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# **Guess and Check, A Powerful Problem Solving Tool**

For more information about the materials you find in this packet, go to the web site above or contact:

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## Solving Problems with Guess and Check

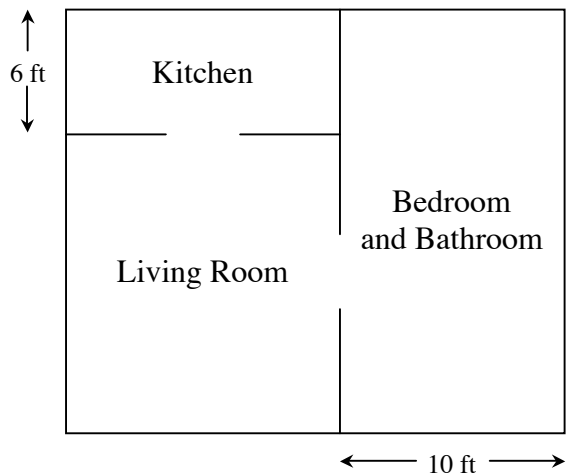
In this lesson, you will work with your team to find a strategy for solving a complex problem. It will be important for you to find ways to organize your work so that other people can follow your process.

### 1-40. THE APARTMENT

Your architecture firm has been hired to design an apartment building. Each of the apartments in the building will be laid out as shown at right so that each room is rectangular.

The building's owners have given you the following specifications.

- The living room of an apartment must have an area of 180 square feet.
- The shorter side of the kitchen must be 6 feet to make room for counter space.
- The shorter width of the bedroom and bath must be 10 feet so that a dresser and king-sized bed will fit.
- The entire area of each apartment must be exactly 450 square feet.



**Your Task:** Find the possible dimensions for every room in the apartment. Be prepared to **justify** your answer (show how you know it works), and show all of your work in a way that someone who is not in your team can read and understand it.

### *Discussion Points*

How can you start?

How can you organize your work?

How can you use the results from one guess to make your next guess better?

## Further Guidance

- 1-41. Charles decided to start this problem by making a guess. He guessed that everything would come out right if one side of the living room were 10 feet.
- Does it matter which side of the living room is 10 feet long? Why or why not? Find the area of the entire apartment twice: once if the base of the living room is 10 feet long, and again if the height of the living room is 10 feet long. Do the results come out the same
  - Is Charles' guess correct? That is, can one of the sides of the living room be 10 feet long? Explain.
  - As you checked Charles' guess, did you organize your work so that anyone could read and understand your thinking? If not, try to find a way to reorganize your work to make it clear.
  - As a team, try another guess for a dimension of the living room. Organize your work to check if that guess is correct.
- 1-42. One way to organize your work in this kind of problem is by using a table. The table can be structured something like this:

Length of _____	?	Total Area

- Use this method to check the rest of your guesses for this problem.
- Continue guessing and checking until you find correct room dimensions. Once you have an answer, organize your work so that it shows the dimensions of all of the rooms and can be read and understood by someone who is not in your team. Be ready to present to the class both your final answer and the different guesses that you tried along the way.

GO-17. Solving a Problem Using Guess and Check

Dr. Tasha McNeil, the local orthodontist, saw eight more patients on Tuesday than on Monday. Dr. McNeil knew that the total number of patients for the two days was 44 because she had a stack of 44 folders. Each patient has exactly one folder. How many folders should be stacked in the Monday pile?



Sometimes the easiest way to get started is to guess what the answer might be and then to check to see if you are right. Guessing is a good strategy for learning more about the problem, and it will lead to algebraic solutions later in the course. The example below leads you through the four steps of a Guess and Check table. Follow along with your teacher and help Dr. McNeil find out how many folders are in the Monday pile.

Step 1: Set up a table. The first column will contain your guess. In this case, you will be guessing the number of folders for Monday. Copy the table as shown below on your paper.

Guess Number of Folders for Monday
------------------------------------

Step 2: Make a guess about the number of folders for Monday. Your first guess should be something reasonable and easy. Since you know the total number of folders is 44, it would be silly to guess anything bigger. Try guessing 10. Place your guess of 10 in the box under the heading “Guess Number of Folders for Monday.”

Guess Number of Folders for Monday
------------------------------------

Step 3: If there were ten folders on Monday, how many folders will be in the Tuesday stack? Label the top row as shown, then write that number in the space below.

Guess # of Folders for Monday	# of Folders for Tuesday
10	

Step 4: Your first guess of 10 folders for Monday means there must be 18 folders for Tuesday. You can use this information to determine your next column. Label the column “Total Number of Folders” and write the total in the space below.

Guess Number of Folders for Monday	Number of Folders for Tuesday	Total Number of Folders
10	18	

>>Problem continues on the next page.>>

Step 5: Next you check to see whether your guess gives you the correct result. The last column should be checking if the total number of folders is equal to 44. If it is not, write in “too high” or “too low.” Add this information to your paper.

