# Functions and Graphing in CPM Algebra 2

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Each course is built around a few core ideas as recommended by the TIMSS Report, which in turn are developed and deepened over the four year program. Specific skills and concepts are tied to these threads and presented in context--both conceptually and in realistic problems--so that students will see the connections among the ideas and make sense of mathematics.

**Math 1 (Algebra 1)**
Problem Solving
Graphing
Writing Equations
Solving Equations
Ratios
Symbol Manipulation

**Math 2 (Geometry)**
Problem Solving
Graphing
Ratios
Geometric Properties
Algebra
Spatial Visualization
Conjecture & Explanation (Proof)

**Math 3 (Algebra 2)**
Problem Solving
Representation & Modeling
Functions & Graphing
Intersections and Systems
Algorithms
Reasoning and Communication

**Math 4 (Mathematical Analysis)**
Problem Solving
Concepts of Calculus
Analysis of Models
Trigonometry
Advanced Functions
Algebraic Fluency & Accuracy

## Thread Snapshot: Functions and Graphing in Algebra 2 (Math 3)

The focus on functions begins immediately in Unit 1. On the second day students learn to use the graphing calculator to investigate a function. Throughout the course they will continue to investigate both new and familiar functions, each time adding new criteria to the investigation.

In Unit 1 students review function notation and what they have learned about linear functions and their graphs. In Unit 2 arithmetic sequences are seen as discrete linear functions and geometric sequences are introduced so they can be related to exponential functions in Unit 3. The focus of Unit 3 is exponential functions and their applications, from the Penny Lab, which introduces the idea of exponential decay, to
a comparison of the depreciation rates of cars, the unit theme problem. In Unit 4 students work with graphs of parabolas to develop the notions of "parent graph" and "families of functions," and then they generalize their findings about parabolas to graphs of other functions including rational, cubic, exponential, and absolute value functions. Students also graph circles and parabolas that are not functions and clarify their definition of function.

Unit 6 focuses on inverse functions. Students algebraically "undo" a selection of linear, quadratic, and cubic functions and investigate the relationship between the graphs of the original functions and their inverses. Posing the question of "undoing" an exponential function leads to the definition of logarithmic functions, and logarithmic functions are added to the set of familiar families of functions. In Unit 7 students explore polynomial functions and their roots, and in Unit 8 they encounter the sine, cosine, and tangent functions. By the end of the course students should be able to sketch the graphs of linear, quadratic, other polynomial, absolute value, exponential, logarithmic, trigonometric functions, some rational functions and, given their graphs or information about their graphs, they should be able to write the equations. They should also be able to identify intercepts, domains, and ranges and find inverse functions and asymptotes when they exist.

In many of the units students use the equations and graphs of functions to model and/or solve the theme problem: a linear function for the sharpening pencils lab in Unit 1, exponential functions for the bouncing ball and the fast cars problems in Units 2 and 3, a quadratic function to approximate the St. Louis Arch in Unit 4, an exponential function to solve the Mystery of the Cooling Corpse in Unit 6, a piece of a polynomial function to solve the Game Tank problem in Unit 7, and the sine function to represent a ride on the "Circle of Terror" in Unit 8.

Answers and teacher notes appear in **bold** print, as they do in the teacher version of the text. The references to student "tool kits" is CPM's way of teaching and requiring students to have study notes for each unit. The tool kit icon means "put this in your tool kit---NOW!" Proof problems are identified by the domino icon.

## *From Unit 1*

EF-12.

> As you begin investigating functions, it is important that you understand what we expect when we ask you to sketch a graph. To **sketch a graph** means you show the approximate shape of the graph in the correct location with respect to your axes and that you clearly label all key points.

Consider the following equation: $y = \sqrt{(4 - x)} - 1$
Use a graphing calculator to help you to answer each of the following questions completely.

a) Sketch the graph from your graphing calculator. Identify the x and y intercepts.
**[ (3,0), (0,1) ]**

b) What are all the possible values of x? Are there any values that will not work for x? What is the largest value you can use for x? Explain.
**[ $x \le 4$ ]**

c) Does the graph ever cross the horizontal line $y = 50$? How about $y = 500$? How do you know?

d) What is the smallest possible value of y? What are all of the possible values of y? **[$y \ge -1$]**

e) Have you found <u>everything</u> about this equation that is important? Explain.

f) Does the line $y = x$ intersect the graph? How could you find the point of intersection?
**[ Yes; Use a calculator to graph both and estimate or solve $x = \sqrt{(4-x)} - 1$. ]**

***In problems that precede EF-73 students have worked with the input and output of "function machines."***

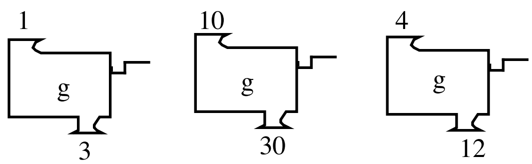
EF-73.

