COLLEGE PREPARATORY MATHEMATICS

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Upstream or Downstream?

Let Parametric Equations Show You How to Paddle Your Kayak

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> **Note to student:** In the next problem you will explore one of the reasons that parametric equations are so powerful—they allow you to work separately with what happens in the horizontal and vertical directions. This depends, of course, on the fact that was just demonstrated: vertical falling velocity is unaffected by horizontal velocity.

PE-3. While sitting on a pole 256 feet above the ground on a windy day, a gull drops an oyster shell. A wind is blowing the shell sideways, but as we've seen from the nickel demonstration, the height is unaffected. Suppose the wind is blowing the shell sideways at 15 mph, which is 22 feet per second, so $x = 22t$. Of course, the shell falls faster and faster as time increases. If we ignore the air resistance as the shell falls, the height y of the shell at time t (seconds) is given by $y = -16t^2 + 256$, where gravity is given by $-16t^2$.

a) Make a table with three columns as shown below for $0 \le t \le 4$. The columns represent the time (in seconds), the x-coordinate of the shell, and the y-coordinate of the shell. We are essentially graphing two equations of t in one table. The second column is filled in by using the equation $x = 22t$. Fill in this column. The third column is filled in by using the equation $y = -16t^2 + 256$. Now fill in the third column.

Time (seconds)	$x = 22t$ x-coordinate (feet)	$y = -16t^2 + 256$ y-coordinate (feet)
0	0	256
0.5	11	252
1	22	240
1.5		

b) Using a graph with the x-axis scaled from 0 to 100, and the y-axis scaled from 0 to 300, plot the position of the shell for times $t = 0, 0.5, 1, 1.5$, etc. up to $t = 4$.

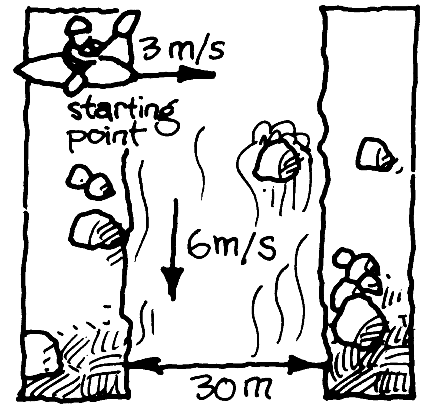
c) What sort of curve do you get?

d) When did the shell hit the ground? How do you know?

e) The parametric equations for the motion of the shell express x and y as functions of t with $x = 22t$ and $y = -16t^2 + 256$. How would the parametric equations be different if the wind were blowing at 10 ft/s? 40 ft/s?

f) On the same graph, but using different colored lines, plot the graph of the motion of the shell if the wind is blowing at 30 mph, if it is blowing at 10 mph, if it is blowing at 5 mph, and if it is not blowing at all. Split up the labor of making the tables in your team. Write a few sentences explaining what you notice.

PE-59. Suppose you are in a kayak and want to cross a river. You can paddle your kayak at 3 m/s. You are trying to cross a river flowing at 6 m/s. You aim your kayak directly across the river from your starting point.



a) You want to know how far down the river you would land. Figure it out, with some help from your team.

b) You also want to know the path of your kayak. To do this, copy the table below onto your paper and fill in the blanks for the 10 seconds it takes you to cross the river.

Time	Distance Across	Distance Upstream
0	0	0
1	3	-6
2		
3		
4		
5		
6		
7		
8		
9		
10		

c) What are the units for t? For x? For y?

d) Find an equation for x in terms of t. Then find another equation for y in terms of t.

PE-60. Now suppose you head upstream at an angle of 40˚.

a) How far did you go horizontally (across the river) in one second

b) How far did you go vertically (down the river) in one second? Remember the current!

c) Find parametric equations for x in terms of t and for y in terms of t.

d) How long will it take you to get across the river if you paddle in this direction?

PE-61. Now we generalize the situation found in the previous problem.

a) You head upstream at an angle θ. Express the time t when the kayak hits the opposite bank in terms of θ?

b) Find the distance downstream that your kayak goes in that time.

c) Now write the downstream distance d as a function of θ.

d) What value of θ gives you the minimum downstream distance?

e) When you graph d as a function of θ, what does the tangent line to the curve look like at this minimum distance?

College Preparatory Mathematics 4
(Mathematical Analysis)

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