Guess Number of Folders for Monday	Number of Folders for Tuesday	Total Number of Folders	Check 44
10	18	28	

Step 6: Start over with a new guess. Since the last guess was too low, we need to guess something higher. Try guessing 20, then repeat steps three through five.

Guess Number of Folders for Monday	Number of Folders for Tuesday	Total Number of Folders	Check 44
10	18	28	too low

Step 7: Twenty folders for Monday turned out to be too high. Now you know the solution is something less than 20 but greater than 10. Try guessing 15 and repeat the process.

Guess Number of Folders for Monday	Number of Folders for Tuesday	Total Number of Folders	Check 44
10	18	28	too low
20	28	48	too high

Step 8: Fifteen is too low. Try guessing 18 folders.

Guess Number of Folders for Monday	Number of Folders for Tuesday	Total Number of Folders	Check 44
10	18	28	too low
20	28	48	too high
15	23	38	too low

Step 9: You have now solved the problem. There is only one thing left to do: write the answer in a complete sentence that restates the question.

Eighteen folders should be stacked in the Monday pile.

**TEACHER NOTE:** In problem GO-18 students will make a Guess and Check table as partners or a team. After they read the problem, have one person copy the problem on one piece of paper for the whole team/pair. The first person fills in the table for the next guess (the first one is done). The second person checks the work and completes the next guess. As each guess is completed, the partners or team must decide on the next guess until the answer is found. Each student must copy the completed table on his/her own paper.

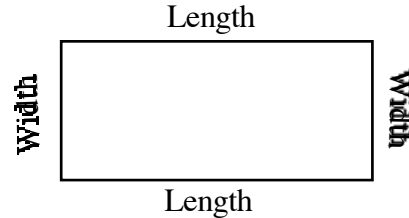
GO-18. Using the Guess and Check table below, find two consecutive numbers that have a sum of 37. (**Consecutive** means “one after another in order,” like 3 and 4.) Answer in a complete sentence.

Guess First Number	Second Number	Total of Both Numbers (Sum)	Check 37
10	$(10) + 1 = 11$	$(10) + (11) = 21$	too low

GO-19. Parts (a) and (b) below help you solve the following problem using a Guess and Check table.

The length of a rectangle is three centimeters greater than the width. If the perimeter (distance around) equals 54 centimeters, find the dimensions (length and width) of the rectangle.

- a) In this case, a diagram would be useful.  
Copy the diagram at right on your paper.
- b) Copy and complete the table below.  
Remember to show all of your calculations and to answer using a complete sentence.



Guess Width	Length of Rectangle	Perimeter of Rectangle	Check 54
10	$(10) + 3 = 13$	$(10) + (13) + (10) + (13) = 46$	too low

GO-33.



### SOLVING PROBLEMS WITH GUESS AND CHECK TABLES

- Read the problem carefully. Make notes or sketch a picture to organize the information in the problem.
- Look at the question being asked. Decide what you are going to guess. Set up a table. Leave extra space for more columns in case you need them.
- Calculate the entry for a column and label the column.
- Continue the table until the check is correct.
- Write the answer in a complete sentence.

Example: The sum of two consecutive numbers is 29. What are the numbers?

Guess First Number	Second Number	Total of Numbers (Sum)	Check 29
10	$(10) + 1 = 11$	$(10) + (11) = 21$	too low
20	$(20) + 1 = 21$	$(20) + (21) = 41$	too high
15	$(15) + 1 = 16$	$(15) + (16) = 31$	too high
14	$(14) + 1 = 15$	$(14) + (15) = 29$	correct

The sum of the two consecutive numbers 14 and 15 is 29.

MC-3. Sometimes you can make a math problem easier by replacing part of it with an **equivalent** quantity. For example, you could describe the distance from San Francisco to Los Angeles as 379 miles or as 2,001,120 feet. Two expressions are equivalent if they represent the same number. Twelve inches is equivalent to 1 foot, 4 quarts is equivalent to 1 gallon, and 379 miles is equivalent to 2,001,120 feet.

- Thirty-two ounces is equivalent to how many pounds?
- One century is equivalent to how many years?
- Ten weeks is equivalent to how many days?
- Marty says her baby weighs 8 pounds. The scale shows 128 ounces. Is 8 pounds equivalent to 128 ounces? Show your work

MC-8.



## EQUATIONS

A mathematical sentence with an equal sign is called an **EQUATION**. It is a relationship showing that two expressions have the same value.

Statements like 16 ounces = 1 pound,  $n = 4$ ,  $3(x - 5) = 16$ , and  $3(2) + 7 = 16 - 3$  are examples of equations.

