

# *Algebra*

*and*

# *Functions*

**Order of Operations**

**Combining Like Terms**

**Rectangular Model of Multiplication & the Distributive Property**

**Solving Word Problems Using Guess and Check Tables**

**Solving Equations**

**Graphing**

**Linear Functions**

**Inequalities**

## ORDER OF OPERATIONS

Students were given problems in class like  $3 + 4 \cdot 2$ . Some students thought the answer was 14 and some thought the answer was 11. There needs to be a method to simplify an expression which involves more than one operation so that everyone can agree on the answer.

There is a set of rules to follow that provides a consistent way for everyone to evaluate expressions. These rules, called the order of operations, must be followed in order to arrive at a correct solution. As the name says, these rules tell the order in which the mathematical operations are done.

For additional information, see Year 1, Chapter 3, problems PR-36 through 40 and PR-45 or Year 2, Chapter 4, problems GC-2, 3, 5, and 6.

The first step is to organize the expression into parts called TERMS. Terms are separated by addition (+) or subtraction (-) symbols unless the addition (+) or subtraction (-) happens inside parentheses.

Examples of numerical terms are: 4,  $3(6)$ ,  $6(9 - 4)$ ,  $2 \cdot 3^2$ ,  $3(5 + 2^3)$ , and  $\frac{16 - 4}{6}$ .

For the problem above,  $3 + 4 \cdot 2$ , the terms are circled at right.

$$\textcircled{3} + \textcircled{4 \cdot 2}$$

Each term is simplified separately, giving  $3 + 8$ . Then the terms are added:  $3 + 8 = 11$ . Thus,  $3 + 4 \cdot 2 = 11$ .

### Example 1

$$2 \cdot 3^2 + 3(6 - 3) + 10$$

- Circle the terms.
- Simplify each term until it is one number.
  - The operations inside the parentheses are done first.
  - Exponents are a form of multiplication.
  - Multiplication and division are done from left to right.
- Finally, add or subtract the terms going from left to right.

$$\textcircled{2 \cdot 3^2} + \textcircled{3(6 - 3)} + 10$$

$$\textcircled{2 \cdot 3^2} + \textcircled{3(3)} + 10$$

$$\textcircled{2 \cdot 9} + \textcircled{3(3)} + 10$$

$$\textcircled{18} + \textcircled{9} + 10$$

$$27 + 10$$

$$37$$

### Example 2

- Circle the terms.
- Simplify inside the parentheses.
- Simplify the exponents.
- Multiply and divide from left to right.
- Finally, add and subtract from left to right

$$\begin{aligned} & 5 - 8 \div 2^2 + 6(5 + 4) - 5 \\ \textcircled{5} - \textcircled{8 \div 2^2} + \textcircled{6(5 + 4)} - \textcircled{5^2} \\ \textcircled{5} - \textcircled{8 \div 2^2} + \textcircled{6(9)} - \textcircled{5^2} \\ \textcircled{5} - \textcircled{8 \div 4} + \textcircled{6(9)} - 25 \\ \textcircled{5} - \textcircled{2} + \textcircled{54} - \textcircled{25} \\ & 3 + 54 - 25 \\ & 57 - 25 \\ & 32 \end{aligned}$$

### Example 3

- Circle the terms.
- Multiply and divide left to right, including exponents.
- Add or subtract from left to right.

$$\begin{aligned} & 20 + \frac{5+7}{3} - 4^2 + 12 \div 4 \\ \textcircled{20} + \textcircled{\frac{5+7}{3}} - \textcircled{4^2} + \textcircled{12 \div 4} \\ \textcircled{20} + \textcircled{\frac{5+7}{3} = \frac{12}{3} = 4} - \textcircled{16} + \textcircled{3} \\ & 24 - 16 + 3 \\ & 8 + 3 \\ & 11 \end{aligned}$$

### Problems

Circle the terms, then simplify the expression.

- $5 \cdot 3 + 4$
- $10 \div 5 + 3$
- $2(9 - 4) \cdot 7$
- $6(7 + 3) + 8 \div 2$
- $15 \div 3 + 7(8 + 1) - 6$
- $\frac{9}{3} + 5 \cdot 3^2 - 2(14 - 5)$
- $\frac{20}{6+4} + 7 \cdot 2 \div 2$
- $\frac{5+30}{7} + 6^2 - 18 \div 9$
- $2^3 + 8 - 16 \div 8 \cdot 2$
- $25 - 5^2 + 9 - 3^2$
- $5(17 - 7) + 4 \cdot 3 - 8$
- $(5 - 2)^2 + (9 + 1)^2$
- $4^2 + 9(2) \div 6 + (6 - 1)^2$
- $\frac{18}{3^2} + \frac{5}{5}^3$
- $3(7 - 2)^2 + 8 \div 4 - 6 \cdot 5$
- $14 \div 2 + 6 \cdot 8 \div 2 - (9 - 3)^2$
- $\frac{27}{3} + 18 - 9 \div 3 - (3 + 4)^2$
- $26 \cdot 2 \div 4 - (6 + 4)^2 + 3(5 - 2)^3$
- $\left(\frac{42 + 3}{5}\right)^2 + 3^2 - (5 \cdot 2)^2$



## Answers

- |        |         |        |         |        |
|--------|---------|--------|---------|--------|
| 1. 19  | 2. 5    | 3. 70  | 4. 64   | 5. 62  |
| 6. 30  | 7. 9    | 8. 39  | 9. 12   | 10. 0  |
| 11. 54 | 12. 109 | 13. 44 | 14. 5   | 15. 47 |
| 16. -5 | 17. -25 | 18. -6 | 19. -10 |        |

## COMBINING LIKE TERMS

The order of operations is used to simplify or evaluate a numerical expression. Algebraic expressions can also be simplified by combining (adding or subtracting) terms that have the same variable(s) into one quantity. Note that "same variable" means that the variable and its exponent are the same. The skill of combining like terms is necessary for solving equations. For additional information, see Year 1, Chapter 8, problems MC-70 through 73 or Year 2, Chapter 4, problems GC-42 and 43.

### Example 1

Combine like terms to simplify the expression  $3x + 5x + 7x$ .

All these terms have  $x$  as the variable, so they are combined into one term,  $15x$ .

### Example 2

Simplify the expression  $3x + 12 + 7x + 5$ .

The terms with  $x$  can be combined. The terms without variables (the constants) can also be combined.

$$\begin{aligned}3x + 12 + 7x + 5 \\3x + 7x + 12 + 5 \\10x + 17\end{aligned}$$

Note that in the simplified form the term with the variable is listed before the constant term.

### Example 3

Simplify the expression  $5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$ .


$$\begin{aligned}5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1 \\4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1 \\6x^2 + 8x + 3\end{aligned}$$


Note that terms with different exponents are not combined and are listed in order of decreasing power of the variable, in simplified form, with the constant term last.

### Example 4







Algebra tiles are used in Year 2 to model how to combine like terms.

The large square  represents  $x^2$ .

The rectangle  represents  $x$ .

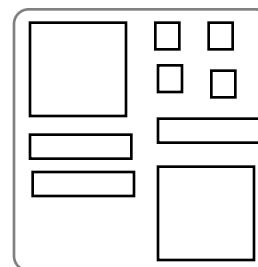
The small square  represents one.

We can only combine tiles that are alike:

 with   with  and  with 

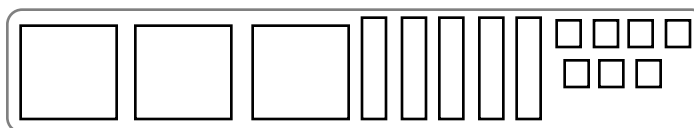
If we use the two sets of algebra tiles at right and write an algebraic expression for each of them, we get  $2x^2 + 3x + 4$  and  $3x^2 + 5x + 7$ . Use the tiles to help combine the like terms:

$2x^2$  (2 large squares) +  $3x$  (3 rectangles) +  $4$  (4 small squares)  
 +  $3x^2$  (3 large squares) +  $5x$  (5 rectangles) +  $7$  (7 small squares)



The combination of the two sets of tiles, written algebraically, is:

$$5x^2 + 8x + 11$$



### Example 5

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite like terms in vertical columns before you combine them:

$$(2x^2 - 5x + 6) + (3x^2 + 4x - 9)$$

$$\textcircled{2x^2} \textcircled{-5x} \textcircled{+6} + \textcircled{3x^2} \textcircled{+4x} \textcircled{-9}$$

$$\begin{array}{r} 2x^2 - 5x + 6 \\ + 3x^2 + 4x - 9 \\ \hline 5x^2 - x - 3 \end{array}$$

This procedure may make it easier to identify the terms as well as the sign of each term.

## Problems

Combine the following sets of terms.

