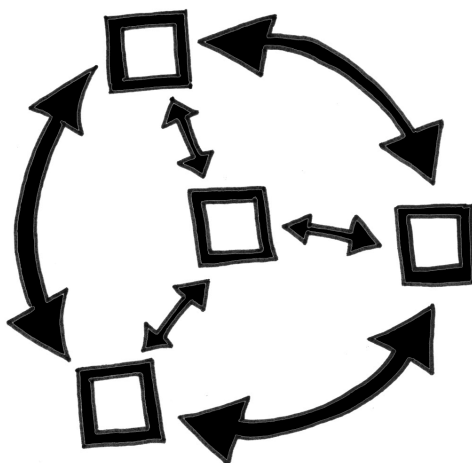


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## Problem Solving Strategies



For more information about the materials you find in this packet, contact:

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## **KWL Strategy**

Referring to the problem solving strategy given to you at your table, write appropriate responses for each of the prompts below.

What I **K**now about my strategy:

What I **W**ant to Know about my strategy:

What I **L**earned about my strategy and the others we explored.

## Using Guess and Check

- PR-2. Read the following problem and try to help Ramon and Kyle. They are weighing in their sheep at the county fair. Ramon notices that his sheep weighs 15 pounds more than Kyle's. Together the two sheep weigh 205 pounds. How much does Kyle's sheep weigh?

Sometimes the easiest way to get started on a word problem is just to guess a possible answer and then check to see how close you are. Guessing is a good strategy if you use the results of each guess to systematically narrow down guesses to reach the correct answer. Use the problem below as a model of how to use a table to organize your guesses and how to use your guesses to find the correct answer.

Step 1: Read the problem again. With some problems it is helpful to sketch a picture.

Step 2: Set up a table. Decide what it is you want to know. In this case, you want to know how much Kyle's sheep weighs. This is what you will be guessing. Put this guess in the first column.

Guess Weight of Kyle's Sheep
------------------------------

Guess the weight of Kyle's sheep. Try guessing 70 pounds. (70 is an easy number with which to work, and it makes sense, since the two sheep together weigh 205 pounds.) Put your guess in the box under the title, "Guess Weight of Kyle's Sheep."

Guess Weight of Kyle's Sheep
70

Step 3: If you know that Kyle's sheep weighs 70 pounds, you can use this number to find how much Ramon's sheep weighs. Since the problem states that it weighs 15 pounds more than Kyle's sheep, calculate the weight of Ramon's sheep, then label the heading for the second column.

Guess Weight of Kyle's Sheep	Weight of Ramon's Sheep
70	$(70) + 15 = 85$

Step 4: Now that you have the possible weights for both Kyle's and Ramon's sheep, you can calculate the total weight and label the third column "Total Weight of Both Sheep."

Guess Weight of Kyle's Sheep	Weight of Ramon's Sheep	Total Weight of Both Sheep
70	$(70) + 15 = 85$	$(70) + (85) = 155$

Step 5: Label and use the last column to check whether the two sheep together weigh 205 pounds. If they do not weigh 205 pounds, write in the box whether the amount is too high or too low. If they do weigh 205 pounds, write “correct.”

Guess Weight of Kyle’s Sheep	Weight of Ramon’s Sheep	Total Weight of Both Sheep	Check 205
70	$(70) + 15 = 85$	$(70) + (85) = 155$	too low

Step 6: Start over with a new guess and use the same columns. Since the last guess was too low, Kyle’s sheep must weigh more than 70 pounds. Try guessing 75 pounds and complete the second row of the table.

Guess Weight of Kyle’s Sheep	Weight of Ramon’s Sheep	Total Weight of Both Sheep	Check 205
70	$(70) + 15 = 85$	$(70) + (85) = 155$	too low
75	$(75) + 15 = 90$	$(75) + (90) = 165$	too low

Step 7: Seventy-five pounds turned out to be a little closer, but since it produced a total weight of 165 pounds, it is still far too low. For the next guess, choose a significantly greater weight: 100 pounds. Complete the third row.

