CPM Educational Program

A California, Non-Profit Corporation

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CPM Courses and Their Core Threads

Each course is built around a few core ideas as recommended by the TIMSS Report, which in turn are developed and deepened over the four year program. Specific skills and concepts are tied to these threads and presented in context--both conceptually and in realistic problems--so that students will see the connections among the ideas and make sense of mathematics.

Math 1 (Algebra 1)
Problem Solving
Graphing
Writing Equations
Solving Equations
Ratios
Symbol Manipulation

Math 2 (Geometry)
Problem Solving
Graphing
Ratios
Geometric Properties
Algebra
Spatial Visualization
Conjecture & Explanation (Proof)

Math 3 (Algebra 2)
Problem Solving
Representation & Modeling
Functions & Graphing
Intersections and Systems
Algorithms
Reasoning and Communication

Math 4 (Mathematical Analysis)
Problem Solving
Concepts of Calculus
Analysis of Models
Trigonometry
Advanced Functions
Algebraic Fluency & Accuracy

CPM Snapshot: Developing the Graphing Thread in Algebra 1 (Math 1)

CPM uses many methods to develop graphing, a major thread through all four courses in the CPM college prep sequence. Students begin by interpreting graphs in the first three units. Graphing of linear and quadratic functions begins in Unit 3. We use tables of values to reinforce the relationship between the points that lie on the line, the equation of the line, and the graph itself. Graphing with tables also allows us to introduce other functions early in the course. We also use contexts that ask students to consider what the relationship between the domain and range mean, especially how to use a graph of two or more relationships to answer comparative questions. Each unit has a theme problem that uses main ideas from the unit. "The Burning Candle" (Unit 3), "Choosing a Phone Plan" (Unit 4), and "The Big Race" (Unit 7) are included in this packet. They illustrate the more extensive kinds of investigations that are designed to last a full period.

Between the introduction of graphing functions in Unit 3 and the formal introduction of slope and slope-intercept form in Unit 7, we reinforce what students have learned, extend the ideas, and use them in connection with other algebraic ideas. See page 7, problem EF-58 (from Unit 5) for an example of using graphing with ratios to foreshadow slope. Unit 7 has various investigations to help students see the relationship between the graph of a linear equation, its slope and y-intercept, and the form of the equation itself. As these ideas are developed, students are learning how to solve systems of equations algebraically as well as graphically. You can extrapolate from some of the skills necessary to do the problems in this packet what else has been taught from the other course threads. The last three units concentrate on quadratic functions. Unit 12 includes lines and parabolas of best fit.

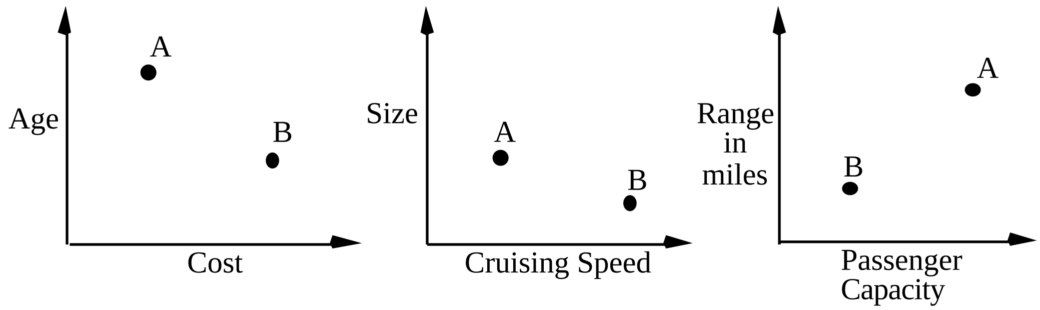
Bold type indicates teacher notes for the problem or lesson and answers, which appear in the Teacher Edition only. References to student "tool kits" is CPM's way of teaching and requiring students to have study notes for each unit. The tool kit icon means "put this in your tool kit--NOW!"

Unit 1: Organizing Data

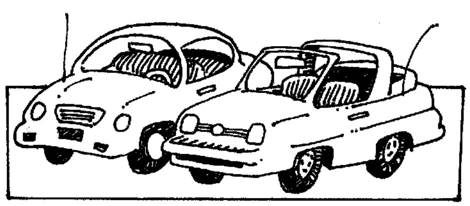
In problem SQ-21, the range of a car depends on its fuel capacity and its fuel efficiency. Expect some discussion about students' interpretations.

SQ-21. CAR COMPARISON

The following three graphs describe two cars, A and B.



Decide whether each of the following statements is true or false. Explain your reasoning.



a) The newer car is more expensive. **[True]**

b) The slower car is larger. **[True]**

c) The larger car is newer. **[False]**

d) The less expensive car carries more passengers. **[T]**

Unit 3: Patterns and Graphs

(Unit 3 begins with an "Algebra Walk," an exercise in "human graphing" where students represent points on an xy-coordinate graph. The activity is explained in detail in the teacher notes, including the necessary resources and how to prepare them in advance of class. Briefly, the teacher chalks a large set of axes somewhere outside on campus before class. In class, students are given a color-coded integer between -4 and 4 (their x-coordinate) and a resource page with the entire activity outlined. When the class goes outside, the teacher calls each member of a color-coded set of human x-coordinates to stand on their respective places on the x-axis. The teacher then tells them to take their number, follow a rule (e.g., "double your number and subtract one"), then walk forward or backward to their new number. In most cases students take care of necessary arithmetic corrections and remind peers about how one "moves about" on a coordinate plane. As each group completes its graph, everyone records the result on their resource page. The activity is reinforced upon return to the classroom by each color-coded set of students placing a matching color sticky dot on a pair of axes that the teacher has pre-drawn on poster board. The teacher then leads a debriefing discussion that pulls together how the xy-coordinate system works as well as an introduction to "input" and "output" values for simple functions.)

BC-8. Complete the table below using $y = 2x - 1$. Plot and connect the points on a graph, then label the graph. Be sure to label the axes and to include the scale.

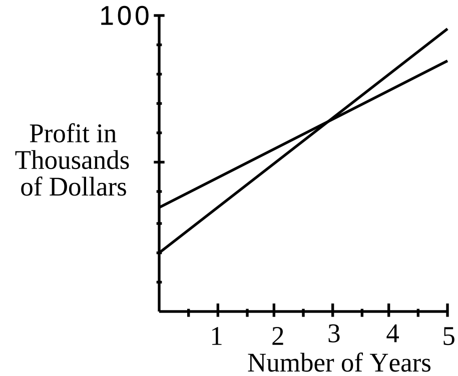
IN (x)	-6	-5	-4	-3	-2	$-^3/_2$	-1	0	0.5	1	2	3	4	5	6
OUT (y)															

BC-9. a) Use your graph of the equation $y = 2x - 1$, to approximate the value of x when y = 8. **[4.5]**

b) Approximate the value of x when y = 0. **[$^1/_2$]**

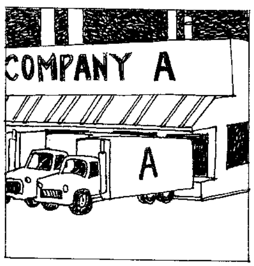
c) Explain how you can use the graph to find this value.
[Look for intersection of vertical and horizontal lines from axes]

BC-4. The two lines at right represent the growing profits of companies A and B in thousands of dollars.