- $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$
- $(3x^2 + x + 4) + (x^2 + 4x + 7)$
- $(8x^2 + 3) + (4x^2 + 5x + 4)$
- $(4x^2 + 6x + 5) - (3x^2 + 2x + 4)$
- $(4x^2 - 7x + 3) + (2x^2 - 2x - 5)$
- $(3x^2 - 7x) - (x^2 + 3x - 9)$
- $(5x + 6) + (-5x^2 + 6x - 2)$
- $2x^2 + 3x + x^2 + 4x - 3x^2 + 2$
- $3c^2 + 4c + 7x - 12 + (-4c^2) + 9 - 6x$
- $2a^2 + 3a^3 - 4a^2 + 6a + 12 - 4a + 2$

## Answers

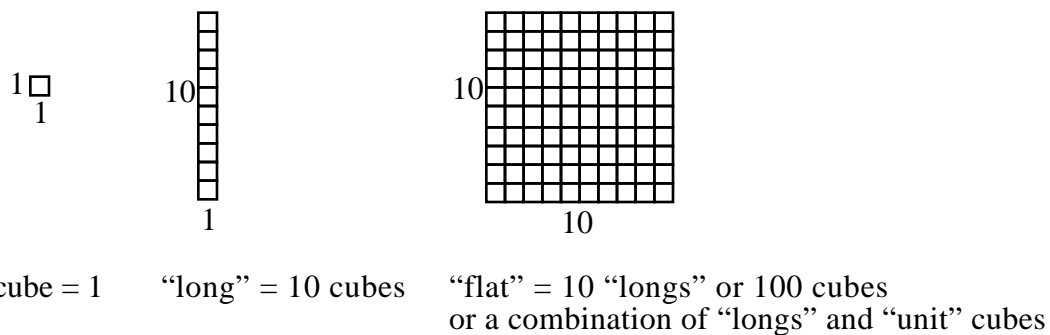
- $6x^2 + 8x + 13$
- $4x^2 + 5x + 11$
- $12x^2 + 5x + 7$
- $x^2 + 4x + 1$
- $6x^2 - 9x - 2$
- $2x^2 - 10x + 9$
- $-5x^2 + 11x + 4$
- $7x + 2$
- $-c^2 + 4c + x - 3$
- $3a^3 - 2a^2 + 2a + 14$

## RECTANGULAR MODEL OF MULTIPLICATION AND THE DISTRIBUTIVE PROPERTY

### BASE TEN MODEL

In Year 1, Chapter 4 the rectangular model is developed to give students a geometric view of multiplication. This approach will lead to multiplying polynomials in algebra. Base ten blocks are used in class to help students build a rectangular model of multiplication.

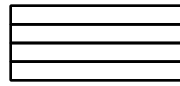
A set of base ten blocks looks like the example shown below:



### Example 1

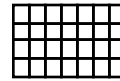
To show the multiplication problem  $4 \cdot 17$ , base ten blocks are used to build the model below.

Build  $4 \cdot 10$ .



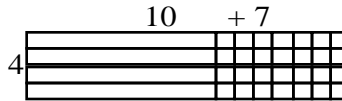
(four base ten longs =  $4 \cdot 10$ )

Build  $4 \cdot 7$ .



(28 base ten cubes =  $4 \cdot 7$ )

Put the two sets of blocks together to show a model of  $4 \cdot (10 + 7)$  or  $4 \cdot 17$ .



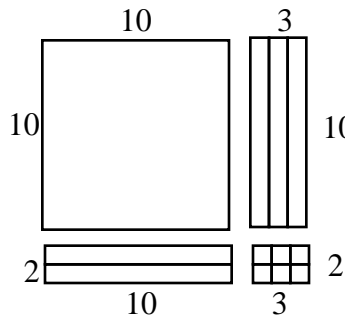
The dimensions of the rectangle are  $4 \cdot (10 + 7)$  or  $4 \cdot 17$ .

The area of the rectangle is  $40 + 28 = 68$  square units.

### Example 2

The model below shows that  $12 \cdot 13$  is equal to  $(10 + 2) \cdot (10 + 3)$ .

The rectangular regions at right are the subproblems (parts of problems) for the multiplication problem.



subproblems

$$\begin{array}{r} 10 \cdot 10 = 100 \\ 10 \cdot 3 = 30 \\ 2 \cdot 10 = 20 \\ 2 \cdot 3 = +6 \\ \hline 156 \text{ square units} \end{array}$$

If the areas of all the rectangular regions are added together, the total area will be 156 square units.

The traditional multiplication algorithm is used. The product looks like this:

$$\begin{array}{r} 13 \\ \times 12 \\ \hline 26 \\ 130 \\ \hline 156 \end{array}$$

### Example 3

The next example shows the generic rectangular models of  $6 \cdot 32$  and  $15 \cdot 25$ .

$$6 \cdot 32 = 6 \cdot (30 + 2)$$

$$6 \begin{array}{|c|c|} \hline 30 & + 2 \\ \hline 180 & 12 \\ \hline \end{array} = 180 + 12 = 192$$

$$15 \cdot 25 = (10 + 5) \cdot (20 + 5)$$

$$\begin{array}{|c|c|} \hline 20 & + 5 \\ \hline 200 & 50 \\ \hline 100 & 25 \\ \hline \end{array} \quad 200 + 50 + 100 + 25 = 375$$

## Problems

Solve the following multiplication problems by drawing generic rectangles.

1.  $2 \cdot 24$
2.  $3 \cdot 92$
3.  $5 \cdot 89$
4.  $11 \cdot 23$
5.  $25 \cdot 32$
6.  $17 \cdot 58$
7.  $27 \cdot 65$
8.  $32 \cdot 32$
9.  $18 \cdot 57$
10.  $11 \cdot 321$
11.  $23 \cdot 252$
12.  $35 \cdot 411$
13.  $115 \cdot 227$
14.  $20 \cdot 115$
15.  $101 \cdot 310$
16.  $215 \cdot 205$
17.  $107 \cdot 35$
18.  $255 \cdot 810$
19.  $909 \cdot 780$

## Answers

1. 48  

$20 + 4$				
2		40		8
2. 276  

$90 + 2$				
3		270		6
3. 445  

$80 + 9$				
5		400		45
4. 253  

$20 + 3$				
10		200		30
+				
1		20		3
5. 800  

$30 + 2$				
20		600		40
+				
5		150		10
6. 986  

$50 + 8$				
10		500		80
+				
7		350		56
7. 1755  

$60 + 5$				
20		1200		100
+				
7		420		35
8. 1024  

$30 + 2$				
30		900		60
+				
2		60		4
9. 1026  

$50 + 7$				
10		500		70
+				
1		400		56
10. 3531  

$300 + 20 + 3$						
10		3000		200		10
+						
1		300		20		1
11. 5796  

$200 + 50 + 2$						
20		4000		1000		40
+						
3		600		150		6
12. 14,385  

$400 + 10 + 1$						
30		12000		300		30
+						
5		2000		50		5
13. 26,105  

$200 + 20 + 7$						
100		2000		2000		700
+						
10		2000		200		70
+						
5		1000		100		35
14. 2300  

$100 + 10 + 5$						
20		2000		200		100
15. 31,310  

$300 + 10$				
100		30000		1000
+				
1		300		10

16. 44,075

	200 + 5	
200	4000	1000
+	2000	50
+	1000	25
5		

17. 3745

	30 + 5	
100	3000	500
+	210	35
7		

18. 206,550

	800 + 10	
200	160000	2000
+	40000	500
+	4000	50
5		

19. 709,020

	700 + 80	
900	630,000	72,000
+	63000	720
9		

## DISTRIBUTIVE PROPERTY WITH INTEGERS

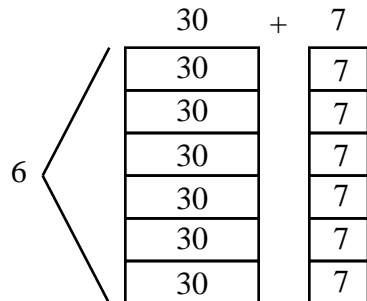
The Distributive Property may be used to multiply numbers by breaking them into smaller parts. Once the smaller numbers are multiplied, the products are then added or subtracted to get the final answer.

- Rewrite the multiplication problem as two or more terms of products to be added or subtracted. Examples:  $4 \cdot 28 = 4 \cdot 20 + 4 \cdot 8$  and  $6 \cdot 372 = 6 \cdot 300 + 6 \cdot 70 + 6 \cdot 2$
- Multiply the separate parts. Examples:  $80 + 32$  and  $1800 + 420 + 12$
- Add or subtract the products. Examples:  $4 \cdot 28 = 112$  and  $6 \cdot 372 = 2232$

For additional information, see Year 2, Chapter 4, problem GC-18.