A convenient way to show what a function machine does is to use **FUNCTION NOTATION**. For Carmichael's machine in problem EF-63, we would write $f(x) = x^2 + 2x + 1$. The f is just the **name** of the function machine; it is not a variable. It could just as well be $Pierce(x) = x^2 + 2x + 1$ if the machine happened to be named Pierce! Warning: f(x) does **not** mean f times x; we read it as "f of x." It means the output of the function f resulting from the input x. In part (c) of problem EF-63, you actually found f(-22.872). Here is another example that shows how to use function notation. Add this information to your Tool Kit.

If $g(x) = 2x^2 - 5x$ then $g(\mathbf{-2}) = 2(\mathbf{-2})^2 - 5(\mathbf{-2}) = 8 + 10 = 18$.

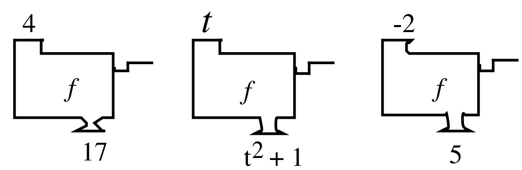
Now use function notation to describe what each of the following machines does to x.

a)




**[ g(x) = 3x ]**

c)




**[ $f(x) = x^2 + 1$ ]**

## *From Unit 3*

***Early in Unit 3 the students take a "function walk" which provides a kinesthetic and visual introduction to exponential functions. Directions for the teacher on leading the function walk follow.***

**The outdoor xy-coordinate system should be marked off before class. For the axes, some teachers use two pieces of rope wrapped with colored tape to mark the units; others use chalk to draw the axes on the pavement.**

**Start with the function $y = 2^x$. Be sure nine students are standing on the x-axis with the mark that corresponds to their number (an integer, -4 to 4) between their feet and facing in the direction of the positive y-axis. Direct students to use their number as x, and calculate the value of y. Then when you say 'go,' they should take that many paces forward.**

**Have the rest of the students record the function (equation) and as much detail as they can about the resulting graph. Three members of each team will have information about each function, but no one will have all four. Then have another group do $y = 3^x$, the next do $y = (1.5)^x$, and then do $y = (0.5)^x$. While each team is "walking" the rest of the students should be recording the other three functions and their graphs.**

**Have students note the Domain as they stand on the x-axis. After they have moved into position have them make a 90˚ turn toward the y-axis and walk straight towards it. The space between 0 and 1 will become a bit crowded, but they should now be able to note the range.**

FX-43. Make a table like the one below for each of the four functions from the function walk and fill in as much as you can. Give y-values in fraction form and look for a pattern.

| x | $2^x$ | y |
|---|---|---|
| -4 | **$2^{-4}$** | **1/16** |
| -3 | **$2^{-3}$** | **1/8** |
| -2 | **$2^{-2}$** | **1/4** |
| -1 | **$2^{-1}$** | **1/2** |
| 0 | $2^0$ | **1** |
| 1 | $2^1$ | 2 |
| 2 | $2^2$ | 4 |
| 3 | $2^3$ | 8 |
| 4 | **$2^4$** | **16** |

FX-45. Draw a graph for each of the four functions from the function walk and give the domain and range of each one.