MC-14. **Cover-Up Method** The Mental Math Club was planning a carnival to raise money. Jacob decided to run the Guess My Number booth. His good friend Carlin wanted to check out the booth before the carnival opened for business. “Try out a problem on me,” said Carlin.

Jacob gave Carlin this problem: “17 more than a number is 52. Guess the number and win a prize.” Carlin pictured it like this:

$$x + 17 = 52$$

Carlin covered up the  $x$  and asked himself, “What number plus 17 equals 52? Aha,” he said, “ $x$  equals 35.”

$$(?) + 17 = 52$$

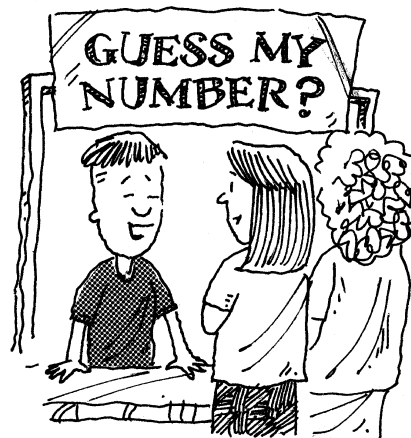
$$x = 35$$

He put 35 back into the equation and discovered he was right, because  $35 + 17 = 52$ .

$$(35) + 17 = 52$$

“That was too easy!” exclaimed Carlin. “We won’t make any money if the problems are this easy. You need to make them harder. Show me the other problems you had in mind.”

Help Carlin check to see if these problems are also too easy. Use Carlin’s cover-up method to solve each problem. Be sure to put your answers back into the original equations to check your work.



a)  $x + 7 = 13$

b)  $x - 6 = 18$

c)  $y + 22 = 39$

d)  $s + 25 = 155$

e)  $r - 12 = 38$

f)  $m - 15 = -60$

g)  $x - 36 = -30$

h)  $x + 40 = -10$

i)  $z + \frac{5}{7} = 3$

**TEACHER NOTE: For the next problems, the students should write three steps and the check. Make sure that they are writing the problems the way they are shown below. Again, demonstrate the steps with the class, showing them how to cover up the unknown, write each step, and check by substitution. Substitution is one part of the mathematical process known as evaluation. Evaluating an equation by substitution is to check that the solution is correct. Create the standard now that the final step for showing work is checking by substitution.**

MC-15. Jacob pulled out his second set of problems. “These are harder. Try this one.”

“Four more than three times the number is 40.”

Carlin pictured the problem like this:  $3x + 4 = 40$

He had to break this problem down into more than one step.

First he covered up the term with the unknown. He asked himself what number plus 4 equals 40. “Oh,” he said, “That’s easy. Thirty-six plus four equals 40, so  $3x$  must equal 36.”

$$\begin{aligned} (?) + 4 &= 40 \\ (36) + 4 &= 40 \\ 3x &= 36 \end{aligned}$$

However, Carlin was not finished. He still had to find out what number the unknown  $x$  represented. So he covered up the  $x$  in  $3x$ , asking himself, “Three multiplied by what number equals 36?” This was not difficult. He knew that 3 times 12 is 36, so  $x$  must equal 12.

$$\begin{aligned} 3 \cdot (?) &= 36 \\ 3 \cdot (12) &= 36 \\ x &= 12 \end{aligned}$$

He checked his answer by substituting 12 for  $x$ . Since this was correct, he knew he had found the correct value for  $x$ .

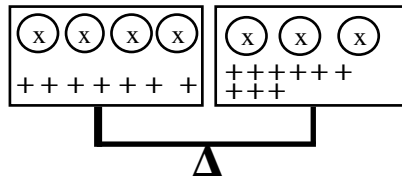
$$\begin{aligned} 3(12) + 4 &= 40 \\ 36 + 4 &= 40 \end{aligned}$$

“These are better,” claimed Carlin. “Let me see the rest of them.” Use Carlin’s cover-up method to check the rest of Jacob’s problems. Write each step.

- |                    |                       |                              |
|--------------------|-----------------------|------------------------------|
| a) $6x - 7 = 35$   | b) $3x + 8 = 23$      | c) $\frac{c}{2} + 20 = 38$   |
| d) $8x + 14 = -10$ | e) $102m + 102 = 306$ | f) $\frac{x}{20} + 17 = 257$ |
| g) $3x - 2 = 13$   | h) $5x + 4 = -26$     | i) $2x + \frac{1}{5} = 1$    |

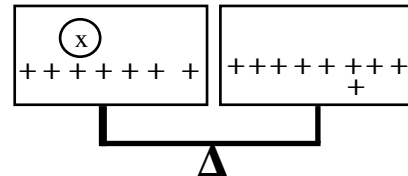
MC-38. You are going to practice a system for solving equations that will be modeled by your teacher.