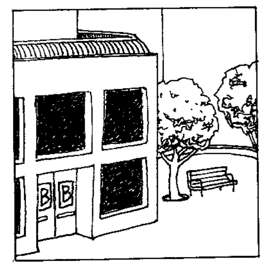
a) Sketch this graph on your paper. If company A started out with more profit than company B, determine which line represents A and which represents B. Label the lines appropriately. **[The steeper line is B]**

b) In how many years will both companies have the same profit? **[≈ 3 years]**



c) What will that profit be? **[≈ $65,000]**

d) Which company has faster growing profits? How can you tell? **[Company B]**



SILENT BOARD GAMES:
We use silent board games to introduce the idea of making a table, finding a pattern and writing an equation to solve a problem.

For example: [3x + 1]

$\frac{1}{2}$	1	5	2	7	-3	9	0	-6	x
	4	16			-8				

Basic Rules:
1) **Students should remain silent during this game. They should raise their hands to be recognized, write an answer on the board, then sit down.**
2) **Leave correct answers on the board or overhead and erase an answer if it is incorrect.**
3) **After each game, students should come up with a rule. For an input of "x" we expect a response of "3x + 1" or "multiply by three and add one" for the game shown above.**

Notice that the entries in the top row of the game board are not in ascending order. We want students to find patterns vertically from the input values to the output values in the bottom row. If the input values are in numerical order, it is too easy to find patterns horizontally, and thus miss the purpose of the game, which is to use patterns to discover and write a function rule.

BC-37. Copy and complete this table. Describe in words the rule you used.
[Double the number and add one.]

x	2	0	-3	$\frac{1}{2}$	-1	0.3	**3**	$-\frac{1}{3}$	**4.5**	-11	-2	x
y	5	1	-5	2	**-1**	**1.6**	7	**$^{1}/_{3}$**	10	**-21**	**-3**	**2x+1**

a) State the rule using a variable, such as x. **[y = 2x + 1]**

b) Graph the **ordered pairs** (x, y). Find two more pairs of points that satisfy the rule without using any more whole numbers.

c) Estimate where the graph crosses the x-axis. This point is called the **x-intercept**. Notice that at this point, the value of y is always 0. **[($-^{1}/_{2}$, 0)]**

d) Find a value for x that makes the equation 2x + 1 = 0 true. If you are unsure how to solve the equation algebraically, use the Guess and Check strategy.

e) How do your answers to parts (c) and (d) compare?
[They should be about the same.]

f) Reading the line from left to right, would you describe this line as going uphill or downhill? Is it very steep? **[Uphill, fairly steep]**

g) What do all the points on the graph of the equation have in common?
[solutions to y = 2x + 1]

h) Why is the point (4, 5) not on the graph? **[not a solution]**

i) What does the graph represent? **[The line represents the input (x) and output (y) sets that will make the equation true.]**

BC-66. On a large set of axes, graph the equation assigned to your team by your teacher. Pay attention to the given domain and scaling information. Be prepared to share your graph with the class.

	Equation	Domain and Axes Scaling
a)	$y = 2^x$	x = -2.0, -1.5, -1, . . . , 2.5, 3.0 Scale the y-axis so that two marks on the graph paper represent one unit.
b)	$y = \sqrt{x}$	x = 0, 0.5, 1, 2, . . . , 8, 9, 9.5, 10 Scale the axes so that two marks on the graph paper represent one unit.
c)	$y = 0.5^x$	x = -4, -3, . . . , 3, 4 Scale the axes so that two marks on the graph paper represent one unit.
d)	$y = \frac{1}{x}$	x = -5, -3, -2, -1.5, -1, -0.5, -0.33, -0.2, 0, 0.2, 0.33, 0.5, 1, 1.5, 2, 3, and 5. Scale the axes so that two marks on the graph paper represent one unit.
e)	$y = x^3$	x = -3, -2, -1.5, -1, -0.5, -0.33, -0.2, 0, 0.2, 0.33, 0.5, 1, 1.5, 2, and 3 Scale the axes so that one mark on the graph paper represent one unit.

BC-72. THE BURNING CANDLE PROBLEM -- An Investigation

It is your friend's birthday. You want to surprise her by walking into the room carrying a piece of cake with a lighted candle. Can you predict when the candle will go out?

To answer this question, we will use a video presentation of a burning candle to collect data, make a graph, and look for a pattern.

a) Gather data from the "Burning Candle" video presentation and place it into a table. Note the mass of the candle at various times during the presentation and write down your observations. You should make at least five observations. Be sure to write down both the time and the associated candle mass.

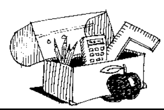
b) Make a new column in your table to show elapsed time, that is, the total time since the candle was lit.

c) On axes (using equal intervals) with elapsed time on the horizontal axis, graph your data, comparing mass, y, to elapsed time, x. Sketch a line or curve by connecting your data points. Compare your graph with those of your team.

d) Use your graph to predict the mass of the candle at the elapsed time of 1 minute and 20 seconds. Predict the mass of the candle at the elapsed time of 2 minutes and 47 seconds.

e) If the candle continued to burn, when do you think it would go out? Discuss this question with your team and carefully explain your answer in complete sentences.

Copy a model of a 'complete graph' into your tool kit or notes. Use the box below as a reference.



TABLES AND GRAPHS

A complete graph has the following components:

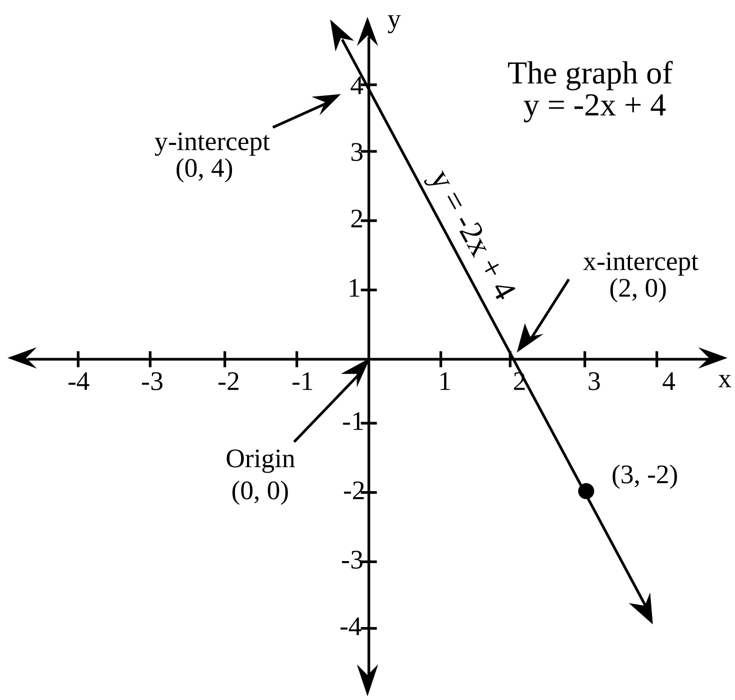
- x-axis and y-axis labeled, clearly showing your scale.
- Equation of the graph near the line or curve.
- Line or curve extended as far as possible on graph.
- x- and y-intercepts labeled.
- **Coordinates** of points are stated in (x, y) format.

x	-1	0	1	2	3
y	6	4	2	0	-2

Tables can be formatted horizontally, like the one above, or vertically, as shown below:

x	y
-1	6
0	4
1	2
2	0
3	-2
4	-4

Throughout this course, we will continue to graph lines and other curves. Be sure to label your graphs appropriately.