### Example 1

$$6 \cdot 37$$



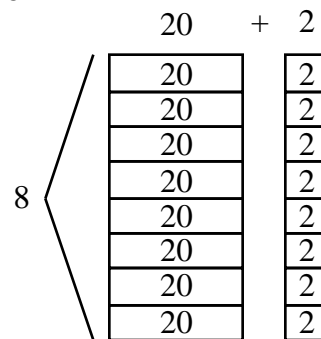
$$6 \cdot 37$$

$$\begin{aligned} &= 6(30 + 7) \\ &= 6 \cdot 30 + 6 \cdot 7 \\ &= 180 + 42 \\ &= 222 \end{aligned}$$

$$6 \cdot 37 = 222$$

### Example 3

$$8 \cdot 22$$



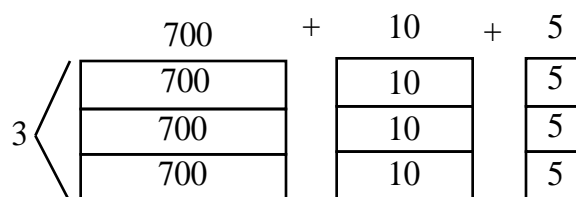
$$8 \cdot 22$$

$$\begin{aligned} &= 8(20 + 2) \\ &= 8(20) + 8(2) \\ &= 160 + 16 \\ &= 176 \end{aligned}$$

$$8 \cdot 22 = 176$$

### Example 2

$$3 \cdot 715$$



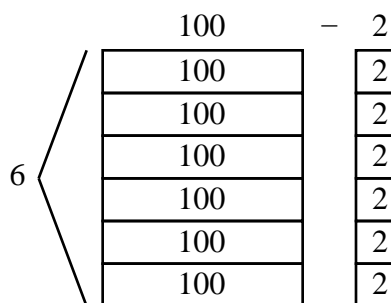
$$3 \cdot 715$$

$$\begin{aligned} &= 3(700 + 10 + 5) \\ &= 3(700) + 3(10) + 3(5) \\ &= 2100 + 30 + 15 \\ &= 2145 \end{aligned}$$

$$3 \cdot 715 = 2145$$

### Example 4

$$6 \cdot 98$$



$$6 \cdot 98$$

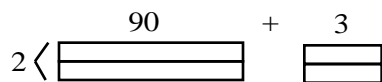
$$\begin{aligned} &= 6(100 - 2) \\ &= 6(100) - 6(2) \\ &= 600 - 12 \\ &= 588 \end{aligned}$$

$$6 \cdot 98 = 588$$

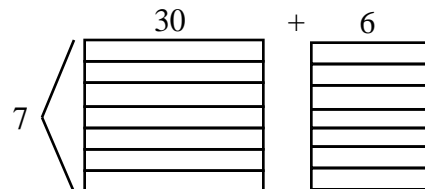
## Problems

Find each product using one of the methods shown in the examples.

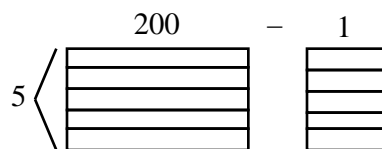
1.  $2 \cdot 93$



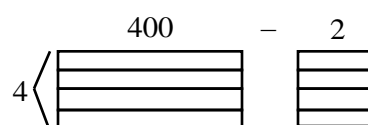
2.  $7 \cdot 36$



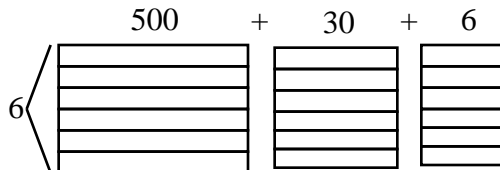
3.  $5 \cdot 199$



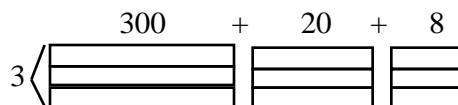
4.  $4 \cdot 398$



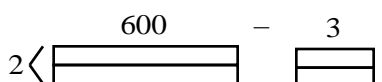
5.  $6 \cdot 536$



6.  $3 \cdot 328$



7.  $2 \cdot 597$



8.  $5 \cdot 43$

9.  $6 \cdot 91$

10.  $8 \cdot 214$

11.  $6 \cdot 537$

12.  $3 \cdot 193$

13.  $50 \cdot 139$

14.  $40 \cdot 27$

15.  $20 \cdot 38$

16.  $-3 \cdot 81$

17.  $-8 \cdot 273$

18.  $15 \cdot 21$

19.  $12 \cdot 39$

20.  $3 \cdot 495$

## Answers

1.  $2(90 + 3)$   
 $2(90) + 2(3)$   
 $180 + 6 = 186$

$2 \cdot 93 = 186$

2.  $7(30 + 6)$   
 $7(30) + 7(6)$   
 $210 + 42 = 252$

$7 \cdot 36 = 252$

3.  $5(200 - 1)$   
 $5(200) - 5(1)$   
 $1000 - 5 = 995$

$5 \cdot 199 = 995$

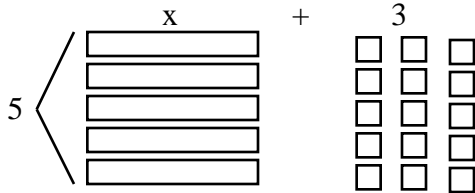
4.  $4(400 - 2)$   
 $4(400) - 4(2)$   
 $1600 - 8 = 1592$

$4 \cdot 398 = 1592$

5.  $6(500 + 30 + 6)$        $6 \cdot 536 = 3216$       6.  $3(300 + 20 + 8)$        $3 \cdot 328 = 984$   
 $6(500) + 6(30) + 6(6)$        $3(300) + 3(20) + 3(8)$   
 $3000 + 180 + 36 = 3216$        $900 + 60 + 24 = 984$
7.  $2(600 - 3)$        $2 \cdot 597 = 1194$   
 $2(600) - 2(3)$   
 $1200 - 6 = 1194$
8.  $5(40 + 3) = 5(40) + 5(3) = 200 + 15 = 215$
9.  $6(90 + 1) = 6(90) + 6(1) = 540 + 6 = 546$
10.  $8(214) = 8(200 + 10 + 4) = 1600 + 80 + 32 = 1712$
11.  $6(500 + 30 + 7) = 6(500) + 6(30) + 6(7) = 3000 + 180 + 42 = 3222$
12.  $3(100 + 90 + 3) = 3(100) + 3(90) + 3(3) = 300 + 270 + 9 = 579$
13.  $50(100 + 30 + 9) = 50(100) + 50(30) + 50(9) = 5000 + 1500 + 450 = 6950$
14.  $40(20 + 7) = 40(20) + 40(7) = 800 + 280 = 1080$
15.  $20(30 + 8) = 20(30) + 20(8) = 600 + 160 = 760$
16.  $-3(80 + 1) = -3(80) + -3(1) = -240 + -3 = -243$
17.  $-8(200) + -8(70) + -8(3) = -1600 + -560 + -24 = -2184$
18.  $15(20 + 1) = 15(20) + 15(1) = 300 + 15 = 315$
19.  $12(30 + 9) = 12(30) + 12(9) = 360 + 108 = 468$
20.  $3(500 - 5) = 3(500) - 3(5) = 1500 - 15 = 1485$

## DISTRIBUTIVE PROPERTY WITH ALGEBRA TILES

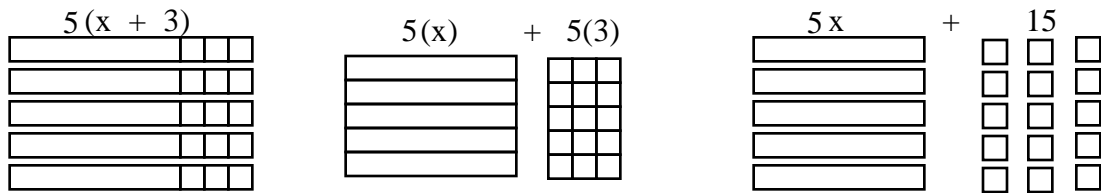
The Distributive Property can be used to multiply expressions containing variables. For example,  $5(x + 3)$  means there are 5 groups. Each group contains one “x” and 3 “ones.” If each group contains one “x” and 3 “ones,” then the 5 groups would contain a total of 5 “x” and 15 “ones.” Using the Distributive Property,  $5(x + 3)$  can be rewritten as  $5(x) + 5(3)$ , which equals  $5x + 15$ . We can use algebra tiles to model this idea.



If you count the total number of tiles, there are 5 x's and 15 “ones.”

$$5(x + 3) = 5(x) + 5(3) = 5x + 15$$

The area of the rectangular arrangement of algebra tiles can be represented three different ways.

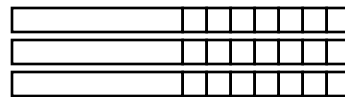


For additional information, see Year 2, Chapter 4, problems GC-56 and 57.

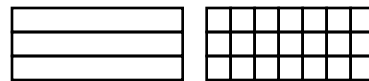
### Example 1

Model the three representations of the expression  $3(x + 7)$  using algebra tiles. Remember that  $3(x + 7)$  means there are three groups, and each group contains one x and 7 ones. If you count the number of each kind of tile used in each of the representations, there are 3 x's and 21 ones. Therefore, we can say  $3(x + 7) = 3(x) + 3(7) = 3x + 21$ .