***Students refer to the graphs of exponential functions as visual validation for the algebraic extension of the rules of exponents to integral and rational exponents.***

## *From Unit 4*

PG-13. For each equation below, predict the vertex, the orientation (open up or down?), and tell whether it is the same as a vertical stretch or compression of $y = x^2$. Before using the graphing calculator, sketch a quick graph based on your predictions.

a) $y = (x + 9)^2$
b) $y = x^2 + 7$
c) $y = 3x^2$
d) $y = \frac{1}{3}(x - 1)^2$
e) $y = -(x - 7)^2 + 6$
f) $y = \frac{5}{2}(x - 2)^2 + 1$
g) $y = 2(x + 3)^2 - 8$
h) $4y = -4x^2$
i) $y = 4x - 4$

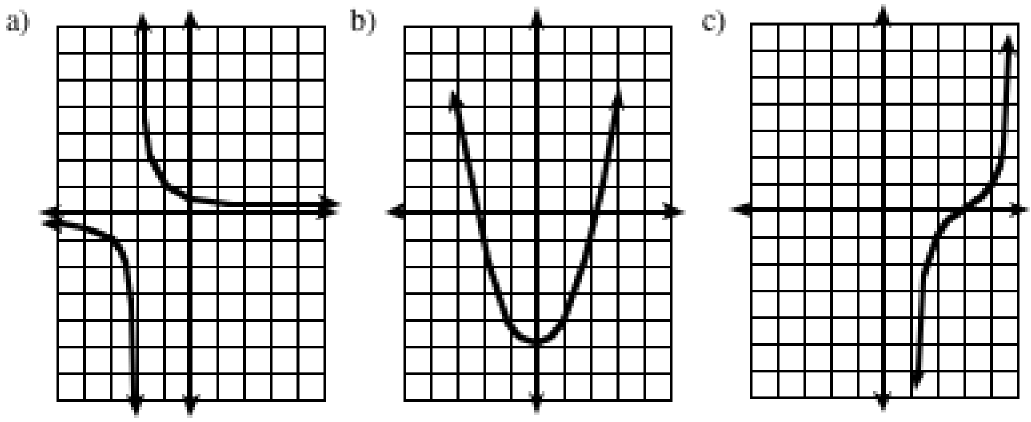
j) Check your predictions for the equations in parts (a) through (i) on your graphing calculator. If you made any mistakes, correct them and briefly describe why you made the mistake (what incorrect idea you had). Then make a neat and accurate graph for each function.

PG-56 Working as a team, sketch a graph of each of these equations and identify each graph with its equation. Label the coordinates of intercepts or any other important points that are relatively easy to find.

a) $y = x^3$
b) $y = 2^x$
c) $y = \frac{1}{x}$
d) $y = \frac{1}{x} - 4$
e) $y = (x - 1)^3$
f) $y = 2^x - 4$
g) $y = 2^{(x-4)} - 3$
*h) $y = \frac{1}{x + 3} + 1$
*i) $y = \frac{2}{x - 3}$
j) $y = (x - 2)^3 + 1$
k) $y = 2^{(x+3)}$
l) $y = \frac{1}{2}(x + 2)^3$

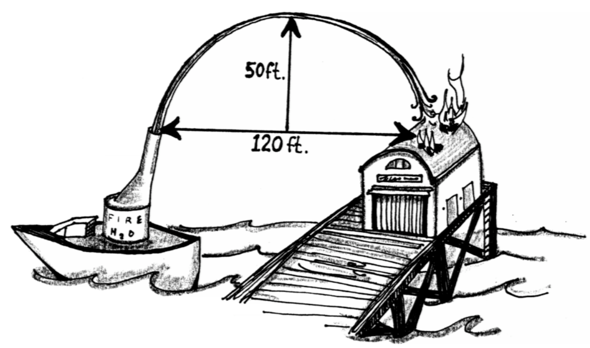
PG-57. Separate your graphs (cut them apart) and then discuss possible methods of sorting the graphs; which graphs seem to belong together? Once your team has decided on a sorting method, sort the graphs into appropriate groups, select the simplest equation for each, and describe the other graphs in the group in relation to the graph of that equation.

PG-79. Write a possible equation of each of these graphs. Assume that one mark on each axis is one unit. When you are in class, check your equations on a graphing calculator and compare your results with your teammates.




a) **[ $y = \frac{1}{x+2}$ ]** b) **[ $y = x^2 - 5$ ]** c) **[ $y = (x - 3)^3$ ]**

PG-97. **The Fireboat Again** The equation below is one possibility for the parabola formed by the water canon on the fireboat in problem PG-33:




$$f(x) = a(x - 60)^2 + 50$$

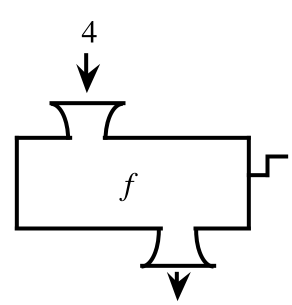
a) Will the value of a be positive or negative? **[ negative ]**

b) Check to see whether you got exactly $-\frac{1}{72}$ for a. If you did not, then calculate the value for a by substituting the coordinates of some known point other than the vertex and then write the complete equation. **[ $y = -\frac{1}{72}(x - 60)^2 + 50$ ]**

c) Why wouldn't it work to substitute the coordinates of the vertex in part (b)? If you're not sure, try it.

