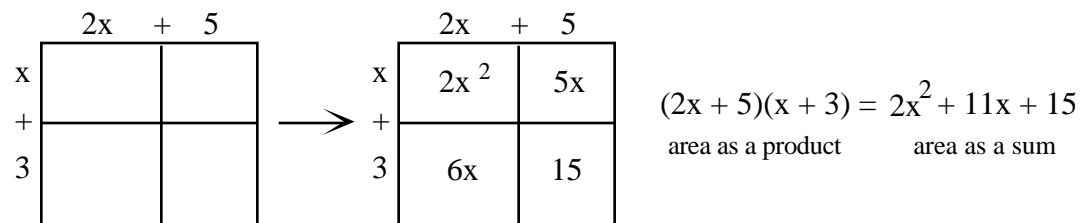
$$x + 2 \quad 7 - 5x \quad 2x - 7 \quad (3x^2 - 17)$$

These are **NOT** binomials:

$$2x \quad 3x^2 \quad -5xy - 2x + 9$$

Write a description of a binomial that would be clear to a new student.

We can use generic rectangles to find various products. We call this process **MULTIPLYING BINOMIALS**. For example, multiply  $(2x + 5)(x + 3)$ :



Note that a generic rectangle helps us organize the problem. It does not have to be drawn accurately or to scale. In the example above, you can see that the region representing  $5x$  is probably not as big as it should be compared to the region for  $6x$ .

WR-82. Algebra tiles can be used to build a series of squares. After you build each square, write the area as a product, sketch the tiles, and write the areas as a sum. Organize your work in a table as shown below.

- a) Complete this chart for the first 6 squares. The first square is  $x + 1$  by  $x + 1$ , the second square is  $x + 2$  by  $x + 2$ , and so on.

Area as a Product	Sketch	Area as a Sum
$(x + 1)^2$		$x^2 + 2x + 1$
		$[ x^2 + 4x + 4 ]$ $[ x^2 + 6x + 9 ]$ $[ x^2 + 8x + 16 ]$ <b>etc.</b>

- b) In the rectangles you drew, explain where you see the squares. [ **opposite corners.** ]
- c) Describe all the patterns you found in the table. [ **answers vary; look at perfect square constants, patterns with the middle coefficient** ]
- d) Using your patterns, and without drawing a sketch, fill in the chart's next two products and areas.
- e) Without drawing a sketch, what would the 10th product and area be?  
 $[ (x + 10)^2 = x^2 + 20x + 100 ]$
- f) Add two more perfect square products and areas to your table without drawing a sketch.

### Unit 7 Slopes and Rates of Change *(Practice and consolidation before factoring.)*

BR-30. Copy these generic rectangles on your paper. Multiply or factor to rewrite these expressions.

a)  $[ 78x - 126 ]$

b)  $[ x^2 - 2x - 15 ]$

BR-67. Write a shorter expression for each of the following polynomials by simplifying.

a)  $(4x^2 + 12x - 5) - (3x^2 + 6)$   
 $[ x^2 + 12x - 11 ]$

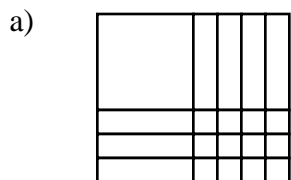
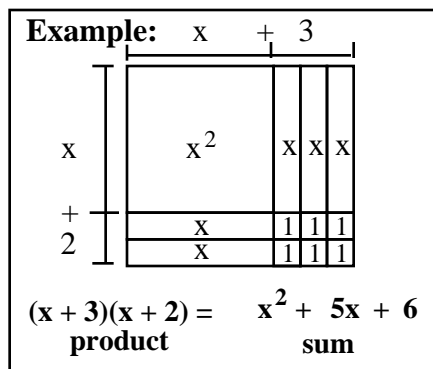
b)  $3(x + 1) + 2(x + 1) + (x + 1)$   
 $[ 6x + 6, \text{ or } 6(x + 1) ]$

BR-68. Multiply. You may find the sketch of a generic rectangle useful.