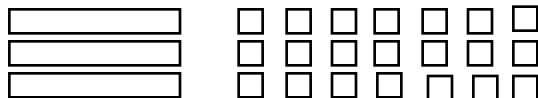
$$3(x + 7)$$



$$= 3(x) + 3(7)$$



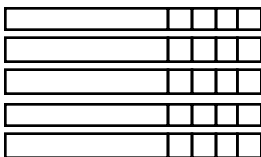
$$= 3x + 21$$



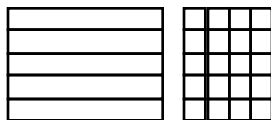
## Example 2

Model the three representations of the expression  $5(x + 4)$ . Remember that  $5(x + 4)$  means there are five groups, and each group contains one  $x$  and 4 ones. You will use 5  $x$ 's and 20 ones each time. Therefore we can say  $5(x + 4) = 5(x) + 3(4) = 5x + 20$ .

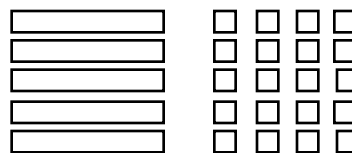
$$5(x + 4)$$



$$= 5(x) + 5(4)$$



$$= 5x + 20$$



## Problems

Show the distributed form of each expression below.

1.  $2(x + 5)$     2.  $4(x + 4)$     3.  $3(x + 2)$     4.  $5(x + 3)$     5.  $9(x + 1)$

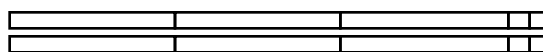
## Answers

1.  $2(x) + 2(5) = 2x + 10$     2.  $4(x) + 4(4) = 4x + 16$     3.  $3(x) + 3(2) = 3x + 6$   
 4.  $5(x) + 5(3) = 5x + 15$     5.  $9(x) + 9(1) = 9x + 9$

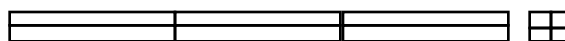
## Example 3

Model the three representations of the expression  $2(3x + 2)$  using algebra tiles. This time there are two groups, and each group contains 3  $x$ 's and 2 ones. You will use a total of 6 rectangular strips and 4 ones for each representation. Therefore we can say  $2(3x + 2) = 2(3x) + 2(2) = 6x + 4$ .

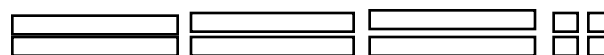
$$2(3x + 2)$$



$$= 2(3x) + 2(2)$$



$$= 6x + 4$$



## Problems

Show the distributed form of each expression below.

1.  $2(2x + 4)$     2.  $5(3x + 1)$     3.  $3(4x + 2)$     4.  $7(2x + 1)$   
 5.  $4(5x + 2)$     6.  $2(6x + 5)$     7.  $3(3x + 6)$

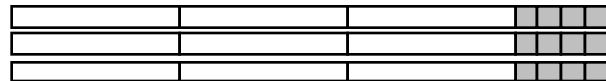
## Answers

- $2(2x) + 2(4) = 4x + 8$
- $5(3x) + 5(1) = 15x + 5$
- $3(4x) + 3(2) = 12x + 6$
- $7(2x) + 7(1) = 14x + 7$
- $4(5x) + 4(2) = 20x + 8$
- $2(6x) + 2(5) = 12x + 10$
- $3(3x) + 3(6) = 9x + 18$

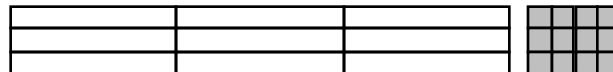
## Example 4

Model the three representations of the expression  $3(3x - 4)$  using algebra tiles. Remember that  $3(3x - 4)$  means there are three groups and each group contains 3  $x$  tiles and 4 negative tiles. You will use a total of 9  $x$  tiles and 12 negative tiles for each representation. Therefore we can say that  $3(3x - 4) = 3(3x) + 3(-4) = 9x - 12$ . Notice that the negative tiles are shaded.

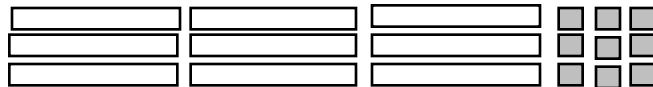
$$3(3x - 4)$$



$$= 3(3x) + 3(-4)$$



$$= 9x - 12$$



## Problems

Show the distributed form of each expression below, that is, simplify each expression.

- $2(x - 4)$
- $5(2x - 1)$
- $6(3x - 3)$
- $4(x - 9)$
- $3(5x - 3)$
- $10(4x - 7)$
- $2(x^2 + 4x - 3)$
- $3(x^2 - 2x + 5)$

## Answers

- $2(x) + 2(-4) = 2x - 8$
- $5(2x) + 5(-1) = 10x - 5$
- $6(3x) + 6(-3) = 18x - 18$
- $4(x) + 4(-9) = 4x - 36$
- $3(5x) + 3(-3) = 15x - 9$
- $10(4x) + 10(-7) = 40x - 70$
- $2(x^2) + 2(4x) + 2(-3) = 2x^2 + 8x - 6$
- $3(x^2) + 3(-2x) + 3(5) = 3x^2 - 6x + 15$

## FACTORING

The Distributive Property is used to simplify expressions in Year 2, Chapter 4. Students need to be able to take the simplified form and reverse the order to put the expressions into the factored form. The factors show what was multiplied before the distribution took place.

For additional information, see Year 2, Chapter 4, problems GC-31 and 109.

### Example 1

$3x + 3$  is the simplified (distributed) form of a product.  $3(x + 1)$  is the factored form. In other words, since  $3x + 3 = 3 \cdot x + 3 \cdot 1$ , the 3 is placed outside the parentheses:  $3(x + 1)$ .

### Example 2

The simplified expression for  $6x + 12$  can be written  $2(3x + 6)$  or  $3(2x + 4)$  or  $6(x + 2)$ . This example has many possible answers because 6 and 12 have many common factors. In general, an expression is completely factored when the greatest common factor is used. Thus, the factored form of  $6x + 12$  is  $6(x + 2)$ .

### Problems

Factor.

- $3x + 18$
- $5x + 15$
- $6b + 24$
- $3x - 6$
- $6a + 3$
- $5x + 60$
- $12x + 48$
- $7x - 21$
- $-3x - 9$
- $-4x - 8$

### Answers

- $3(x + 6)$
- $5(x + 3)$
- $6(b + 4)$
- $3(x - 2)$
- $3(2a + 1)$
- $5(x + 12)$
- $12(x + 4)$
- $7(x - 3)$
- $3(-x - 3)$  or  $-3(x + 3)$
- $-4(x + 2)$  or  $4(-x - 2)$

## GUESS AND CHECK

### WHY WE USE GUESS AND CHECK TABLES

Guess and Check is one method that students can use to solve many types of problems, especially word problems. When students create a Guess and Check table, it provides a record of the student's thinking. The patterns in the table lead directly to writing algebraic equations for the word problems.

Writing equations is one of the most important algebra skills students learn. Guess and Check tables help make this skill accessible to all students. In order to help students see the relationships in a word problem, we require them to include at least four entries (rows) in their Guess and Check tables. The repetition of the operations is needed to see how the columns are related. After students have several weeks of practice using Guess and Check tables to solve problems, we begin generalizing from the patterns in the table of guesses to write an equation that represents the relationships in the problem. During the first few weeks, we intentionally use problems that might be solved with fewer than four guesses so that students can focus on constructing effective tables and later using the patterns to write equations. Thus early in the course, if the correct answer is found earlier than the fourth entry, a number of incorrect entries should be written to complete the table.

We also believe that writing the answer in a sentence after the table is complete is important because many students forget what the question actually was. The sentence helps the student see the "big picture" and brings closure to the problem.

In Year 1, refer to Chapter 3, problem PR-2, for a detailed, step-by-step demonstration of a Guess and Check problem similar to the one below and to problem PR-5 for the Tool Kit entry for Guess and Check tables.

In Year 2, refer to Chapter 1, problem GO-17, for a detailed, step-by-step demonstration of a Guess and Check problem similar to the one below and to problem GO-33 for the Tool Kit for Guess and Check tables.

#### Example 1

A box of fruit has three times as many nectarines as grapefruit. Together there are 36 pieces of fruit. How many pieces of each type of fruit are there?

Step 1: Guess the number of fruit you know the least about.

Guess Number of Grapefruit	
11	

Step 2: What else do we need to know?

The number of nectarines, which is three times the number of grapefruit.

Guess Number of Grapefruit	Number of Nectarines	
11	$3(11) = 33$	

Step 3: What else do we need to know?

The total number of pieces of fruit.

Guess Number of Grapefruit	Number of Nectarines	Total Pieces of Fruit	
11	33	$11 + 33 = 44$	

Step 4: What else do we need to know?

We need to check the total pieces of fruit based on a guess of 11 grapefruit and compare it to the total given in the problem.

Guess Number of Grapefruit	Number of Nectarines	Total Pieces of Fruit	Check 36
11	33	44	too high

Step 5: Determine our next guess. Our total was 44; the total needed is 36, so our guess was too high and our next guess should be lower.

Guess Number of Grapefruit	Number of Nectarines	Total Pieces of Fruit	Check 36
11	33	44	too high
10	30	40	too high

Step 6: Determine our next guess. Our total was 40; the total needed is 36, so our guess was too high and our next guess should be still lower.