Guess Weight of Kyle’s Sheep	Weight of Ramon’s Sheep	Total Weight of Both Sheep	Check 205
70	$(70) + 15 = 85$	$(70) + (85) = 155$	too low
75	$(75) + 15 = 90$	$(75) + (90) = 165$	too low
100	$(100) + 15 = 115$	$(100) + (115) = 215$	too high

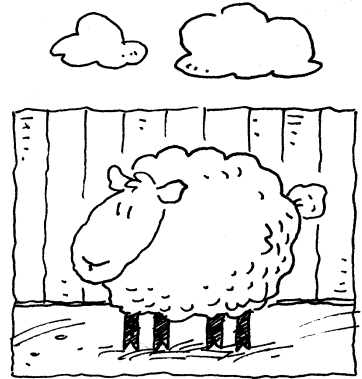
Step 8: One hundred pounds is too high, but now we know that the answer is between 75 and 100 pounds and that 100 pounds is closer to the correct total weight. Try 95 pounds.

Guess Weight of Kyle’s Sheep	Weight of Ramon’s Sheep	Total Weight of Both Sheep	Check 205
70	$(70) + 15 = 85$	$(70) + (85) = 155$	too low
75	$(75) + 15 = 90$	$(75) + (90) = 165$	too low
100	$(100) + 15 = 115$	$(100) + (115) = 215$	too high
95	$(95) + 15 = 110$	$(95) + (110) = 205$	correct

Step 9: Congratulations! You have found the correct answer. Now you have only one thing left to do. Write a complete sentence that answers the question asked in the problem: Kyle’s sheep weighs 95 pounds.

PR-59.

Sohan built a rectangular pen for his sheep with an area of 242 square feet. He only had 66 feet of fencing to use. He wanted the length to be twice as long as the width. What were the dimensions of the pen? Draw a picture of the pen and label the length and width, then fill in the table and solve the problem using Guess and Check.



Guess Width of the Pen	Length of Pen	Area of Pen	Check _____

PR-3. I am thinking of a number. If you add 3 to the number and then multiply the sum by 4, you get 48. What is my number? Solve by copying and continuing the Guess and Check table below and continuing the process to solve the problem.

Guess the Number	Add 3 to the Number	Multiply Sum by 4	Check 48
5	$(5) + 3 = 8$	$4 \cdot (8) = 32$	too low

PR-4. Copy and complete the Guess and Check table below to solve the following problem. Remember to write the answer in a sentence.

I am thinking of a number. First multiply it by 5, then subtract 8. The result is 77. What is my number?

PR-5.



### SOLVING PROBLEMS WITH GUESS AND CHECK TABLES

Step 1: Read the problem carefully. Make notes or sketch a picture to organize the information in the problem.

Step 2: Look at the question being asked. Decide what you are going to guess. Set up a table. Leave extra space for more columns in case you need them.

Step 3: Calculate the entry for a column and label the column.

Step 4: Continue the table until the check is correct.

Step 5: Write the answer in a complete sentence.

Example:

1. Kaitlin went to the fair. It costs \$5 to get in. Tickets for the rides cost \$1.25 each. How many rides did she go on if she spent a total of \$26.25?

2.	Guess Number of Rides	Money Spent on Rides	Total Money Spent at Fair	Check \$26.25
3.	10	$(10) \cdot 1.25 = \$12.50$	$(\$12.50) + \$5 = \$17.50$	too low
4.	20	$(20) \cdot 1.25 = \$25.00$	$(\$25.00) + \$5 = \$30.00$	too high
	15	$(15) \cdot 1.25 = \$18.75$	$(\$18.75) + \$5 = \$23.75$	too low
	17	$(17) \cdot 1.25 = \$21.25$	$(\$21.25) + \$5 = \$26.25$	correct