- a) On the balance scale below, the circles represent cups with an unknown number of positive or negative tiles inside. The cups all have the same number of tiles in them. How many cups can we remove from both sides and still maintain balance? Use the resource page to complete the problem

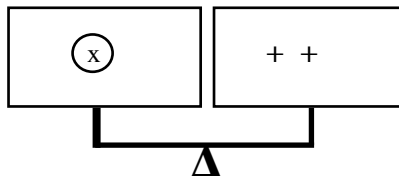


- b) The balance scale now looks like this:

What can we remove from (or add to) both sides to maintain the balance



- c) In part (b) you should have found that the scale balanced as shown below.



This situation can be represented as  $x = 2$ . Is this the solution to the equation represented by the cups and tiles in part (a)?

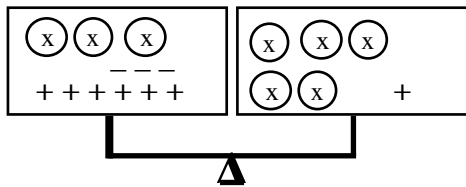
- d) Write an equation that represents the cups and tiles on the balance scale in part (a). Check to see if  $x = 2$  is the solution to the equation in part (a) by replacing the  $x$  with 2 in the original equation.

- MC-39. We want to solve the following algebra problem for  $x$ :  $x + 6 = 5x + 2$ . Use the resource page to complete the problem.
- Draw the cups and tiles on the balance scale, then write the algebraic equation on the line.
  - Simplify the problem on the balance scale to figure out what is in each cup. Then explain your thinking.
  - What is the solution to the equation?
  - The final step is to check your work by substituting your solution into the original equation.

- MC-40. Use the resource page to solve the equation  $2x + 3 = x + 7$ .
- Draw the picture and write the equation.
  - Simplify the problem on the balance scale to figure out what is in each cup, then explain in words what you did.
  - Now you have solved the equation. Check your solution by substituting the value you found into the original equation.

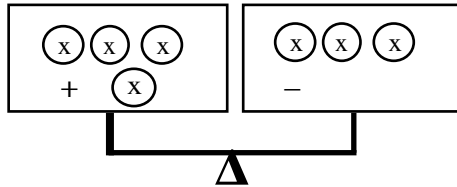
- MC-54. Solve the equation  $2x + 9 = 4x + 5$  using the balance scales on your resource page.
- Show the simplest way to keep the scales balanced.
  - Explain your reasoning in the “think clouds” and record the algebraic equations. Check your solution.

- MC-55. Solve the equation  $3x + 6 - 3 = 5x + 1$  using the balance scales on your resource page.



**TEACHER NOTE:** To solve the next equation, students will either need to add a zero pair to the right to solve by subtraction or add a negative tile to each side creating a zero pair.

MC-56. As you use the resource page to do the problem,  $4x + 1 = 3x - 1$ , remember that adding a zero pair (+ and -) does not change the value of the equation or the balance.



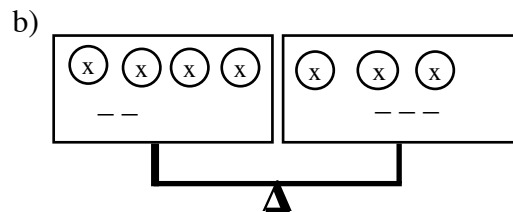
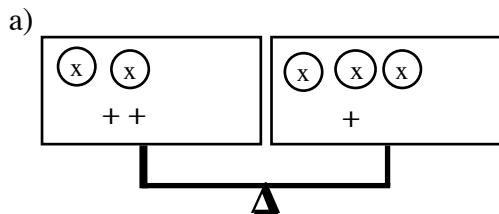
MC-57. Solve each of the equations below, then substitute your solution for  $x$  back into the original equation to be sure that your solution is correct.

a)  $2x - 3 = 4x - 9$

b)  $4x + 6 = 5x - 7$

Remember that you can add an opposite to make zero pairs.

MC-58. Solve each of the following problems. Be sure to show all of the steps in each solution and substitute your solution for  $x$  back into the original equation. Notice that one of the pan diagrams below has positive (+) signs, while the other has negative (-) signs.



MC-59. Carlin and Jacob just finished the last problem.

*Carlin:* Hey, Jacob! I just noticed something. In that last problem you took three  $x$ 's from each balance pan and got  $x - 2 = -3$ . Then all you have to do is use the cover-up method to finish the problem. You can just say  $x$  minus 2 is negative 3, so  $x$  has to be negative 1.

*Jacob:* You're right. Let's try that system for the next problems.

Solve the following equations for  $x$ . Show all steps.

a)  $2x - 2 = 4x - 8$

b)  $5x + 6 = 2x + 24$

c)  $4x - 5 = 2x + 27$

d)  $3x - 7 = 6x - 16$