Guess Number of Grapefruit	Number of Nectarines	Total Pieces of Fruit	Check 36
11	33	44	too high
10	30	40	too high
8	24	32	too low

Step 7: Determine our next guess. Our total was 32; the total needed is 36, so our guess was too low and our next guess should be higher than 8 but lower than 10.

Guess Number of Grapefruit	Number of Nectarines	Total Pieces of Fruit	Check 36
11	33	44	too high
10	30	40	too high
8	24	32	too low
9	27	36	correct

There are 9 grapefruit and 27 nectarines in the box.

## Example 2

The perimeter of a rectangle is 120 feet. If the length of the rectangle is ten feet more than the width, what are the dimensions (length and width) of the rectangle?

Step 1: Guess the width because, of the two required answers, it is the one we know the least about. The length is 10 feet more than the width, so add 10 to the guess.

Guess Width	Length	Perimeter	Check 120
10	20	$(10 + 20) \cdot 2 = 60$	too low

Step 2: Since the guess of 10 resulted in an answer that is too low, we should increase the guess. Pay close attention to each guess and its result. Each result helps determine the next guess as you narrow down the possible guesses to reach the answer. Note: as students get more experience with using Guess and Check tables, they learn to make better educated guesses from one step to the next to solve problems quickly or to establish the pattern they need to write an equation.

Guess Width	Length	Perimeter	Check 120
10	20	$(10 + 20) \cdot 2 = 60$	too low
20	30	100	too low
30	40	140	too high
25	35	120	correct

The dimensions are 25 and 35 feet.

## Example 3

Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth \$2.40. How many dimes and quarters does Jorge have?

NOTE: This type of problem is more difficult than others because the number of things asked for is different than their value. Separate columns for each part of the problem must be added to the table as shown below. Students often neglect to write the third and fourth columns.

Guess Number of Quarters	Number of Dimes	Value of Quarters	Value of Dimes	Total Value	Check \$2.40
10	20	2.50	2.00	4.50	too high
8	18	2.00	1.80	3.80	too high
6	16	1.50	1.60	3.10	too high
4	14	1.00	1.40	2.40	correct

Jorge has four quarters and 14 dimes.

## HELPFUL QUESTIONS TO ASK YOUR CHILD

When your child is having difficulty with a Guess and Check problem, it may be because he/she does not understand the problem, not because he/she does not understand the Guess and Check process. Here are some helpful questions to ask when your child does not understand the problem. (These are useful in non-Guess and Check situations, too.)

1. What are you being asked to find?
2. What information have you been given?
3. Is there any unneeded information? If so, what is it?
4. Is there any necessary information that is missing? If so, what information do you need?

## TIPS ABOUT COLUMN TITLES

1. Guess the answer to the question. Ten or the student's age are adequate first guesses.
2. Continue establishing columns by asking, "What else do we need to know to determine whether our guess is correct or too low or too high?"
3. After the guess, only put the answer to one calculation in each column. Students sometimes try to put the answer to several mental calculations in one column. (See NOTE in Example 3 on the previous page.)

## Problems

Solve these problems using Guess and Check tables. Write each answer in a sentence.

1. A wood board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?
2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?
3. Tomas is thinking of a number. If he triples his number and subtracts 13, the result is 305. Of what number is Tomas thinking?
4. Two consecutive numbers have a sum of 123. What are the two numbers?
5. Two consecutive even numbers have a sum of 246. What are the numbers?
6. Joe's age is three times Aaron's age and Aaron is six years older than Christina. If the sum of their ages is 149, what is Christina's age? Joe's age? Aaron's age?
7. Farmer Fran has 38 barnyard animals, consisting of only chickens and goats. If these animals have 116 legs, how many of each type of animal are there?

8. A wood board 156 centimeters long is cut into three parts. The two longer parts are the same length and are 15 centimeters longer than the shortest part. How long are the three parts?
9. Juan has 15 coins, all nickels and dimes. This collection of coins is worth 90¢. How many nickels and dimes are there? (Hint: Create separate column titles for, “Number of Nickels,” “Value of Nickels,” “Number of Dimes,” and “Value of Dimes.”)
10. Tickets to the school play are \$ 5.00 for adults and \$ 3.50 for students. If the total value of all the tickets sold was \$2517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?
11. A wood board 250 centimeters long is cut into five pieces: three short ones of equal length and two that are both 15 centimeters longer than the shorter ones. What are the lengths of the boards?
12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There is an equal amount of nickels and quarters but three times as many dimes. If the entire collection is worth \$ 9.60, how many nickels, dimes, and quarters are there?

## Answers

- |   |  |
|---|--|
| 1. The lengths of the boards are 37 cm and 63 cm. | 2. Thu is 28 years old and her brother is 23 years old.                  |
| 3. Tomas is thinking of the number 106.           | 4. The two consecutive numbers are 61 and 62.                            |
| 5. The two consecutive numbers are 142 and 144.   | 6. Christine is 25, Aaron is 31, and Joe is 93 years old.                |
| 7. Farmer Fran has 20 goats and 18 chickens.      | 8. The lengths of the boards are 42, 57, and 57 cm.                      |
| 9. Juan has 12 nickels and 3 dimes.               | 10. There were 255 adult and 355 student tickets purchased for the play. |
| 11. The lengths of the boards are 44 and 59 cm.   | 12. Conrad has 16 nickels and quarters and 48 dimes.                     |

## WRITING EQUATIONS FROM A GUESS AND CHECK TABLE

Guess and Check is a powerful strategy for solving problems. In fact, some types of problems in Algebra 2 are easier to solve this way than by writing an equation. However, solving complicated problems with a Guess and Check table can be time consuming and it may be difficult to find the correct solution if it is not an integer. The patterns developed in the table can be generalized by using variables to write equations. If you can write and solve the equation, it is often more efficient than guessing and checking.

Additional examples are found in the Year 2 text, Chapter 4, problem GC-83. Consider two of the examples from the previous Guess and Check discussion.

### Example 1

Guess Number of Grapefruit	Number of Nectarines	Total Pieces of Fruit	Check 36
11	33	44	too high
10	30	40	too high

After several guesses and checks establish a pattern in the problem, you can generalize it using a variable. Since we could guess any number of grapefruit, use  $x$  to represent it. The pattern for the number of nectarines is three times the number of grapefruit or  $3x$ . The total pieces of fruit is the sum of column one and column two, so our table becomes:

Guess Number of Grapefruit	Number of Nectarines	Total Pieces of Fruit	Check 36
$x$	$3x$	$x + 3x$	$= 36$

Since we want the total to agree with the check, our equation is  $x + 3x = 36$ . Simplifying this yields  $4x = 36$ , so  $x = 9$  (grapefruit) and then  $3x = 27$  (nectarines).

The pattern and structure of the Guess and Check table allowed us to write and solve an equation, so guessing was no longer necessary.

### Example 2

Guess Width	Length	Perimeter	Check 120
10	20	$(10 + 20) \cdot 2 = 60$	too low
20	30	100	too low

Again, since we could guess any width, we labeled this column  $x$ . The pattern for the second column is that it is 10 more than the first:  $x + 10$ . Perimeter is found by multiplying the sum of the width and length by 2. Our table now becomes:

Guess Width	Length	Perimeter	Check 120
$x$	$x + 10$	$(x + x + 10) \cdot 2$	$= 120$

Solving the equation:  $(x + x + 10) \cdot 2 = 120$   
 $2x + 2x + 20 = 120$   
 $4x + 20 = 120$   
 $4x = 100$  So  $x = 25$  (width) and  $x + 10 = 35$  (length)

## Problems

Write an equation for each Guess and Check table from the previous Problems section, numbers 1-12.

### Answers (may vary)

1.  $x + (x + 26) = 100$

2.  $x + (x + 5) = 51$

3.  $3x - 13 = 305$

4.  $x + (x + 1) = 123$

5.  $x + (x + 2) = 246$

6.  $x + (x + 6) + 3(x + 6) = 149$

7.  $2x + 4(38 - x) = 116$

8.  $x + (x + 15) + (x + 15) = 156$

9.  $0.05x + 0.10(15 - x) = 0.90$

10.  $\$5x + \$3.50(x + 100) = 2517.50$

11.  $3x + 2(x + 15) = 250$

12.  $0.05x + 0.25x + 0.10(3x) = 9.60$

## SOLVING EQUATIONS

### BALANCE SCALE METHOD

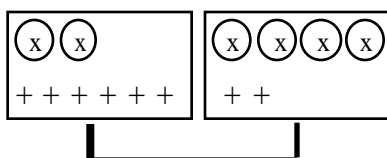
An equation is a mathematical sentence with an equal sign.

One aid to understanding solving equations is using the idea of a balance scale. Sometimes equations have unknowns on both sides. The idea is to keep each side of the scale even or equal. No matter what equation is given, it is treated like a balance scale. Negative and positive signs or tiles are used to signify negative and positive numbers. Cups (unknowns) are used to hide the amount of tiles used on either side of the equation (balance scale).

Additional information is in Year 2, Chapter 5, problems MC-38, 39, and 56 through 58.