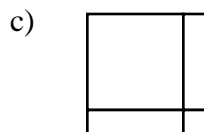
a)  $(x + 2)(x + 8)$   $[ x^2 + 10x + 16 ]$     b)  $(x - 2)(x + 8)$   $[ x^2 + 6x - 16 ]$

## Unit 8 Factoring Quadratics

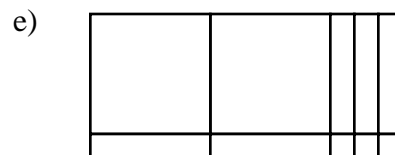
AP-3. Write an algebraic equation for the area of each of the following rectangles as shown in the example below.



$$[ (x + 3)(x + 4) = x^2 + 7x + 12 ]$$

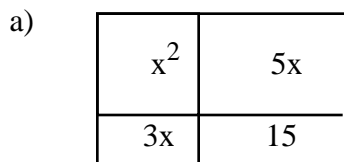


$$[ (x + 1)(x + 1) = x^2 + 2x + 1 ]$$

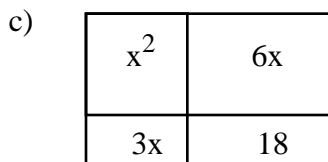


$$[ (x + 1)(2x + 3) = 2x^2 + 5x + 3 ]$$

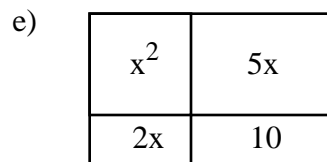
AP-11. Find the dimensions of each of the following generic rectangles. The parts are not necessarily drawn to scale. Use Guess and Check to write the area of each as both a sum and a product.



$$(x + 5)(x + 3)$$



$$(x + 6)(x + 3)$$

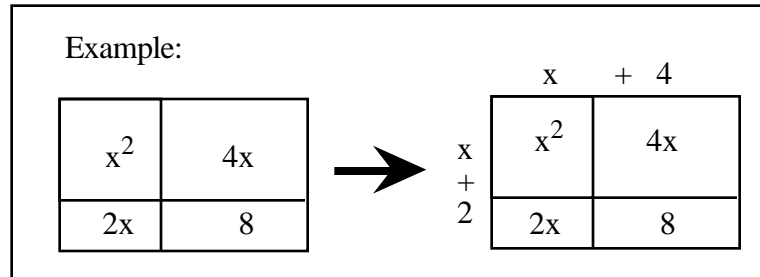


$$(x + 5)(x + 2)$$

**AP-18. USING ALGEBRA TILES TO FACTOR**

What if we knew the area of a rectangle and we wanted to find the dimensions? We would have to work backwards. Start with the area represented by  $x^2 + 6x + 8$ . Normally, we would not be sure whether the expression represents the area of a rectangle. One way to find out is to use Algebra Tiles to try to form a rectangle.

You may find it easier to record the rectangle without drawing all the tiles. You may draw a generic rectangle instead. Write the dimensions along the edges and the area in each of the smaller parts as shown below.



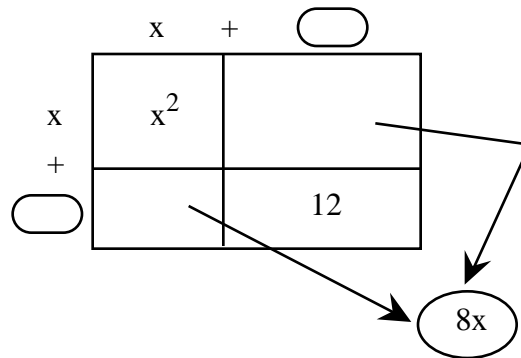
We can see that the rectangle with area  $x^2 + 6x + 8$  has dimensions  $(x + 2)$  and  $(x + 4)$ .