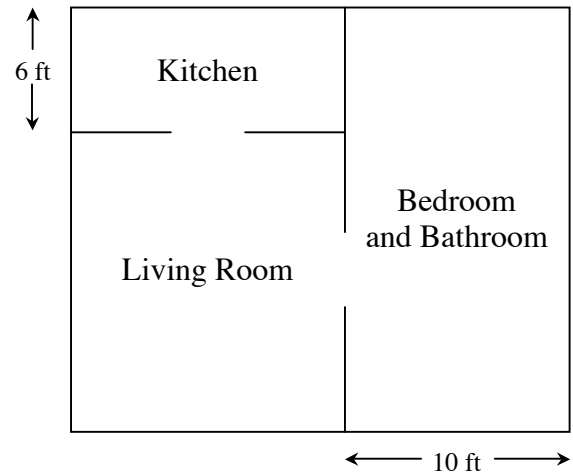
5. Kaitlin went on 17 rides and her total cost at the fair was \$26.25.

1-40. THE APARTMENT

Your architecture firm has been hired to design an apartment building. Each of the apartments in the building will be laid out as shown at right so that each room is rectangular.

The building's owners have given you the following specifications.

- The living room of an apartment must have an area of 180 square feet.
- The shorter side of the kitchen must be 6 feet to make room for counter space.
- The shorter width of the bedroom and bath must be 10 feet so that a dresser and king-sized bed will fit.
- The entire area of each apartment must be exactly 450 square feet.



**Your Task:** Find the possible dimensions for every room in the apartment. Be prepared to **justify** your answer (show how you know it works), and show all of your work in a way that someone who is not in your team can read and understand it.

### *Discussion Points*

How can you start?

How can you organize your work?

How can you use the results from one guess to make your next guess better?

## *Further Guidance*

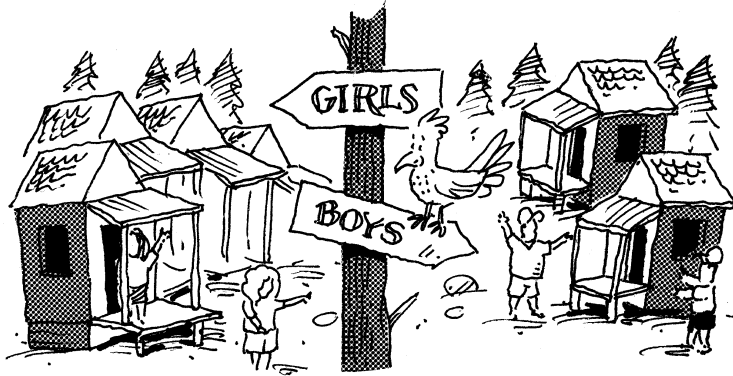
- 1-41. Charles decided to start this problem by making a guess. He guessed that everything would come out right if one side of the living room were 10 feet.
- a. Does it matter which side of the living room is 10 feet long? Why or why not? Find the area of the entire apartment twice: once if the base of the living room is 10 feet long, and again if the height of the living room is 10 feet long. Do the results come out the same?
  - b. Is Charles' guess correct? That is, can one of the sides of the living room be 10 feet long? Explain.
  - c. As you checked Charles' guess, did you organize your work so that anyone could read and understand your thinking? If not, try to find a way to reorganize your work to make it clear.
  - d. As a team, try another guess for a dimension of the living room. Organize your work to check if that guess is correct.
- 1-42. One way to organize your work in this kind of problem is by using a table. The table can be structured something like this:

Length of _____	?	Total Area

- a. Use this method to check the rest of your guesses for this problem.
- b. Continue guessing and checking until you find correct room dimensions. Once you have an answer, organize your work so that it shows the dimensions of all of the rooms and can be read and understood by someone who is not in your team. Be ready to present to the class both your final answer and the different guesses that you tried along the way.

## Writing Equations

- GC-80. The number of girls at camp was six more than twice the number of boys, and altogether there were 156 campers.