## Example

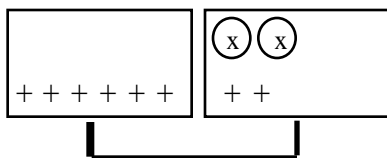
Solve the following equation using the balance scale method. (Remember that the cups are each hiding the same amount of tiles.) An unknown must be the same number within each equation.



$$2x + 6 = 4x + 2$$

This equation represents the balance scale at left.

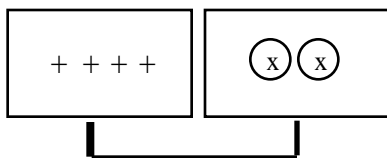
We can remove two cups from both sides of the scale ( $-2x$ ).



Subtract  $2x$  from both sides.

$$6 = 2x + 2$$

We can remove two tiles from each side ( $-2$ ).

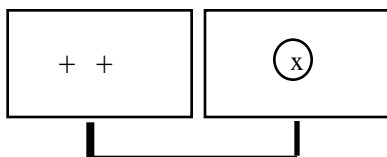


Subtract 2 from both sides.

$$4 = 2x$$

Divide both sides by 2:  $\frac{4}{2}$  and  $\frac{2x}{2}$ .

If 4 tiles equal 2 cups, there must be 2 tiles in each cup.



$$2 = x$$

Since you have found  $x = 2$ , put 2 back in the original equation and check.

$$\begin{aligned} 2x + 6 &= 4x + 2 \\ 2(2) + 6 &= 4(2) + 2 \\ 4 + 6 &= 8 + 2 \\ 10 &= 10 \end{aligned}$$

## Problems

Solve these equations. Remember to check your answers.

1.  $4x + 3 = 2x + 7$
2.  $4x - 3 = 7x + 9$
3.  $5b - 7 = 9b + 13$
4.  $12c + 3 = 17c - 2$
5.  $9x - 4 = 6x - 7$
6.  $13n + 12 = 19n - 6$
7.  $13n - 12 = 6n + 9$
8.  $4c + 16 = 3c + 15$
9.  $-5c - 16 = 2c + 5$
10.  $10x + 18 = 8x - 12$

## Answers

1.  $x = 2$
2.  $x = -4$
3.  $b = -5$
4.  $c = 1$
5.  $x = -1$
6.  $n = 3$
7.  $n = 3$
8.  $c = -1$
9.  $c = -3$
10.  $x = -15$

## COVER-UP METHOD

Another method for solving simple equations is the cover-up method. Students cover up the unknown part of the equation and ask what would have to be uncovered to make the equation true.

For additional information, see Year 1, Chapter 2, problem GS-74 or Year 2, Chapter 5, problems MC-14 and 15.

### Example 1

Use the cover-up method to solve the one-step equation below.

$$\begin{array}{l} x + 5 = 17 \\ (?) + 5 = 17 \\ x = 12 \\ (12) + 5 = 17 \end{array} \quad \begin{array}{l} \text{Cover up the } x. \\ \text{What number plus 5 equals 17?} \\ \text{Put 12 where the } x \text{ was.} \end{array}$$

### Example 2

Use the cover-up method to solve the two-step equation below.

$$\begin{array}{l} 2x + 4 = 10 \\ (?) + 4 = 10 \\ (6) + 4 = 10 \\ 2x = 6 \\ 2 \cdot (?) = 6 \\ 2 \cdot (3) = 6 \\ x = 3 \end{array} \quad \begin{array}{l} \text{Cover the term with } x \text{ in it.} \\ \text{What number plus 4 equals 10?} \\ \text{6 plus 4 equals 10, so } 2x \text{ must equal 6.} \\ \text{2 times what number equals 6?} \\ \text{2 times 3 equals 6, so} \\ \text{ } x \text{ must equal 3.} \end{array}$$

## Problems

Solve these equations. Remember to check your answers.

1.  $2x + 3 = 15$
2.  $10n - 13 = 37$
3.  $14 - 7n = 42$
4.  $72 = 4x + 4$
5.  $3 = 2x - 11$
6.  $402n - 412 = 2000$
7.  $25y + 6 = 181$
8.  $24 = 2x + 10$
9.  $-42y + 24 = -102$
10.  $11m + 321 = 409$

## Answers

1.  $x = 6$
2.  $n = 5$
3.  $n = -4$
4.  $x = 17$
5.  $x = 7$
6.  $n = 6$
7.  $y = 7$
8.  $x = 7$
9.  $y = 3$
10.  $m = 8$

## INVERSE OPERATIONS

The algebraic approach to solving equations uses inverse operations to “undo” what has been done to create the equation. By systematically working backward, the value of the variable can be found. Multiplication and division are inverse operations, as are addition and subtraction.

The order of operations in the equation  $5x + 7 = 37$  is to multiply by 5 and then add 7 to get 37. To solve the equation, work backward using inverse operations. First subtract 7 to get 30, then divide by 5 to get 6.

The examples below model the steps your child should use to solve equations algebraically. For additional information, see Year 1, Chapter 2, problem GS-75.

### Example 1

$5x$  means 5 times some number  $x$ . For example, suppose  $5x = 20$ . To solve, we do the inverse of multiplying by 5, which is dividing by 5. So  $20 \div 5 = 4$ ,  $x = 4$ .

### Example 2

$$3x = 30$$

Divide by 3 to solve.  
The result is  $x = 10$ .

$$\frac{x}{3} = 30$$

To solve, do the inverse of dividing by 3; that is, multiply by 3. The result is  $x = 90$ .

### Example 3

$x + 3 = 30$  Solve by subtracting 3:  $x = 27$ .

### Example 4

$x - 3 = 30$  Solve by adding 3:  $x = 33$ .

### Example 5

$$\begin{aligned}3x + 9 &= 15 \\3x &= 6 \\x &= 2 \\3(2) + 9 &= 15\end{aligned}$$

Subtract 9.  
Divide by 3.  
Substitute 2 for  $x$  to  
check your answer.

### Example 6

$$\begin{aligned}\frac{n}{3} + 6 &= 10 \\ \frac{n}{3} &= 4 \\ n &= 12 \\ \frac{(12)}{3} + 6 &= 10\end{aligned}$$

Subtract 6.  
Multiply by 3.  
Substitute to check.

### Problems

Solve these equations. Remember to check your answers.

1.  $2x + 3 = 7$
2.  $5x - 3 = 7$
3.  $4x + 6 = 10$
4.  $3x - 6 = 6$
5.  $x - 10 = 25$
6.  $x - 10 = -5$
7.  $\frac{x}{2} + 3 = 8$
8.  $\frac{n}{3} - 6 = 9$
9.  $201n + 602 = 1808$
10.  $10x + 15 = 165$
11.  $12x + 12 = 144$
12.  $23x - 9 = 106$
13.  $401c - 201 = 3007$
14.  $43 = 4y - 5$
15.  $16 + 9y = 88$
16.  $32 + 6y = 74$
17.  $301 + 302c = 1207$
18.  $55n - 52 = 553$
19.  $\frac{x}{5} - 6 = -9$
20.  $\frac{x}{20} - 7 = -2$

### Answers

1.  $x = 2$
2.  $x = 2$
3.  $x = 1$
4.  $x = 4$
5.  $x = 35$
6.  $x = 5$
7.  $x = 10$
8.  $n = 45$
9.  $n = 6$
10.  $x = 15$
11.  $x = 11$
12.  $x = 5$
13.  $c = 8$
14.  $y = 12$
15.  $y = 8$
16.  $y = 7$
17.  $c = 3$
18.  $n = 11$
19.  $x = -15$
20.  $x = 100$

## DISTRIBUTIVE PROPERTY

You can also solve equations that involve the Distributive Property.

### Example 1

Solve the equation  $3(5x + 2) = 8x + 20$ .

$$\begin{array}{l} \overbrace{3(5x + 2)} \\ 3 \cdot 5x + 3 \cdot 2 = 8x + 20 \end{array}$$

Use the Distributive Property to eliminate the parentheses.

$$\begin{array}{r} 15x + 6 = 8x + 20 \\ -6 \quad -6 \end{array}$$

Subtract 6 from both sides of the equation.

$$\begin{array}{r} 15x = 8x + 14 \\ -8x \quad -8x \end{array}$$

Subtract 8x from both sides of the equation.

$$\frac{7x}{7} = \frac{14}{7}$$

Divide both sides by 7 to get one x.

$$x = 2$$

Substitute the solution into the original equation to check your answer.

$$3(5 \cdot 2 + 2) = 8 \cdot 2 + 20$$

Use the order of operations and simplify.

$$3(12) = 36$$

### Example 2

$$2(x + 3) = 4(x + 1)$$

$$2 \cdot x + 2 \cdot 3 = 4 \cdot x + 4 \cdot 1$$

$$\begin{array}{r} 2x + 6 = 4x + 4 \\ -4 \quad -4 \end{array}$$

$$\begin{array}{r} 2x + 2 = 4x \\ -2x \quad -2x \end{array}$$

$$\frac{2}{2} = \frac{2x}{2}$$

$$1 = x$$

Check:

$$2(1 + 3) = 4(1 + 1)$$

$$2(4) = 4(2)$$

$$8 = 8$$

## Problems

Solve these equations. Remember to check your answers.