## *From Unit 6*

CC-2. The function machine to the right, f, follows the rule:




$$f(x) = 5x + 2.$$

a) What is f(4)? **[ 22 ]**

b) If the crank is turned backwards, what number should be pulled up into the machine in order to have a 4 come out the top? **[ 22 ]**

c) Keiko wants to build a new machine that will undo what f does. What must Keiko's machine do to 17 to undo it and make it a 3? Write your rule in function notation and call it g(x). **[ Subtract 2 and divide by 5; $g(x) = \frac{x-2}{5}$ ]**

CC-16.

> The formal mathematical name for an undoing function is **INVERSE**. Record this in your Tool Kit.

Find the inverse for each of the functions below. Then graph each function and its inverse on the same set of coordinate axes. In other words, use one set of axes for part (a), a new set for part (b), etc. This is NOT a good time to divide up the graphing among your teammates. Make sure each of you does the work for each part. Check with each other as you go.

a) $f(x) = 2x + 4$ **[ $y = \frac{x-4}{2}$ ]**

b) $f(x) = -\frac{2}{3}x$ **[ $y = -\frac{3}{2}x$ ]**

c) $y = \frac{1}{3}x + 2$ **[ $y = 3x - 6$ ]**

d) $y = x^3 + 1$ **[ $g(x) = \sqrt[3]{x - 1}$ ]**

CC-17. When you have completed all the pairs of graphs, look for patterns in the graphs. What relationships do you see between the graph of a function and the graph of its inverse? Do the pairs of graphs have a line of symmetry? Justify your answer.
**[ Yes, each pair has the line $y = x$ as a line of symmetry. ]**

CC-69. If $x = 2^y$, how can you solve for y and make it say "y ="? Discuss this with your team and be prepared to report to the class what you think.
**[ You should be prepared for answers like "take the $y^{th}$ root" or other ideas. It shows students are trying! ]**

CC-70.

When mathematicians cannot solve a problem or do not have the tools to do a particular problem, do you know what they usually do? They invent a tool to do what they want! Of course this invention has to be consistent with all the existing mathematical rules. In this case they invented a new operation, the inverse exponential function, base 2. Henceforth they said (at one point in history) the inverse exponential function base 2 will be known as **LOGARITHM, BASE 2**. Why did they select "logarithm" when thousands of other names were possible? That is a question to investigate on the internet. You graphed this function in problem CC-58 (we called the graph the inverse exponential function, base 2). Find your graph and label it "logarithm, base 2." Add the graph and its equation $y = \log_2 x$ to your Parent Graph Tool Kit.

Logarithm base 2 is usually abbreviated as log, base 2 and is usually written $\log_2$. When we see this symbol, we read it aloud as "log, base 2." The comma means we pause a bit when we say it. Since it is a function, we can use function notation, such as $g(x) = \log_2(x)$. Notice that the base number, 2 in this case, is always written a little lower, which is called a subscript.

Use your graph of $g(x)$ to find each of the missing values:

a) $\log_2(32) = ?$ b) $\log_2\left(\frac{1}{2}\right) = ?$ c) $\log_2(4) = ?$

## *From Unit 7*

CF-16. **Polynomial Functions Lab (CF-16 & CF-17).** We are going to be doing an investigation of polynomial functions, but first we will do one example as a class.
The example function we will work with is $P_1(x) = (x - 2)(x+ 5)^2$.