- Use a Guess and Check table to find the number of boys and the number of girls at camp. Write your answer in a complete sentence.
  - Use your Guess and Check table and a variable to write an equation.
- GC-81. Johanna is planting tomatoes in the camp garden this year. At the nursery, tomato plants come in packs of six. She needs 80 plants in the garden and already has 26. How many packs of plants will she need?
- Use a Guess and Check table to solve this problem. Remember to write your answer as a complete sentence.
  - Use your Guess and Check table to write an equation for the problem.

- GC-82. Guess Jeremiah's favorite number. He tells the campers, "If you multiply my number by 28 and then subtract 8, you end up with 160."

- Use a Guess and Check table to solve for his number. Be sure to write your answer as a sentence.
- Use your Guess and Check table to write an equation that represents this situation.





### WRITING EQUATIONS FROM A GUESS AND CHECK TABLE

- Make a Guess and Check table and solve the problem. Use at least four guesses, show all of your work, and write your answer in a complete sentence.
- Put an  $x$  in your Guess column and use  $x$  to write expressions in each of the other columns.
- Write your equation outside the Guess and Check table. You may need to use the Distributive Property and/or combine like terms.

Example: Noel and his sister, Brigitte, are both at camp. Noel is 3 years older than Brigitte, and the sum of their ages is 21. How old is Noel and how old is Brigitte? Use a Guess and Check table to solve for their ages. Write an equation.

Guess Brigitte's Age	Noel's Age	Sum of Noel and Brigitte's Ages	Check 21
10	$(10) + 3 = 13$	$(10) + (13) = 23$	too high
5	$(5) + 3 = 8$	$(5) + (8) = 13$	too low
8	$(8) + 3 = 11$	$(8) + (11) = 19$	too low
9	$(9) + 3 = 12$	$(9) + (12) = 21$	correct
$x$	$x + 3$	$(x) + (x + 3) = 21$	

$$(x) + (x + 3) = 2x + 3 = 21$$

Noel is 12 years old and Brigitte is nine years old.

### Using Algebra Puzzles

- AR-4. **Algebra Puzzles** Algebra puzzles give you a new way to apply the number skills you have already learned. Here is the first one.

Decide which number belongs in the blank to make the equation true. If you cannot find the answer just by looking, try several different numbers until you find the answer. Notice the dot in each part of this problem. The dot means that you multiply the first number by the number that goes in the blank that follows it. You have used an  $x$  for multiplication in the past. You will no longer do so. For now you should use the dot or parentheses for multiplication. If you continue to use  $x$  for multiplication, you could confuse it with the letters used in algebra.

a)  $3 \cdot \underline{\quad} + 2 = 14$

b)  $4 \cdot \underline{\quad} + 1 = 21$

c)  $6 \cdot \underline{\quad} + 7 = 25$

d)  $5 \cdot \underline{\quad} - 3 = 42$

AR-48. **Algebra Puzzles** Decide which number belongs in the blank to make the equation true. If you cannot find the answer just by looking, try several different numbers. Work with your partner using the “pairs check” strategy outlined below.

**Pairs Check:**

- i) Play a game of Rock-Paper-Scissors with your partner to decide who does the first problem.
- ii) Whoever goes first explains his or her thinking while working. The other partner listens, asks questions, and helps (when asked).
- iii) Then the paper and pencil go to the other partner who does part (b).
- iv) After two problems are done, each partner checks the other’s work.
- v) Then take turns on parts (c) and (d) and check them.

a)  $6 \cdot \underline{\quad} - 1 = 71$

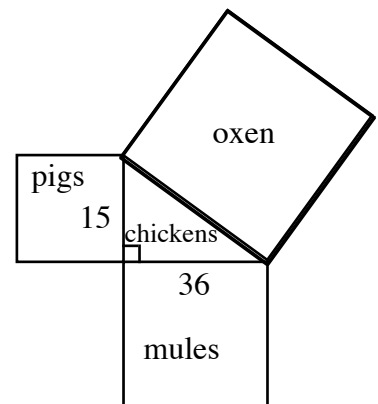
b)  $6 \cdot \underline{\quad} - 4 = 20$