Unit 4: Writing and Solving Equations *(Practice with Graphing continues in homework.)*

CP-12. Make a table with eight values for x between -3 and 3 and find the corresponding y-values. Then, graph the following equations on the same set of axes.

1) $y = x$ 2) $y = x + 2$

a) Where do the graphs of the two equations meet? **[They do not meet.]**

b) List any similarities and differences you see in the graphs and equations.

CP-69. Graph each of the following equations on the same set of axes. Remember to label the axes and each graph.

a) $y = x$ b) $y = x - 3$ c) $y = 3x$

d) Compare the graphs in parts (a) and (b): How did the "3" in part (b) change the graph from part (a)? **[Lowered the line three units; same slant (slope)]**

e) Compare the graphs in parts (a) and (c): How did the "3" in part (c) change the graph from part (a)? **[Line became steeper]**

CP-92. CHOOSING A PHONE PLAN *(Anticipates solving systems algebraically.)*

You have decided to try and convince your parents to let you get your own telephone line. After finding out that the monthly rate is $13.50, which you can afford, you offer to pay the cost yourself. However, just before you place your order, you see an advertisement that the local telephone companies are discontinuing the flat rate of $13.50 per month and will only offer metered service. Their ads are shown below:

Your parents think that you need to do a little research before making a decision. What would you do to help make your decision?

a) About how many minutes of phone calls do you make per month?
(Refer to CP-91) What is the largest number of minutes from your team?

b) For the flat rate and for each company make a table comparing total minutes to total cost. Choose at least four reasonable amounts to represent the number of minutes per month. It will be helpful to include 0 minutes and the largest number of minutes from your team. You will have three tables when you are done.

c) We are now going to plot our data. Carefully plan how to scale your axes. Label the vertical axis "cost per month" and the horizontal axis "minutes of calls" per month. Plot and connect the points for each table in part (b). Be sure to label the graph of each company plan.

d) When do both companies charge the same amount? When is AT^2 a better choice? When is Pa Bell a better choice? Discuss how your graphs relate to the solution of the problem. **[AT^2 costs less when $x < 550$; Pa Bell costs less when $x > 550$ minutes.]**

e) Consider the options using the number of minutes you call per month. Which plan would you choose? Will you save money over the flat rate?**[Answers vary; both are better than the flat rate unless you talk more than 8.75 hours.]**

f) For each option, use the pattern in the table to write an equation which represents the cost in a given month in terms of the minutes of phone calls.
[Flat = 13.50, $AT^2 = 3 + 0.02x$, and Pa Bell = $8.50 + 0.01x$.]

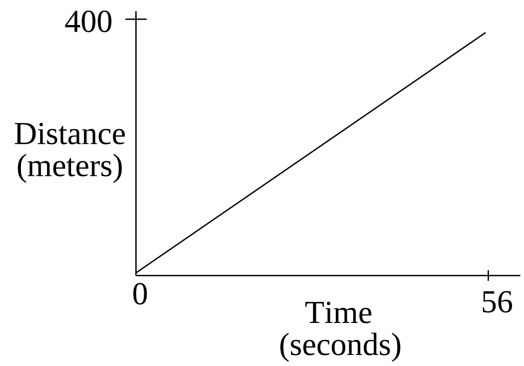
g) For each person in your study team, write down their name and how many minutes they estimate they spend on the phone. Which phone option should each person in your study team use and why? **[Answers vary.]**

EF-27. a) Graph the curve $y = x^2 - 3$ for $-3 \le x \le 3$.

b) Estimate the x- and y-intercepts. **[$(\pm\sqrt{3}, 0) \approx (\pm 1.73, 0)$; $(0, -3)$]**

(EF-58 ties current Unit 5 study back to graphing of Unit 3 and foreshadows slope in Unit 7.)

EF-58. The 4 x 400 m relay is a track event in which four runners each run 400 meters. Each 400 m portion is called a "leg." In a recent race, Maisha ran the second leg of a 4 x 400 m relay in 56 seconds. Her speed is graphed at right. Copy this graph onto your paper. Label your axes completely.



a) What is the ratio of the distance to time at the end of Maisha's leg?
[400 meters : 56 seconds]

b) Assume that Maisha ran at a constant speed. How long did it take Maisha to go 200 m?
[28 seconds]

c) What is the ratio of distance to time for the first 200m of the leg? **[200 meters : 28 seconds]**

d) What is the ratio of distance to time for the first 100m of the leg? **[100 meters : 14 seconds]**

e) Review the ratios you found. Write each ratio in simplest form and compare the results. **[The ratio of distance to time is the same for the entire leg. 50 meters : 7 seconds]**

f) Draw similar right triangles on the graph at 56 seconds and 28 seconds.

g) What can you conclude about the ratio of the variables (in this case distance on the y-axis and time on the x-axis) on the line of a graph?
[The ratio of the points on the graph stayed the same.]

EF-66. We wish to graph the line y = 2. Since this equation does not force x to equal any specific or unique number, then y will always equal 2 no matter what x is.

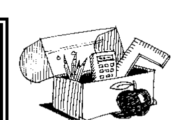
Below is a table for the equation y = 2. Use it to graph your line.

x	-3	-2	-1	0	1	2	3
y	2	2	2	2	2	2	2

a) Add the lines y = -1 and y = 6 to your graph.

b) Add the line x = 3 to your graph.

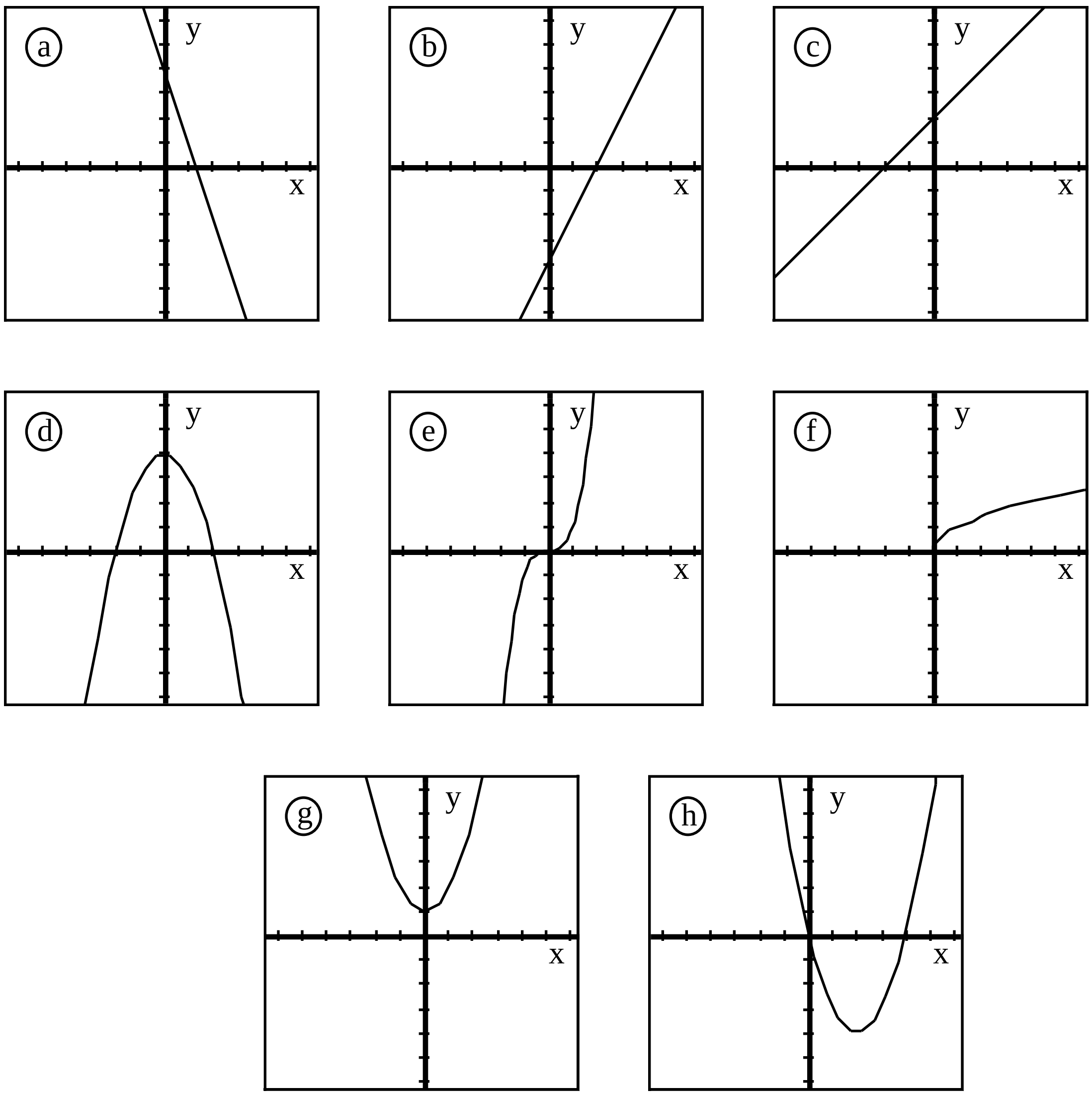
c) In your tool kit, make a sketch for y = -3 and x = 5. Be sure to label your lines and write a brief general observation about the equations of **horizontal** and **vertical** lines.



