Introduction and Overview

Geometry Connections

Geometry Connections is the second in a five-year sequence of college preparatory mathematics courses. It emphasizes several big ideas in an integrated algebra/geometry context. The key concepts addressed in this course are:

- Transformations (reflection, rotation, translation, dilation) and symmetry
- Relationships between figures (such as similarity and congruence)
- Properties of plane figures (such as equal or perpendicular sides or diagonals)
- Measurements of plane figures (such as area, perimeter, and angle measure)
- Measurements of three-dimensional shapes (such as volume and surface area)
- Tools for analyzing and measuring shapes (such as the Pythagorean Theorem, trigonometric ratios, the Laws of Sines and Cosines, and coordinate geometry)
- Investigation and proof (having found patterns, students conjecture and prove)
- Geometric construction (with compass and straightedge)
- Algebra (with substantial review of writing and solving equations and graphing)
- Probability

The course is structured around problems and investigations that build spatial visualization skills, conceptual understanding of geometry topics, and an awareness of connections between different ideas. Students are encouraged to investigate, conjecture, and then prove to develop their reasoning skills.

Lessons are structured for students to collaborate actively by working in study teams. During class time, students work in study teams on challenging problems that introduce new material. In several circumstances, an investigation or challenge will be presented with a Task Statement and Further Guidance structure. These activities are designed to provide teachers with the freedom of deciding how structured or open to leave a mathematical challenge.

The homework in the "Review & Preview" section of each lesson reinforces previously-learned skills and concepts and prepares students for new ones. The homework problems also allow students to apply previously-learned concepts and skills in new contexts and deepen their understanding by solving the same type of problem in different ways. CPM offers homework support through Hotmath (www.hotmath.com) and also provides teachers with the answers to problems.

Geometry Goals for Students

Upon completing this geometry course, students should be able to:

- pose mathematical questions, such as "What if…?," meaningfully and appropriately.
- make conjectures and test their validity.
- recognize and represent patterns mathematically or in prose.
- appreciate geometry as a connected, systematic branch of mathematics.
- apply geometry to solve problems in both mathematical and real-world contexts.
- critique a logical argument.
- communicate their mathematical understanding effectively and formulate complete, logical arguments to support their conclusions.
- use algebra to formulate and solve equations arising from geometric situations both on and off a coordinate grid.
- exhibit creativity and perseverance in mathematical problem solving, with the ability to determine when an approach is not working and a new direction is needed.

Course Outline of Major Content Strands

This course contains several content threads that extend through multiple chapters and help to highlight connections between ideas. Chapter 1 begins with five big problems that motivate students and anticipate major themes of the course: shape, transformations, measurement, patterns and reasoning, and symmetry. Chapter 2 then focuses on measurement of and relationships between parts of shapes. The chapter addresses angle relationships, area and perimeter, and the Pythagorean Theorem.

Chapter 3 introduces students to the concept of similarity and the conditions that allow us to show that triangles are similar. At this point, congruence is considered as a special case of similarity. In Chapter 6, students focus on congruence more directly, discovering the conditions that show triangles are congruent. Ratios between the perimeters and areas of similar figures are investigated in Chapter 8, and the concept of similarity is extended to three-dimensional figures in Chapter 9.

Triangle similarity also initiates the development of the tangent ratio in Chapter 4. This development is extended to sine and cosine ratios in Chapter 5, followed by the Laws of Sines and Cosines. Students apply these powerful tools to analyze triangles and quadrilaterals in Chapters 6 and 7.

Throughout this development, the book returns to perimeter and area frequently, then expands to volume in Chapter 9. Following an introduction in Chapter 1, area algorithms for basic shapes are developed in Chapter 2, along with the Pythagorean Theorem as a tool for finding perimeter. The triangle tools, developed by students in Chapters 4 and 5, are applied to calculations of perimeter and area. In Chapter 8, students find perimeters and areas of regular polygons and circles. The surface area and volume of prisms and pyramids are investigated in Chapter 9, followed by cylinders, cones and spheres in Chapter 11.

Probability is first introduced in Chapter 1, in the context of the various shapes students are becoming familiar with. Chapter 4 introduces multiple models for analyzing probabilistic situations, and gives students practice choosing and applying an appropriate model. This chapter also addresses the distinction between theoretical and experimental probability. Chapter 10 then picks up the probability thread with an introduction to expected value.

The concepts of Pattern and Reasoning have long been considered the heart of a geometry course. This course emphasizes a three-step procedure of investigating, conjecturing, then proving. The investigations are often quite open-ended, giving students space to be creative and therefore more motivated. This approach requires the teacher to channel a wide variety of student ideas appropriately, moving them toward the formulation of substantive conjectures.

Because proof is such an important mathematical concept, this thread is developed carefully and slowly. The opening problems of Chapter 1 have students looking for patters and considering what makes reasoning convincing. Students begin to use conditional statements in Chapter 2, then are introduced to flowcharts as a method of organizing their reasoning in Chapter 3. Chapter 4 discusses counter-examples, while converses of conditionals are addressed in Chapter 6. While students have investigated and conjectured throughout the course, Chapter 7 focuses the lengthiest and most explicit attention on proof. Students prove properties of quadrilaterals and are introduced to the two-column proof format. By the time they reach Chapter 7, students have had ample practice with the foundational elements of proof and are able to adapt to the new format quickly. Chapter 9 then applies students' proof skills in the context of geometric constructions.

To keep algebra skills fresh in students' minds, algebra is reviewed continuously. Systems of equations, quadratic equations, and other algebra topics are threaded throughout the course. New geometry tools and concepts are also applied in a coordinate geometry context. In Chapter 7, students are given quadrilaterals on a coordinate grid and asked to analyze these shapes using both their algebraic skills (finding slopes, equations of lines, and intersection points) and their knowledge of geometry. Sections on equations of circles (Chapter 10) and conic sections (Chapter 12) continue to develop connections between algebra and geometry.

The Course Structure

Chapters are divided into sections that are organized around core topics. Within each section, lessons include activities, challenging problems, investigations, and practice problems. Teacher notes for each lesson include a "suggested lesson activity" section with ideas for lesson *introduction*, specific tips and strategies for lesson *implementation* to clearly convey core ideas, and a means for bringing the lesson to *closure*.

Core ideas are synthesized in "Math Notes" boxes for students who are absent or who want additional review. These notes are placed in a purposeful fashion, often falling several lessons after the initial introduction of a concept. This allows students time to explore and build deeper understanding of an idea before they are presented with a formal definition or an algorithm. "Math Notes" include specific vocabulary definitions and instructions about notation, and occasionally interesting extensions or real-world applications of mathematical concepts.

Technology is integrated throughout the course to allow students to see and explore concepts in a dynamic way after they have developed some initial conceptual understanding. The course assumes that classes have access to at least one of these three technology setups: a full computer lab with computers for each student, a classroom computer equipped with projection technology, or Cabri Jr. software on a graphing calculator (such as a TI-83+). These dynamic investigations allow students to test a conjecture on a wide variety of cases. The ability to view many examples quickly and easily allows students to make conjectures more quickly and have greater confidence in the conjectures they make.

Learning Log reflections appear periodically at the end of lessons to allow students to synthesize what they know and identify areas that need additional explanation.