c)  $7 \cdot \underline{\quad} + 2 = 44$

d)  $7 \cdot \underline{\quad} - 3 = 53$

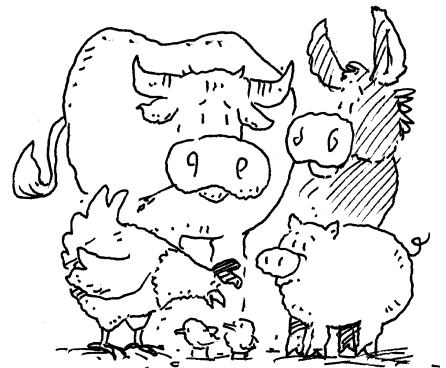
**Using Subproblems**

GS-42. The Claude-Hopper family has decided to build pens for their animals. To keep the different animals separated from each other, they need to build four pens. The pigs are to be kept in a square pen with a side length of 15 feet. The mule pen will also be square and will measure 36 feet on each side. The third square pen is for the oxen. In the middle is a right triangular pen for the chickens. Find the total area of the four pens.



- a) Copy and label the drawing of each animal pen.
- b) The calculation of the area of each pen is a **subproblem** that helps answer the question. Solve each of the five subproblems.

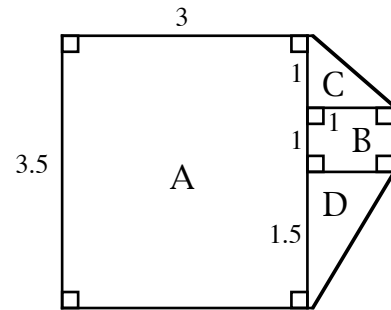
- i) What is the area of pig pen?
- ii) What is the area of the mule pen?
- iii) What is the hypotenuse of the chicken pen?
- iv) What is the area of the chicken pen?
- v) What is the area of the oxen pen?



- c) Use your answers from part (b) to find the total area of the farm pens.

- GS-43. Use subproblems to solve the problem below. Copy and label the drawing. List the subproblems, then solve each one.

Regal Ranch has four agricultural sectors:  
Sectors A and B are rectangles; sectors C and D  
are right triangles. Measurements are shown in  
miles.



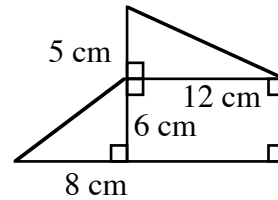
- a) What is the total area of the ranch in square miles? Show the subproblems you use.
- b) One square mile is equal to 640 acres. How many acres is Regal Ranch?

GS-45.

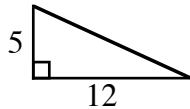


Breaking a large or complex problem into smaller parts is a problem solving strategy called **SUBPROBLEMS**. This strategy means to solve each part and put the smaller parts back together to answer the original question.

Example: Find the area of this figure.



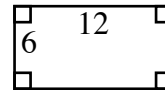
Subproblem 1:



$$\frac{1}{2} \cdot 12 \cdot 5 = 30$$

area of first triangle =  $30 \text{ cm}^2$

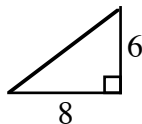
Subproblem 2:



$$6 \cdot 12 = 72$$

area of rectangle =  $72 \text{ cm}^2$

Subproblem 3:



$$6 \cdot 8 = 48$$

$$48 \div 2 = 24$$

area of second triangle =  $24 \text{ cm}^2$

Subproblem 4:

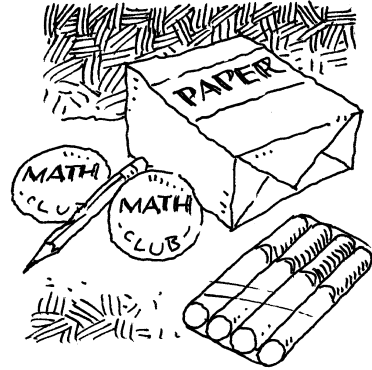
$$30 + 72 + 24 = 126$$

Sum of the areas =  $126 \text{ cm}^2$

## Drawing a Diagram

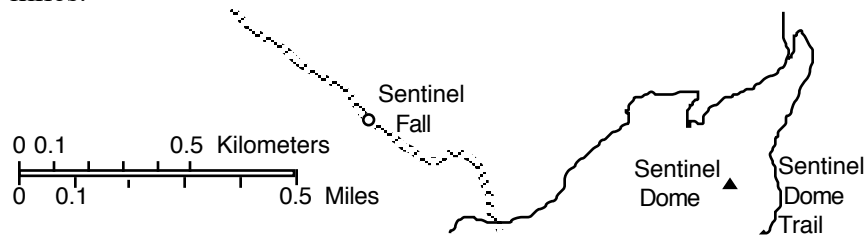
GS-44. Clem and Clyde set up a tent at the base of Lookout Peak. They pounded a stake in the ground five feet from the center pole so they could attach a rope to it. If the rope they attached to the very top of the center pole was 13 feet long, how tall was the center pole? Draw and label a diagram. Show all your work as you solve this problem

MC-43. Diedre did some trading and learned that two math club buttons plus a pencil would get her one set of markers. She also knew that one set of markers, two math club buttons, and a pencil would get her two reams of paper to make fliers announcing the t-shirt sale. She only wanted one ream of paper and had lots of markers. What did she have to trade to get one ream of paper? Use pictures or symbols to show your reasoning.



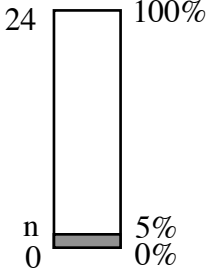
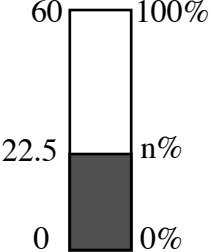
## Using Proportions

MB-82. Use your ruler to verify that for the scale shown on the map,  $1 \frac{3}{8}$  inches represents 0.5 miles.



- What is the distance from Sentinel Fall to Sentinel Dome in inches?
- Convert your answer from part (a) into a decimal.
- Write and solve a proportion to calculate the distance in miles from Sentinel Fall to Sentinel Dome.

MB-101. Here is a graphical look at proportions and percents. Copy and complete the table.

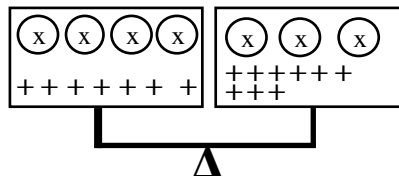
Picture	Problem in Words / Proportion	Answer
	<p>What is 5% of 24?</p> <p>_____ = _____</p>	
	<p>State the question in words.</p> <p>_____ = _____</p>	

MB-103. Julie scored 136 out of 200 on her midterm. Write and solve a proportion that will give you Julie's percentage.

## Using Manipulatives

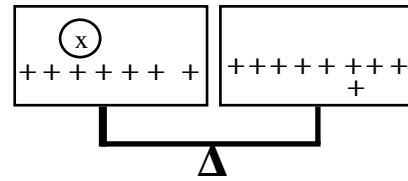
MC-38. You are going to practice a system for solving equations that will be modeled by your teacher.

- a) On the balance scale below, the circles represent cups with an unknown number of positive or negative tiles inside. The cups all have the same number of tiles in them. How many cups can we remove from both sides and still maintain balance? Use the resource page to complete the problem.

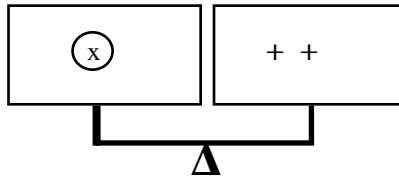


- b) The balance scale now looks like this:

What can we remove from (or add to) both sides to maintain the balance



- c) In part (b) you should have found that the scale balanced as shown below.



This situation can be represented as  $x = 2$ . Is this the solution to the equation represented by the cups and tiles in part (a)?