Unit 6: Graphing and Systems of Linear Equations

WR-1. MATCH-A-GRAPH, Part One

Obtain the resource page from your teacher. **SAVE THIS PAGE; YOU WILL NEED IT LATER IN THIS UNIT**. Use what you know about graphing to match each of the following graphs with its equation below. As a team, decide how you are all going to work together to solve this problem.

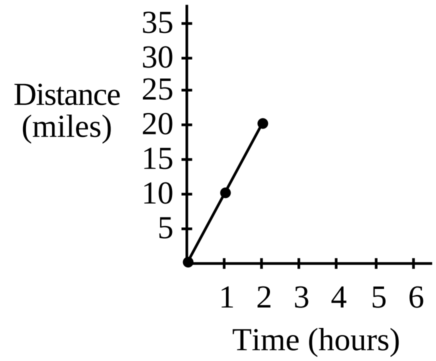


1) $y = 2x - 4$ **[b]** 3) $y = x + 2$ **[c]** 5) $y = 4 - 3x$ **[a]** 7) $y = x^2 + 1$ **[g]**

2) $y = \sqrt{x}$ **[f]** 4) $y = x^2 - 4x$ **[h]** 6) $y = x^3$ **[e]** 8) $y = 4 - x^2$ **[d]**

WR-2. Carefully describe what methods your team used to solve Match-A-Graph.

WR-21. Jose, Racquel, and Lena are planning a bicycle riding vacation. They will ride approximately 75 miles everyday. Jose averages 10 mph on his bike while Racquel averages 15 mph. Lena averages 5 mph because she has a cast on her foot which slows her down. Lena plans to bike for only 3 hours and then will catch a ride in a van which travels at 30 mph.



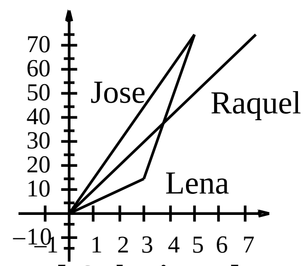
a) Copy this graph onto graph paper. Using a ruler, extend the distance axis to 75 miles and time to 10 hours. The line on the graph shows the first two hours of Jose's trip for one day.

b) Complete Jose's graph. On the same axes, graph Racquel's and Lena's trips. Use different markings (or colors) for each person's graph so that you can analyze this data. Be sure that Lena's graph reflects both the bike and van portions of her trip.

c) How long did each person's trip take? **[Jose 7.5 , Racquel 5 , and Lena 5 hrs.]**

d) Jose's and Racquel's graphs are straight lines. Which graph is steeper? Explain. **[Racquel's is steeper because she is going faster.]**

e) Compare Lena's graph to the other graphs. Is it steeper? Explain. **[Lena's is the least steep for the first 3 hours, but then becomes the steepest after that. At first she is traveling the slowest, but in the van she is going the fastest.]**



[Solution:]

f) If someone rode in the van the entire time, describe what that graph would look like and how long the trip would take. **[It would be a steep, straight line above Racquel's; 2.5 hours]**

Some teachers find it helpful to instruct students to pick one positive and one negative x-value for WR-29. This enables a better estimate of the y-intercept.

WR-29. So far this year you have graphed equations by selecting many different values for x, then calculating the corresponding y-values. Each of the equations has been written in **y-form**; that is, y is alone on the left-hand side of the equation while x and numbers are on the right. Problems WR-29 through WR-31 will help you develop a faster method for graphing linear equations.

Here are two equations in **y-form**:

(1) $y = 2x + 3$ (2) $y = 2x - 1$

a) Carefully graph each equation on the same set of axes by using any <u>two</u> inputs (x-values). Check with your team so that each person uses two different points to graph each line. Compare your graphs. Are your graphs the same? (Remember that lines continue infinitely in each direction.) Do you need more than two points to graph a line?

b) Recall that the x-coordinate of any point on the y-axis is zero. Thus any point on the y-axis can be written in the form (0, y). Examine each of your graphs to find the **y-intercept** (the point where the line crosses the y-axis). **[(0, 3); (0, -1)]**

<<Problem continues on next page.>>

c) Based on your observations, write a conjecture about how to find a line's y-intercept from the equation without graphing the line. If necessary, refer to your tool kit entry from Unit Five which defines the term "conjecture".

d) In the equation y = 2x + 3, "3" is referred to as a **CONSTANT**. Compare the y-intercept for each line in part (a) with the constant in the corresponding equation. Compare what you now observe to your conjecture in part (c).
[The y-coordinate of the y-intercept is the constant of the equation in y-form.]

WR-30. Here are some linear equations in y-form. Use your observation in the previous problem to write the coordinates for the y-intercept of the graph of each equation.

a) $y = 2x + 3$ **[(0, 3)]**

b) $y = -x - 1$ **[(0, -1)]**

c) $y = 2x - 4$ **[(0, -4)]**

d) $y = -3x + 2$ **[(0, 2)]**

e) $y = \frac{1}{2}x$ **[(0, 0)]**

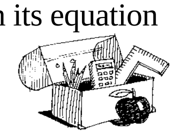
WR-31. "WHAT TWO POINTS SHOULD WE USE?"

a) What is the y-intercept of $y = 2x - 5$? **[(0, -5)]**

b) Now choose **any** x-value **other than zero** and find its corresponding y-value.

c) Plot the two points you found in parts (a) and (b) on a pair of coordinate axes and carefully use a ruler to draw a line through them. Label the line with its equation

d) Pick another x-value, find its corresponding y-value, and check the graph to verify that the point is on your line.



TWO-POINT GRAPHING METHOD

You can graph linear equations by using the quick, efficient **two-point graphing method**.