Use Algebra Tiles to build rectangles with each of the following areas. Draw the complete picture or a generic rectangle and write the dimensions algebraically as in the example above. Be sure you have written both the product and the sum.

- a)  $x^2 + 6x + 8$  [  $(x + 4)(x + 2)$  ]      d)  $x^2 + 7x + 12$  [  $(x + 3)(x + 4)$  ]  
 b)  $x^2 + 5x + 4$  [  $(x + 1)(x + 4)$  ]      e)  $2x^2 + 8x$  [  $x(2x + 8)$  or

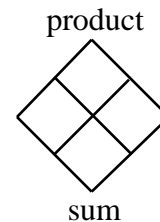
**AP-19. USING DIAMOND PROBLEMS TO FACTOR**

Using Guess and Check is not the only way to find the dimensions of a rectangle when we know its area. Patterns will help us find another method. Start with  $x^2 + 8x + 12$ . Draw a generic rectangle and fill in the parts we know as shown at right.



We know the sum of the areas of the two unlabeled parts must be  $8x$ , but we do not know how to split the  $8x$  between the two parts. The  $8x$  could be split into **sums** of  $7x + 1x$ , or  $6x + 2x$ , or  $3x + 5x$ , or  $4x + 4x$ . However, we also know that the numbers that go in the two ovals must have a **product** of 12.

- a) Use the information above to write and solve a Diamond Problem to help us decide how the  $8x$  should be split.  
 [ **product of 12, sum of 8; 2, 6** ]  
 b) Complete the generic rectangle and label the dimensions.  
 [  $(x + 2)(x + 6)$  ]



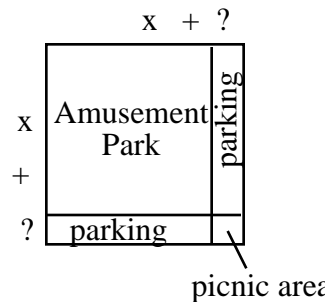
AP-79. AMUSEMENT PARK PROBLEM

The city planning commission is reviewing the master plan of the proposed Amusement Park coming to our city. Your job is to help the Amusement Park planners design the land space.



Based on their projected daily attendance, the planning commission requires 15 rows of parking. The rectangular rows will be of the same length as the Amusement Park. Depending on funding, the Park size may change so planners are assuming the park will be square and have a length of  $x$ . The parking will be adjacent to two sides of the park as shown below.

Our city requires all development plans to include “green space” or planted area for sitting and picnicking. See the plan below.



- a) Your task is to list all the possible configurations of land use with the 15 rows of parking. Find the areas of the picnic space for each configuration. Use the techniques you have learned in this unit. There is more than one way to approach this problem, so show all your work.  
 [ **Area = 14, 26, 36, 44, 50, 54, 56** ]
- b) Record the configuration with the minimum and maximum picnic area. Write an equation for each that includes the dimensions and the total area for the project. Verify your solutions before moving to part (c).  
 [  **$(x + 14)(x + 1) = x^2 + 15x + 14$ ;  $(x + 7)(x + 8) = x^2 + 15x + 56$  ]**
- c) The Park is expected to be a success and the planners decide to expand the parking lot by adding 11 more rows. Assume that the new plan will add 11 additional rows of parking in such a way that the maximum original green space from part (b) will triple. Show all your work. Record your final solution as an equation describing the area of the total = product of the new dimensions.  
 [  **$x^2 + 26x + 168 = (x + 12)(x + 14)$  ]**
- d) If the total area for the expanded Park, parking and picnic area is 2208 square units, find  $x$ . Use the dimensions from part (c) to write an equation and solve for the side of the Park. [  **$2208 = (x + 12)(x + 14)$ ; using Guess & Check  $x = 34$  ]**

The factoring thread is completed in units 9-12, where further work is done with multiplying binomials, factoring trinomials with a  $-1$ , the Quadratic Formula, completing the square, and generating the equation of a parabola from its roots.