- d) Write an equation that represents the cups and tiles on the balance scale in part (a). Check to see if  $x = 2$  is the solution to the equation in part (a) by replacing the  $x$  with 2 in the original equation.

MC-39. We want to solve the following algebra problem for  $x$ :  $x + 6 = 5x + 2$ . Use the resource page to complete the problem.

- Draw the cups and tiles on the balance scale, then write the algebraic equation on the line.
- Simplify the problem on the balance scale to figure out what is in each cup. Then explain your thinking.
- What is the solution to the equation?
- The final step is to check your work by substituting your solution into the original equation.

MC-40. Use the resource page to solve the equation  $2x + 3 = x + 7$ .

- Draw the picture and write the equation.
- Simplify the problem on the balance scale to figure out what is in each cup, then explain in words what you did.
- Now you have solved the equation. Check your solution by substituting the value you found into the original equation.

MC-41. Use the resource page to solve the equation  $x + 14 = 5x + 2$ .

- Draw the picture and write the equation.
- Simplify the problem on the balance scale to figure out what is in each cup. Explain in words what you did.
- Now you have solved the equation. Check your solution by substituting the value you found into the original equation.

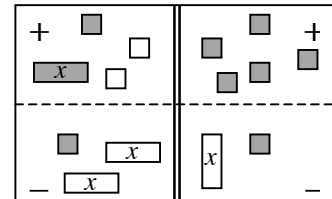
MC-42. Solve the following problems. Use the resource page to show your subproblems as well as how you checked your solution.

- $4x + 3 = 3x + 5$
- $5x + 8 = 3x + 12$
- $3x + 12 = 6x + 9$

## 2-75. SOLVING FOR $X$

Later in the course, you will learn more about situations like parts (a) and (b) in the preceding problem, called “inequalities.” For now, assume that the left expression and the right expression are equal in order to learn more about  $x$ . The two expressions will be brought together on one mat to create an **equation mat**, as shown in the figure below. The double line down the center of an equation mat represents the word “equals.” It is a wall that separates the left side of an equation from the right side.

- Obtain the “Equation Mat” resource page from your teacher. Build the equation represented by the equation mat at right using algebra tiles. Simplify as much as possible and then solve for  $x$ . Be sure to record your work.



- Build the equation  $2x - 5 = -1 + 5x + 2$  using your tiles by placing  $2x - 5$  on the left side and  $-1 + 5x + 2$  on the right side. Then use your simplification skills to simplify this equation as much as possible so that  $x$  is alone on one side of the equation. Use the fact that both sides are equal to solve for  $x$ . Record your work.

2-76. Now **apply** this new solving skill by building, simplifying, and solving each equation below for  $x$ . Record your work.

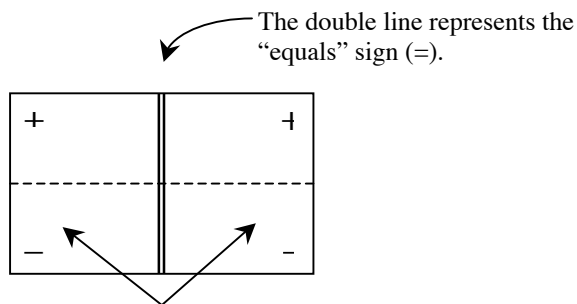
- $3x - 7 = 2$
- $1 + 2x - x = x - 5 + x$
- $3 - 2x = 2x - 5$
- $3 + 2x - (x + 1) = 3x - 6$
- $-(x + 3 - x) = 2x - 7$
- $-4 + 2x + 2 = x + 1 + x$



# METHODS AND MEANINGS

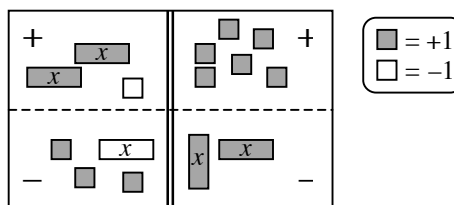
## Using an Equation Mat

An **equation mat** can help you visually represent an equation with algebra tiles.