1. Find the y-intercept and mark it on an xy-coordinate graph.
2. Choose any value for x, other than zero, and substitute this value into the equation to find y. Plot this second point on the graph
3. Use a ruler to carefully draw the line through the two points. Remember to extend the line beyond each point.
4. Pick a third x-value, find its corresponding y-value. Check the graph to verify that this third point is on your line.

WR-32. Choose three different equations from problem WR-30 and graph each one using the two-point method. Remember to use a third x-value to verify your graph.

WR-59. Latanya and George both want to buy new bicycles. The bicycles cost $300. Latanya opened a savings account with $50. She just got a job and is determined to save $30 a week. George started a savings account with $75. He is able to save $25 a week.

a) Write two equations that represent their savings plans. What do your variables represent? **[x = number of weeks, y = total saved; y = 30x + 50; y = 25x + 75]**

b) Solve these equations using substitution. Explain what your solution means. **[(5, 200) This is the point where George and Latanya have the same amount in their savings.]**

c) Graph each situation. Discuss with your team the best way to scale your axes. Label the lines.

d) What does the point of intersection on your graph represent? **[In 5 weeks they each have $200 in their savings]**

e) How long will it take Latanya to save enough money to buy the bike? **[$8^1/_3$ weeks: 9 weeks if she gets paid at the end of the week]**

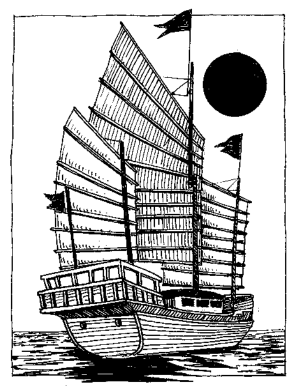
f) How long will it take George to save enough money to buy the bike? **[9 weeks]**

g) Do you think this is a realistic problem? Is there anything that could possibly interfere with George's and Latanya's plans to buy bikes? Explain.

Unit 7: Slopes and Rates of Change

The next problem is intended as a pre-problem to slope and y-intercept. After completion of this problem, students will be able to recognize the difference between positive and negative slopes. They will also begin to recognize a relationship between steepness and rate of change. The y-intercept will represent the initial balance. This problem offers a context to the relationship between an equation and its graph.

BR-12. MONEY MATTERS, Part One



Six cousins were each given $20.00 in a new bank account for the Lunar New Year. On the same set of axes, graph their bank account balances. Use a different marking (or different color pen or pencil) for each person. Let x represent time in days and y represent money in the bank. You should look for the comparative steepness (or slope) of the lines as you graph the different financial scenarios. We will formally define slope on another day. For the purposes of this problem, slope is how steeply the line slants across the graph.

a) May saves two dollars every day. Graph her situation. Label the line with her equation and name. **[May's balance = 2x + 20]**

b) Ling saves five dollars every day. Graph her situation. Label the line with her equation and name. **[Ling's balance = 5x + 20]**

c) Tuan saves fifty cents each day. Graph his situation. Label the line with his equation and name. Hint: Do not forget the starting balance.
[Tuan's balance = $^1/_2$x + 20 = 0.50x + 20]

d) Compare May's, Ling's and Tuan's graphs and equations. How are they alike? Different? **[They are all going uphill; Ling's is steeper]**

<<Problem continues on next page.>>

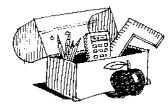
e) Minh spends two dollars a day. Graph his situation. Label the line with his equation and name. **[Minh's balance = -2x + 20]**

f) Tam spends three dollars every day. Graph her situation. Label the line with her equation and name. **[Tam's balance = -3x + 20]**

g) Compare Minh's and Tam's graphs. How are they the same? How are they different? **[They are both going downhill; Tam's is steeper]**

h) Now compare May's, Ling's and Tuan's graphs to Minh's and Tam's. How are they different? **[May's, Ling's and Tuan's are going uphill while Minh's and Tam's are going down]**

i) Kim is not going to spend or save any money. Graph her situation. What does this graph look like? What is her equation? **[a horizontal line; Kim's balance = 20 or y = 20]**



BR-13. Add this definition to your tool kit, if you have not done so already:

X- and Y-INTERCEPTS

Recall that the **X-INTERCEPT** of a line is the point where the graph crosses the x-axis; that is, where y = 0. To find the x-intercept substitute 0 for y and solve for x. Its coordinate is (x, 0).

Similarly, recall that the **Y-INTERCEPT** of a line is the point where the graph crosses the y-axis, which happens when x = 0. Its coordinate is (0, y).

BR-16. MONEY MATTERS, Part Two

a) May's, Ling's and Tuan's graphs have a **positive slope**. Minh's and Tam's graphs have a **negative slope.** Explain why this makes sense. What would Kim's graph represent? Explain. **[Students should relate increasing balance to "positive", decreasing balance with "negative", and no change to neither positive or negative]**

b) Who is spending money at the fastest rate? How does the graph show this? How does the equation show this? **[Tam is spending the fastest. She has the steepest downhill line. The number of weeks is multiplied by the largest number.]**

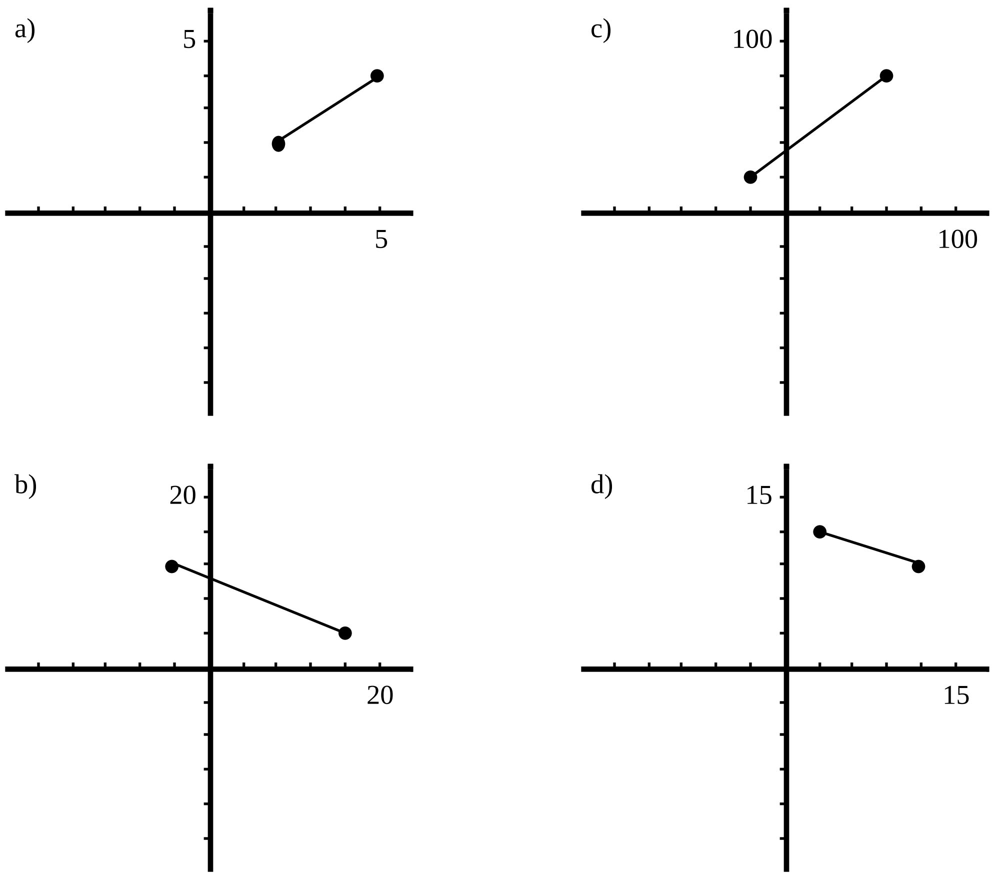
c) How does May's line differ from Minh's? How does the equation show this? **[The lines are the same steepness with one going up and the other down. They spend or save at the same rate.]**