a) Graph the function on a graphing calculator using the standard viewing window. **[ The first grid on the resource page is for this graph. ]**

b) As you can see, we do not get the best view of the function in the standard window. We will use the zoom features of the graphing calculator to obtain a better view of the graph. What should we be looking for when we zoom? Start by zooming out. You may have to do this more than once to get the complete graph. Next use the box feature to select the portion of the graph which you would like to view. Sketch this graph on the second set of axes. It helps to label the important points of the graph such as the intercepts. **[ Look for where the graph curves, the parts of the graph that are "cut out". ]**

c) Find the roots. **[ roots at x = -5 and x = 2 ]**

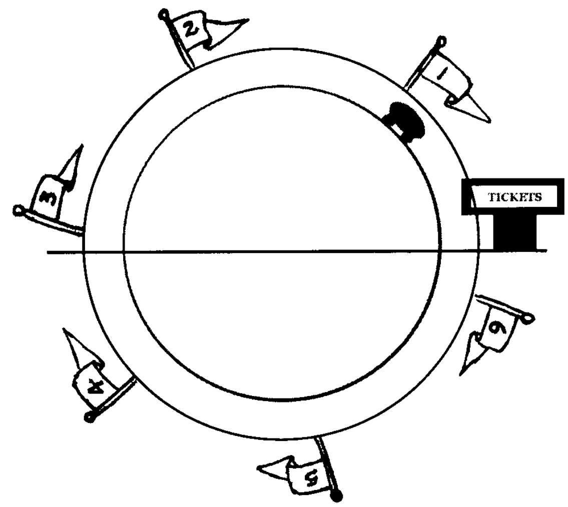
d) On the number line mark the roots with open circles and then shade the regions where the function outputs are positive (the graph is above the x-axis).

e) Describe the graph and its relation to the equation. Make sure that you include the degree and the intercepts as part of your description. Pay close attention to the way the function curves. **[ In this case, the function turns three times or goes three directions: up, then down, and then up again. Sketching the directions will help the students see this property. ]**

## *From Unit 8*

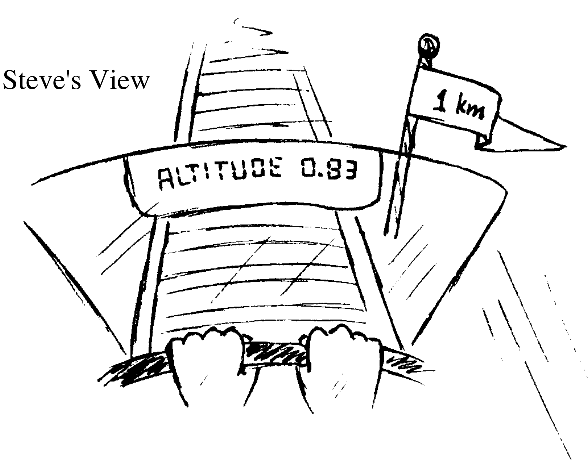
CT-1. It's January 1, 2100, and Great Hemisphere Amusement Park has a new ride for the new century: The Circle of Terror. It is on a circular track, half above ground and half below, and it rises to a height of 1 kilometer and descends to a depth 1 km below the ground. Sally dares Steve to try it. Steve boards the ride at ground level and glances up to see a display that says, "ALTITUDE 0 km." As the ride begins, Steve clutches the rail in front of him and focuses on the track ahead. He notices that the distance along the track is marked every kilometer, and to distract himself from his rising anxiety he begins to pay attention to the relationship between the distance he's traveled along the track and the altitude above or below ground level displayed on the panel overhead. At home later he found himself wondering about the relationship he observed while on the ride.




a) Is the relationship between the distance traveled and the altitude a function? Does it matter which quantity is the independent variable and which is the dependent?
**[ It is a function only if the independent variable is distance traveled, and the dependent variable is altitude. ]**

b) There are four points at which you know Steve's exact altitude. What are they, and how far had he traveled when he was at each of those altitudes? Use your knowledge of circles to express these distances in exact form.
**[ Top, bottom and the 2 points at ground level; altitudes 1, -1, and 0. He traveled $\frac{\pi}{2}$ km when at height 1 km, 1.5π km when at altitude -1, and 0, π, and 2π km when at height 0. ]**

Steve's View




c) Make a table of values for the function Steve observed. Use the diagram on the resource page provided to estimate Steve's altitude at the distances indicated and include the four points you found in part (b). Use negative numbers to represent altitude below ground level.

d) We'll call Steve's function ***S(x)***. Continue investigating this function. Be sure to include <u>**all**</u> of the usual information involved with investigating functions.
**[ Domain 0≤ x ≤ 2π; Range -1 ≤ y ≤ 1; y-intercept (0, 0); x-intercepts (0, 0), (π, 0), (2π, 0) ]**

***Through further investigation students learn that S(x) = sin x, and they go on to use transformations of the sine, cosine, and tangent functions to model additional real and imagined situations.***