For each side of the equation, there is a positive and a negative region.

For example, the equation  $2x - 1 - (-x + 3) = 6 - 2x$  can be represented by the equation mat at right. (Note that there are other possible ways to represent this equation correctly on the equation mat.)



## Using a Table to Write a Rule

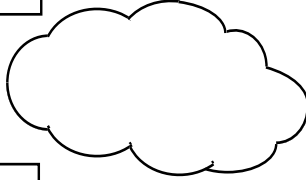
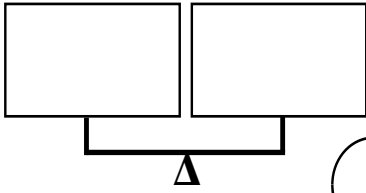
3-99. MR. WALLIS IS BACK! After much consideration, Mr. Wallis decided to use the tip table below to help him estimate what a 15% tip would be for various costs of dinner.

<b>Cost of Dinner</b>	\$10	\$20	\$30	\$35	\$40	\$45	\$50	\$100
<b>Amount of Tip</b>	\$1.50	\$3	\$4.50	\$5.25	\$6	\$6.75	\$7.50	\$15

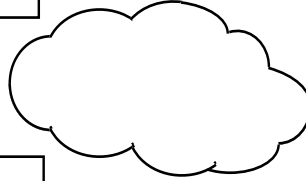
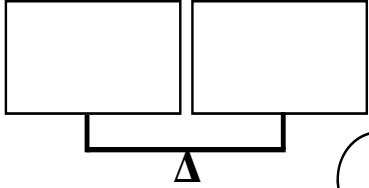
- Find a rule for his table. That is, find a rule that calculates the amount of tip ( $y$ ) based on the cost of the dinner ( $x$ ). How did you find your rule?
- During the date, Mr. Wallis was so distracted that he forgot to write down the cost of the meal in his checkbook. All he remembers is that he left a \$9 tip. What was the original cost of the meal before he paid the tip? Use your equation from part (a) to answer this question. Show all work.
- What was the total cost of the meal?

Name \_\_\_\_\_

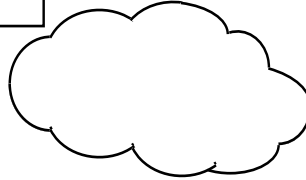
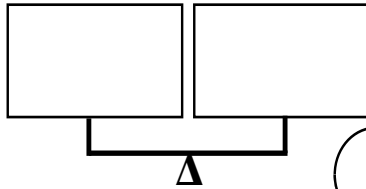
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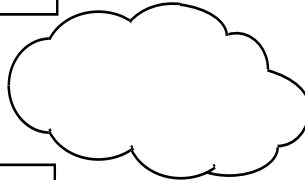
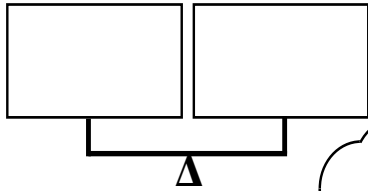


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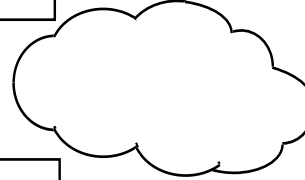
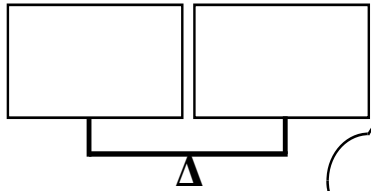


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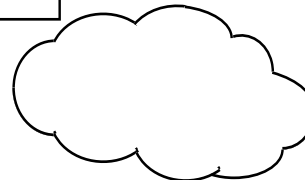
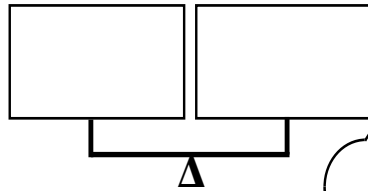
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