d) Where would you expect the line for Kai, who saves $10 per day to be? Where would you expect his line to be if he spent $10 per day? **[The lines will be much steeper; saving - positive, spending - negative]**

e) Predict what would happen to Ling's line if she had started with $10 instead of $20. What would happen to Tam's line if she started with $30 instead of $20? **[The y-intercept changes. Ling's moves down, Tam's moves up.]**

f) Beside each graph, write the rate of savings or spending for each cousin. For example, May saves at a rate of $2 per day. This can be written as $2/day. Write these rates on the graph. Is there a relationship between the rates and the equations?
[$2/day, $5/day, $0.50/day, -$2/day, -$3/day; Yes, the slope is in the equation.]

BR-23. On the Resource Page provided by your teacher, draw slope triangles and find the slope of the line segments below. Be sure to examine the scale carefully. Write the slope as a ratio. Compare your answers in your study team. **[$^2/_3$, $^{-2}/_5$, $^3/_4$, $^{-1}/_3$]**



BR-37. Refer back to your Money Matters graph, BR-12. Determine the slope for each person's line. Determine the y-intercept for each line. How do these relate to the equations for each person? Be sure to consider whether the slope is positive or negative. **[slopes: May: 2, Ling: 5, Tuan: $^1/_2$, Minh: -2, Tam: -3, Kim: 0; all have a y-intercept of 20]**

BR-38. Graph the line $y = -2x + 3$.

a) Mentally verify that this line goes through the points O' = (0, 3), A' = (2, -1), B' = (3, -3), C' = (-1, 5), D' = (-2, 7) and E' = (1000, -1997).

b) Compute the slope of this line. **[-2]**

c) What is the y-intercept of $y = -2x + 3$? **[(0, 3)]**

d) How do the answers to parts (b) and (c) compare with the original equation?
[Slope is the coefficient of x, y-intercept is the constant.]

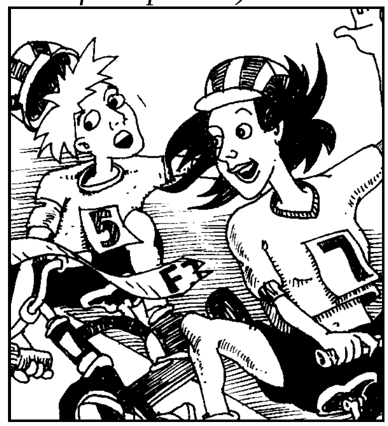
Today is the Big Race. This game is intended as the culminating activity involving graphing with slope and y-intercept and systems of equations. You may need to begin with a reminder to students of the rules when working cooperative logic card problems.

These rules are also printed in the student text for reference during the game. Monitor team progress to be sure that study teams are discussing the problem and its solution.

BR-91 contains questions that will process the today and the important concepts. Each student needs to leave the room with the graph of the race. Distribute the answer graph Resource Page at the end of the day to ensure that the graph is accurate and legible.

****** Resource Page Provided** *(The resource pages are at the end of this packet.)*

BR-90. THE BIG RACE, Part One



Today is finally the Big Race. Obtain a fresh BR-0 Resource Page for this problem. You and the rest of your study team will compete against Leslie and Elizabeth at today's rally in the gym. Determine who will win the race.

Rules:

- Your study team must work cooperatively to answer all the questions on the cards.
- Each member of the team will select rider A, B, C, or D.
- You may not show your card to your team. You may only communicate the information contained on the card.
- Elizabeth's and Leslie's cards will be shared by the entire team.

Remember your guidelines for study teams. Be sure everyone on your team discusses the entire problem and its solution.

BR-91. THE BIG RACE, Part Two

a) Use a table to organize your information. Find the slope for each rider. Find the y-intercept for each rider. Write the equation for each rider.
[Leslie: $y = 2x - 4$; Elizabeth: $y = \frac{1}{4}x + 10$; A: $y = \frac{3}{2}x$; B: $y = \frac{1}{2}x + 6$; C: $y = x + 8$; D: $y = \frac{11}{4}x$]

b) Who won the race? What was the speed of this rider? **[Rider D: $\frac{11}{4}$ mps]**

c) In what order did the participants finish the race? List their names (or letters) with the time it took them to finish.
[D: 9, Leslie: 14.5, A: 16 $\frac{2}{3}$, C: 17 , B: 38, Elizabeth: 60]

d) After 8 seconds, which tricyclist had traveled the shortest distance from the starting line? Who had traveled the farthest distance? **[B; D]**

e) Locate and label three times when one rider passed another rider. Is there a point where more than two tricyclists are tied?
[answers vary; At 8 seconds, Elizabeth, Leslie, and A are tied.]

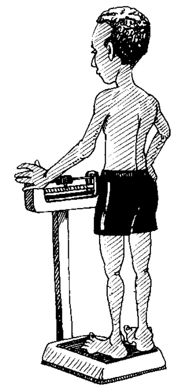
f) If the race were only 20 meters long, does the order of the winners change? How? **[Yes; Leslie and C tie, A is 4th.]**

g) After 16 seconds, how far had each rider traveled from the starting line?
[Leslie: 28m; Elizabeth: 14m; A: 24m; B: 14m; C: 24m; D: 44m]

h) Extension: How long does the race have to be for "C" to be able to win? **[Using substitution, length is between 10.67m and 12.57m. Students can approximate these values instead from the graph.]**

i) Extension: If the race is 22 meters long, how much of a head start does Elizabeth need to beat "D"? **[20 meters]**

BR-99. GETTING IN SHAPE

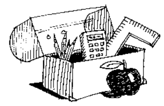


Frank weighs 160 pounds and is on a diet to gain two pounds a week so that he can make the football team. John weighs 208 pounds and is on a diet to lose three pounds a week so that he can be on the wrestling team in a lower weight class. If they can meet these goals with their diets, when will Frank and John weigh the same, and how much will they weigh?

a) Solve the problem. **[9.6 weeks; 179.2 pounds]**

b) Clearly explain your method.

BR-25. Include this information in your tool kit.



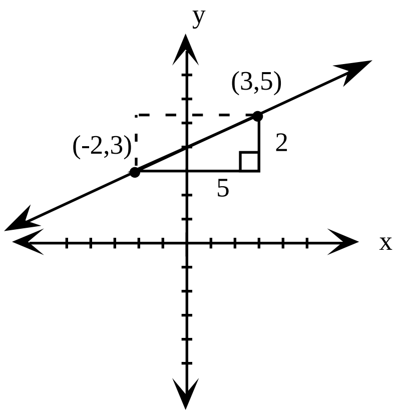
THE SLOPE OF A LINE

The **SLOPE** of a line is a measure of the rate at which something changes. It represents both "steepness" and direction.