1.  $3(2x - 5) = 21$
2.  $2(4x + 5) = 26$
3.  $7(2x - 4) = 42$
4.  $3(2x + 2) = 12$
5.  $5(10x - 6) = 300$
6.  $-4(2x - 6) = 24$
7.  $2(3x - 2) = 4(x + 2)$
8.  $-5(3x + 2) = -3(x - 2)$
9.  $4(3x - 2) = -12$
10.  $-4(-3x - 2) = 12$

## Answers

1. 6
2. 2
3. 5
4. 1
5. 6.6
6. 0
7. 6
8.  $-1\frac{1}{3}$
9.  $-\frac{1}{3}$
10.  $\frac{1}{3}$

## RATIO EQUATIONS (PROPORTIONS)

To solve proportions, use cross multiplication to remove the fractions and then solve in the usual way. Cross multiplication may only be used to solve proportions; that is, cross multiplication may only be used when each side of the equation is a ratio. Multiply the numerator of the left ratio by the denominator of the right ratio, write an equal sign, then write the product of the other numerator and denominator.

For additional information, see Year 1, Chapter 6, problem MB-8 or Year 2, Chapter 6, problem RS-23.

### Example 1

$$\begin{aligned}\frac{m}{6} &= \frac{15}{9} \\ 9 \cdot m &= 6 \cdot 15 \\ 9m &= 90 \\ m &= 10\end{aligned}$$

### Example 2

$$\begin{aligned}\frac{6}{\$9.75} &= \frac{8.5}{w} \\ 6w &= \$82.875 \\ w &= \$13.82\end{aligned}$$

### Example 3

$$\begin{aligned}\frac{x + 1}{3} &= \frac{x - 2}{5} \\ 5(x + 1) &= 3(x - 2) \\ 5x + 5 &= 3x - 6 \\ 2x + 5 &= -6 \\ 2x &= -11 \\ x &= \frac{-11}{2} = -5.5\end{aligned}$$

## Problems

Solve these equations. Remember to check your answers.

1.  $\frac{2}{5} = \frac{y}{15}$

2.  $\frac{x}{36} = \frac{4}{9}$

3.  $\frac{2}{300} = \frac{6}{m}$

4.  $\frac{5}{8} = \frac{x}{100}$

5.  $\frac{6}{\$9.75} = \frac{9}{x}$

6.  $\frac{8}{2000} = \frac{m}{3000}$

7.  $\frac{8}{\$13.25} = \frac{x}{\$24.95}$

8.  $\frac{8}{14} = \frac{4}{m}$

9.  $\frac{20}{30} = \frac{50}{x}$

10.  $\frac{40}{100} = \frac{45}{x}$

11.  $\frac{x-1}{4} = \frac{7}{8}$

12.  $\frac{3y}{5} = \frac{24}{10}$

13.  $\frac{x}{x+1} = \frac{3}{5}$

14.  $\frac{3}{y} = \frac{6}{y-2}$

15.  $\frac{1}{x} = \frac{5}{x+1}$

## Answers

1.  $y = 6$

2.  $x = 16$

3.  $m = 900$

4.  $x = 62.5$

5.  $x = \$14.63$

6.  $m = 12$

7.  $x = 15.06$

8.  $m = 7$

9.  $x = 75$

10.  $x = 112.5$

11.  $x = 4.5$

12.  $y = 4$

13.  $x = 1\frac{1}{2}$

14.  $y = -2$

15.  $x = \frac{1}{4}$

## COMBINING LIKE TERMS BEFORE SOLVING

### Example 1

$$\begin{aligned}3x + 2x - 8 &= -x - 8 + 12 \\5x - 8 &= -x + 4 \\6x - 8 &= 4 \\6x &= 12 \\x &= 2\end{aligned}$$

### Example 2

$$\begin{aligned}-2(x - 3) + 4x &= -(-x + 1) \\-2x + 6 + 4x &= x - 1 \\2x + 6 &= x - 1 \\x + 6 &= -1 \\x &= -7\end{aligned}$$

## Problems

Solve these equations.

1.  $3x + 2x + 2 = -x + 14$

2.  $x + 2(x - 1) = x + 10$

3.  $6x + 4x - 2 = 15$

4.  $6x - 3x + 2 = -10$

5.  $x + 8 + x - 6 = 3(x - 5)$

6.  $3(m - 2) = -2(m - 7)$

7.  $4 - 6(w + 2) = 10$

8.  $6 - 2(x - 3) = 12$

## Answers

1.  $x = 2$
2.  $x = 6$
3.  $x = 1.7$
4.  $x = -4$
5.  $x = 17$
6.  $m = 4$
7.  $w = -3$
8.  $x = 0$

## EQUATIONS WITH MORE THAN ONE VARIABLE (FORMULAS OR LITERAL EQUATIONS)

Follow the same procedures for solving multi-variable equations as shown in the preceding sections of the guide.

For additional information, see Year 2, Chapter 7, problems CT-112 through 114.

### Example 1

$$\begin{array}{l} x + 3 = 5 \\ \text{subtract 3} \\ x = 5 - 3 = 2 \end{array}$$

$$\begin{array}{l} x + b = 5 \\ \text{subtract } b \\ x = 5 - b \text{ (done)} \end{array}$$

### Example 2

$$\begin{array}{l} 2y = 17 \\ \text{divide by 2} \\ y = \frac{17}{2} = 8\frac{1}{2} \end{array}$$

$$\begin{array}{l} dy = 17 \\ \text{divide by } d \\ y = \frac{17}{d} \text{ (done)} \end{array}$$

### Example 3

$$\begin{array}{l} 2x - 1 = 5 \\ 2x = 6 \\ x = 3 \end{array}$$

$$\begin{array}{l} 2x - b = 7 \\ 2x = 7 + b \\ x = \frac{7 + b}{2} \text{ (done)} \end{array}$$

## Problems

Solve each equation for the indicated variable.

1.  $y + b = 7$  for  $y$
2.  $y + b = 7$  for  $b$
3.  $rt = 8$  for  $t$
4.  $ax = b$  for  $x$
5.  $2x + b = c$  for  $x$
6.  $2x + b = c$  for  $b$
7.  $3x - d = m$  for  $x$
8.  $17q = r$  for  $q$
9.  $a + t = b$  for  $t$
10.  $3x + 2y = 10$  for  $y$
11.  $5x + 2y = 8$  for  $x$
12.  $d = c$  for  $d$

## Answers

1.  $7 - b$
2.  $7 - y$
3.  $\frac{8}{r}$
4.  $\frac{b}{a}$
5.  $\frac{c - b}{2}$
6.  $c - 2x$
7.  $\frac{m + d}{3}$
8.  $\frac{r}{17}$
9.  $b - a$
10.  $\frac{10 - 3x}{2}$
11.  $\frac{8 - 2y}{5}$
12.  $d = c$  for

## EQUATIONS WITH FRACTIONAL COEFFICIENTS

To remove a fraction multiplied by the variable, you may divide by the fraction or multiply by its reciprocal.

For additional information, see Year 2, Chapter 7, problem CT-41. There are also additional examples of dividing fractions by fractions in the "Dividing Fractions Using Reciprocals" section of this guide.

### Example 1

Solve the equation  $\frac{2}{3}x = 8$ .

Using division:

$$\begin{aligned}\frac{2}{3}x &= 8 \\ \frac{\frac{2}{3}x}{\frac{2}{3}} &= \frac{8}{\frac{2}{3}} \\ x &= 8 \div \frac{2}{3} \\ x &= 8 \cdot \frac{3}{2} \\ x &= 12\end{aligned}$$

Using reciprocals:

$$\begin{aligned}\frac{2}{3}x &= 8 \\ \left(\frac{3}{2}\right)\frac{2}{3}x &= 8\left(\frac{3}{2}\right) \\ 1x &= 12 \\ x &= 12\end{aligned}$$

### Example 2

Solve the equation  $-\frac{1}{5}x - 1 = -3$ .

Add 1 to isolate the variable.

Use division or reciprocals to solve.

$$\begin{aligned}-\frac{1}{5}x - 1 &= -3 \\ -\frac{1}{5}x &= -2 \\ x &= -2 \div -\frac{1}{5} \quad \text{or} \quad -2 \cdot (-5) = 10\end{aligned}$$

### Problems

Solve each equation.

1.  $\frac{1}{5}x = 25$

2.  $\frac{1}{3}x = 20$

3.  $\frac{1}{4}x = 9$

4.  $\frac{3}{4}x = 9$

5.  $\frac{2}{3}x = \frac{1}{2}$

6.  $-\frac{1}{2}x = 10$

7.  $-\frac{5}{2}x = -10$

8.  $1\frac{1}{3}x = 8$

9.  $2\frac{1}{5}x = -33$

10.  $\frac{1}{3}x - 7 = -3$

11.  $\frac{1}{4}x + 2 = 6$

12.  $-\frac{2}{3}x - 7 = -10$

## Answers

- |                  |        |        |                    |
|------------------|--------|--------|--------------------|
| 1. 125           | 2. 60  | 3. 36  | 4. 12              |
| 5. $\frac{3}{4}$ | 6. -20 | 7. 4   | 8. 6               |
| 9. -15           | 10. 12 | 11. 16 | 12. $4\frac{1}{2}$ |

## GRAPHING

Graphing is a major thread of mathematics, so this section of the parent guide has many subtopics. The fundamentals of an xy-coordinate grid are taught in Year 1, Chapter 2, problems GS-24 through GS-27 and Year 2, Chapter 1, problem GO-66.