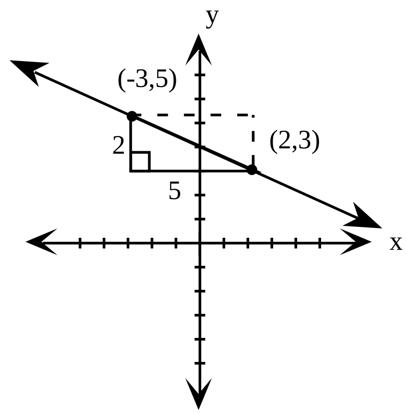
$$\textbf{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in y-values}}{\text{change in x-values}}$$

Note that lines that go **upward** from left to right have **positive** slope, while lines that go **downward** from left to right have **negative** slope. The slope of a line is often denoted by the letter "m." Some texts refer to the vertical change as the "rise" and the horizontal change as the "run."

To calculate the slope of a line, graph two points on the line, draw the slope triangle (as shown in the examples), write the ratio, and lastly check to see if the slope is positive or negative.



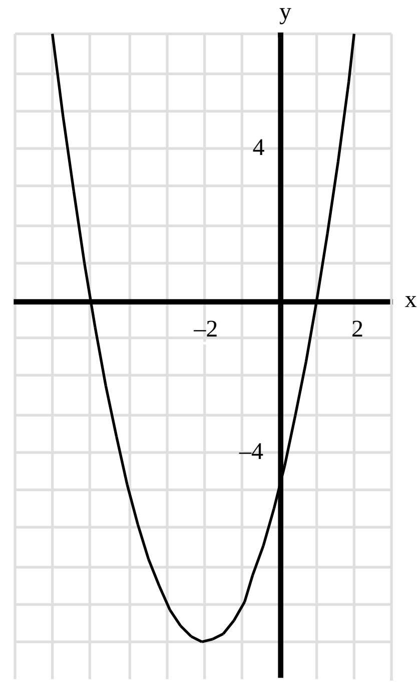
$m = \frac{2}{5}$



$m = -\frac{2}{5}$

Unit 8: Factoring Quadratics

AP-40. We have been working with quadratic expressions such as $x^2 + 4x - 5$. We will now explore the graphs of quadratic expressions and see how factoring relates to the graph.



a) Highlight the line $y = 0$ on the resource page provided by your teacher. Where does the line $y = 0$ intersect the parabola $y = x^2 + 4x - 5$? **[(-5, 0) and (1, 0)]**

b) Factor $x^2 + 4x - 5$. **[(x - 1) (x + 5)]**

c) Use part (b) and Guess and Check to solve $0 = (x - 1)(x + 5)$. **[x = -5, 1]**

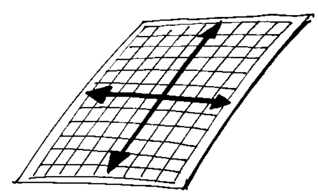
d) What do you observe about the factored form of the equation and the solution in part (c)?
[x is the opposite of the constants.]

(Problem AP-41 develops the Zero Product Property using the relationship demonstrated in AP-40.)

Unit 9: Using Diagrams to Write Equations

(BP-15 extends graphing to another family of curves in the unit where students work with radicals.)

BP-15. GRAPHING SQUARE ROOTS



Obtain the graph resource page from your teacher. (You need to keep track of this page because you will complete a portion of it each day for the next several days).

a) Graph $y = \sqrt{x}$. Record your points in the table on the resource page.

b) Try $x = -3$ in the rule. What is the result? Explain.
[Error; No real square root exists for negative values.]

c) Describe the possible values for x that will yield solutions for the rule $y = \sqrt{x}$.
[Domain: $x \geq 0$]

d) What y values are possible? **[$y \geq 0$]**

e) Describe your graph in words. **[A “banana” curve in quadrant one only.]**

Unit 10: Exponents and Quadratics

YS-15. Graph these two equations on the same set of axes and estimate where they intersect. Use $-2 \le x \le 2$ in increments of 0.5 (e.g., -2, -1.5, ..., 1.5, 2).
[(0, 1), ≈ (-1.4, 0.6), and ≈ (1.4, 3.4)]

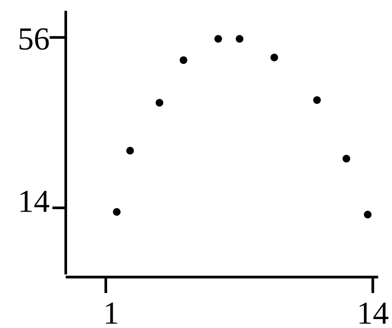
$$y = x^3 - x + 2 \qquad \text{and} \qquad y = x + 2$$

Unit 11: More Diagrams and Subproblems

GG-109. Draw at least four different rectangles on graph paper using integer lengths so that the perimeter of each rectangle is 30 units.

a) Make a table for all of the rectangles that includes the length, width, perimeter, and area of each rectangle. **[The sum of the base and the height must be 15. The largest area (56 sq units) occurs with dimension 8 x 7. The least area (14 sq units) occurs with dimensions 1 x 14.]**

b) Graph your data from part (b) with "height" along the horizontal axis and "area" along the vertical axis.
[Solution shown at right.]



c) Use your graph from part (c) to estimate the maximum area for a rectangle with perimeter 30 units. **[a little more than 56 sq units (exactly 56.25)]**

d) What was the maximum perimeter of the rectangles?
[30 units; it does not change.]

RS-1. The Math Club has been fundraising for months in order to buy the rocket. Bomani, the treasurer, expects an excellent rocket to cost roughly $250 and wants to plan ahead. Use the information shown below from the club's bank statement for the first 10 weeks of school to estimate when they will have enough money to buy the rocket.

a) Plot the data as ten ordered pairs (x, y) with x representing the week. Scale the axes carefully.

week number	balance
0	$ 7.50
1	$ 53.00
2	$ 60.22
3	$ 85.64
4	$ 92.88
5	$ 99.41
6	$ 116.67
7	$ 122.72
8	$ 134.60
9	$ 150.53

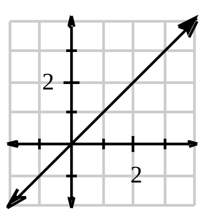
b) Use a ruler to draw the line that best fits the data. Your line may not actually pass through any of the data points. In particular, do not connect the dots with short segments. **[≈ $y = 12x + 40$]**

c) Write an equation for your line of best fit. **[≈ $y = 12x + 40$]**

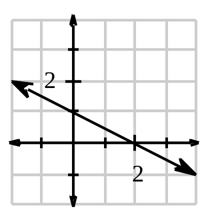
d) Use your equation from part (c) to predict when the Math Club will have enough money to buy the rocket. **[≈ 18 weeks]**

RS-9. One of the most important skills needed to work with graphs is the ability to write an equation for a given line. Write an equation in the form $y = mx + b$ for each of the graphs shown. You may need to estimate to the nearest 0.5.

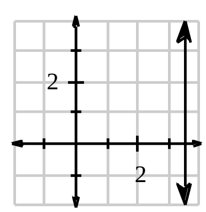
a) **[$y = x$]**



d) **[$y = {}^{-1}/_{2}x + 1$]**



f) **[$x = 3.5$]**



RS-12. Nine students did a science experiment by rolling a marble down a slanted board and timing how long it took.

Adrián found that the marble took 10 seconds to roll 50 cm, but Mícheál's took only 8.9 seconds to roll 40 cm. Monica's results were 14.1 seconds to roll 100 cm, while Karla's were 9.5 seconds to roll 45 cm. Tammy got 12.6 seconds to roll 80 cm. Allan reported 0 seconds for 0 cm and George found the ball took 4.5 seconds to go 10 cm. Marni and Rebecca reported 11 seconds to roll 60 cm and 13.4 seconds to roll 90 cm, respectively.

a) Organize the information.

b) Plot the data as nine ordered pairs (x, y) with x representing time and y representing distance. Scale the axes carefully.

c) Draw a parabola of best fit for the data.

d) Use your graph to predict how long it will take the ball to roll 120 cm, 5 cm, and 15 cm. **[15.5 sec, 3.2 sec, 5.5 sec]**

e) Use your graph to predict how far the ball will travel after 3 seconds, 6 seconds, and 15 seconds. **[4.5 cm, 18 cm, 112.5 cm]**

f) Write an equation for the parabola you drew in part (c). **[$y = 0.5x^2$ is close]**

RS-22. On graph paper, graph $y = x^2 - x - 12$.

a) Find the x-intercepts. **[(4, 0), (-3, 0)]**

b) Solve the equation: $x^2 - x - 12 = 0$. **[x = 4 or -3]**

c) Using complete sentences, describe how you can find the x-intercepts of a parabola <u>without</u> graphing. **[Set equal to zero, factor, then use the Zero Product Property or use the Quadratic Formula.]**

RS-23. Find an equation of a parabola with x-intercepts:

a) (-2, 0) and (1, 0). **[Any equation of the form: y = a(x + 2)(x - 1)]**

b) (6, 0) and the origin.
[Any equation of the form: y = a (x - 6)(x - 0) or y = ax(x - 6)]

c) (b, 0) and (c, 0). **[Any equation of the form: y = a (x - b)(x - c)]**

RS-24. Is there more than one parabola with x-intercepts (5, 0) and (-3, 0)? Draw a sketch to demonstrate your answer. **[yes]**

RS-37. Make a large pair of coordinate axes on a sheet of graph paper by placing the origin at the center of the paper. Label your graph clearly. Graph $y = x^2$.

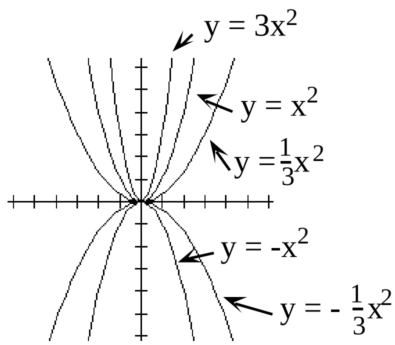
a) Each member of your study team should choose one of the equations from the list below. Neatly graph the parabolas on the same set of axes as above by making tables and assigning values to x from -4 through 4. Share the results of your graph with your team members.

1) $y = -x^2$ 3) $y = \frac{1}{3}x^2$

2) $y = -\frac{1}{3}x^2$ 4) $y = 3x^2$

b) After verifying their accuracy, copy all the parabolas onto your graph. Label each parabola with its equation.

Solution:



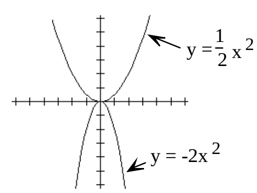
c) Write a few sentences to describe some patterns you see in the relationship between the structure of quadratic equations and their graphs.
[upward: a > 0, downward: a < 0; wide a > 1 or a < 1, narrow: -1 < a < 1]

RS-44. Use your conclusions from RS-37 to decide with your team how to sketch each of the following equations

a) $y = \frac{1}{2}x^2$
[It lies between the graphs of $y = x^2$ and $y = {}^1/_3x^2$.]

b) $y = -2x^2$
[It lies beneath the graph of $y = -x^2$.]

Solution:



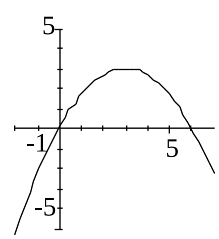
RS-45. a) Find an equation of a parabola with x-intercepts at (0, 0) and (6, 0). **[$y = ax(x - 6)$]**

b) Find the x-coordinate of the vertex. **[$x = 3$]**

c) Find a coefficient of your parabola so that the y-coordinate of the vertex is 3. **[$a = {}^{-1}/_3$]**

d) Sketch this parabola using the x-intercepts and the vertex.

Solution:



The Graphing Thread in Geometry, Algebra 2, and Math Analysis

Geometry (Math 2)

Students begin Geometry with a coordinate geometry activity that immediately reviews and refreshes their ability to work successfully with an xy-coordinate system. Unit 1 reviews and expands graphing linear equations, slope, slope-intercept form, and writing equations from graphs. These and other concepts are introduced using grid triangles, that are also useful to introduce transformations, trigonometry, and similarity later in the year. Units 2 and 3 extend linear graphing to inequalities. Units 4 and 5 graph non-linear functions, with most attention focused on quadratics and the relationship between the factored equation and its roots. There are extensions of slope from Unit 1 that deal with parallel and perpendicular lines.

In the second half of the text, students tie the slope of a line to the tangent ratio, gain experience graphing non-linear functions besides quadratics, and graph circles with centers at the origin. Throughout the course students do problems in a graphing context that applies the geometric concept under study at the moment.

Algebra 2 (Math 3)

Students begin the Algebra 2 course with an introduction to functions and what it means to *investigate* a function using a graphing calculator. Problems include gathering and organizing data, fitting functions to data, and using skills such as graphing in the context of larger problems. Then in Unit 2 we develop sequences as functions and arithmetic sequences are used to illustrate the differences between discrete and continuous functions. Unit 3 builds on geometric sequences as discrete exponential functions and uses the graphing calculator on the theme problem: comparing the depreciation and appreciation of selected types of cars.

Unit 4 deals with stretching, shrinking, and moving basic graphs of functions. In Unit 5, systems of equations evolve to inequalities and the use of linear programming. This unit adds another way to connect equations, graphs, and intersections. In Unit 6, inverse functions are introduced and log graphs join the other functions as translations are given further attention.

In Unit 7 the graphing calculator is an essential tool for exploring polynomials in relation to their intersections with $y = 0$ and their non-intersection leads to the development of complex numbers. Subsequent units use the graphing calculator to transform trigonometric graphs and identities. Students explore the inverses and reciprocals of trig functions graphically, leading to the discovery of key trig identities from their graphs.

Math Analysis (Math 4)

(Pre-Calculus)

Math Analysis is currently in the developmental stage. One example of the graphing thread, which is used extensively in the course, is the concept of the integral, which is developed as area under a curve. To use this concept, we also develop at some length the concept that the area under the curve of the velocity function will represent the distance traveled. This allows students to see more clearly in later units why the slope of the line (derivative) of the distance traveled up to time t will be the velocity.

The Big Race

Your Name:

You ride your tricycle three meters every two seconds and start at the starting line. The race is 25 meters long.

Who is going to win the race?

A

The Big Race

Your Name:

You get a six meter head start and ride twice as fast as Elizabeth.

Who is going to finish last?

B

The Big Race

Your Name:

You ride half as fast as Leslie but get an eight meter head start.

Who is going come in second?

C

The Big Race

Your Name:

You start at the starting line and catch up to Elizabeth in four seconds.

Who is going to win the race?

D

The Big Race

Elizabeth

Elizabeth rides one meter for every four seconds (because she has a flat tire) and gets a 10 meter head start.

Who is going to win the race?

The Big Race

Leslie

Leslie rides two meters per second and starts two seconds after the start of the race.

Who is going to win the race?