## SCALING THE AXES OF GRAPHS

The characteristics of a complete graph are listed in the Took Kit, Year 1, Chapter 1, problem AR-45 and Year 2, Chapter 1, problem GO-10. The axes of the graph must be marked with equal sized spaces called intervals. The numbers on the axes are the scaling of the axes. The difference between consecutive markings tells the size of each interval.

Sometimes the axis or set of axes is not provided. A student must count the number of usable spaces on the graph paper. How many spaces are usable depends in part on how large the graph will be and how much margin will be given for labeling beside each axis.

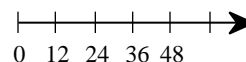
Follow these steps to scale each axis of a graph.

1. Find the difference between the smallest and largest numbers (the range) you need to put on an axis.
2. Count the number of intervals (spaces) you have on your axis.
3. Divide the range by the number of intervals to find the interval size.
4. Label the marks on the axis using the interval size.

Sometimes dividing the range by the number of intervals produces an interval size that makes it difficult to interpret the location of points on the graph. The student may exercise judgment and may round the interval size up (always up, if rounded at all) to a number that is convenient to use. Interval sizes like 1, 2, 5, 10, 20, 25, 50, 100, etc. work well.

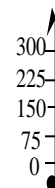
### Example 1

1. The difference between 0 to 60 is 60.
2. The number line is divided into 5 equal intervals.
3. 60 divided by 5 is 12
4. The marks are labeled with multiples of the interval size 12.



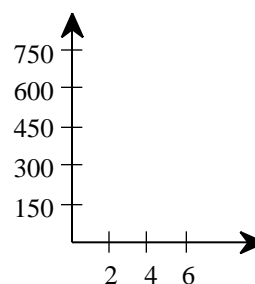
### Example 2

1. The difference between 300 and 0 is 300.
2. There are 4 intervals
3.  $300 \div 4 = 75$
4. The axis is labeled with multiples of 75.



### Example 3

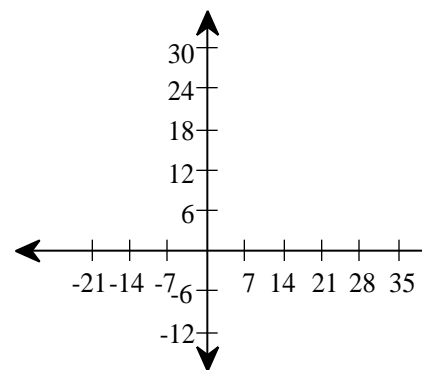
1. The difference on the vertical axis is  $750 - 0 = 750$ . (The origin is  $(0,0)$ .) On the horizontal axis the range is  $6 - 0 = 6$ .
2. There are 5 spaces vertically and 3 spaces horizontally.
3. The vertical interval size is  $750 \div 5 = 150$ . The horizontal interval is  $6 \div 3 = 2$
4. The axes are labeled appropriately.



### Example 4

Sometimes the axes extend in the negative direction. Use the positive direction origin to determine the interval size and then extend the intervals in the negative direction.

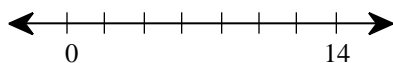
1. The vertical range  $30 - 0 = 30$ .  
The horizontal range is  $35 - 0 = 35$
2. There are 5 intervals in each positive direction.
3. Vertical interval size is  $30 \div 5 = 6$ .  
Horizontal interval size is  $35 \div 5 = 7$ .
4. Label the axes.



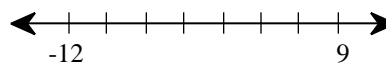
### Problems

Scale each axis:

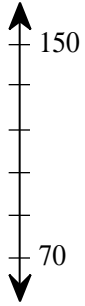
1.



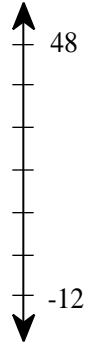
2.



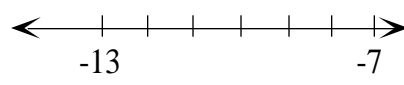
3.



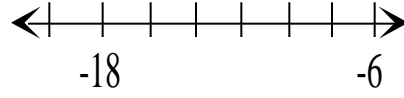
4.



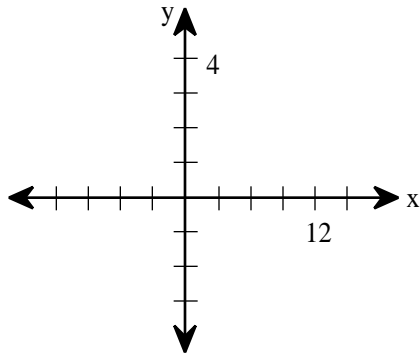
5.



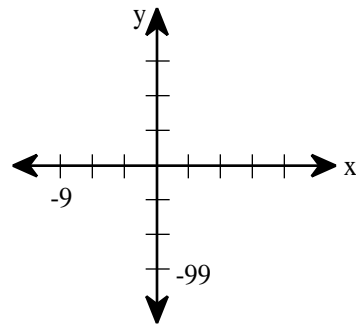
6.



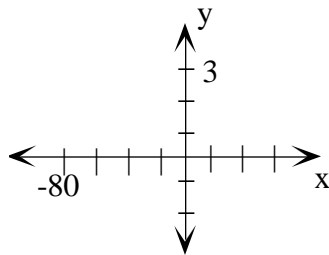
7.



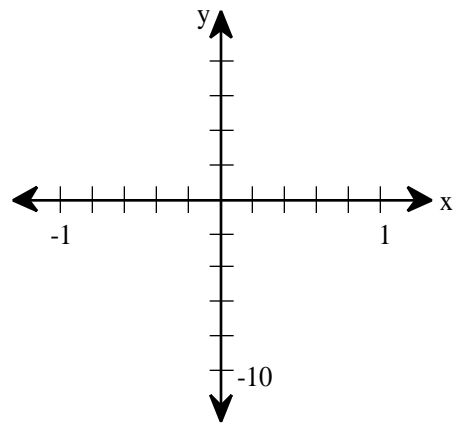
8.



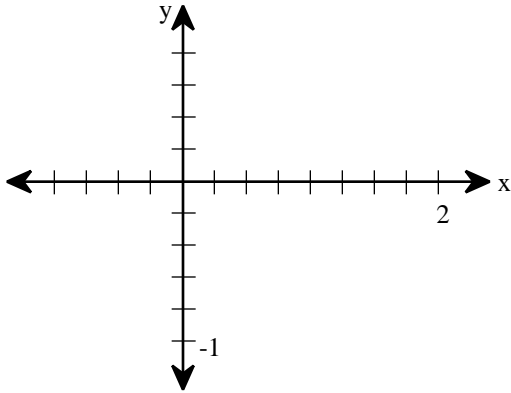
9.



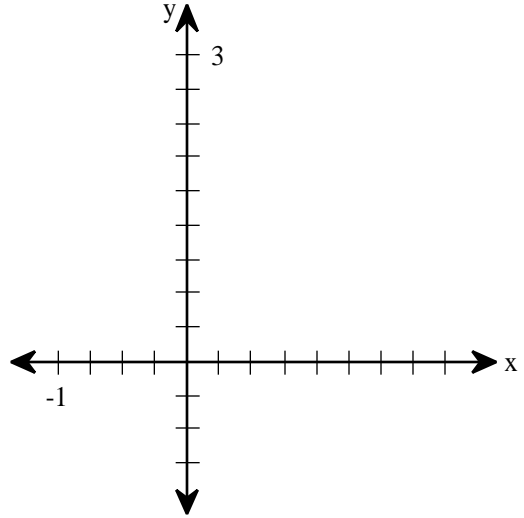
10.



11.



12. Use fractions.



## Answers

1. 2, 4, 6, 8, 10, 12
2. -9, -6, -3, 0, 3, 6
3. 86, 102, 118, 134
4. -2, 8, 18, 28, 38
5. -12, -11, -10, -9, -8
6. -20, -16, -14, -12, -10, -8
7. x: -12, -9, -6, -3, 0, 3, 6, 9, 15  
y: -15, -10, -5, 0, 5, 10, 15
8. x: -6, -3, 0, 3, 6, 9, 12  
y: -66, -33, 0, 33, 66, 99
9. x: -60, -40, -20, 0, 20, 40, 60  
y: -2, -1, 0, 1, 2
10. x: -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8  
y: -8, -6, -4, -2, 0, 2, 4, 6, 8, 10
11. x: -1, -0.75, -0.50, -0.25, -0, .25, .50, .75, 1.00, 1.25, 1.50, 1.75  
y: -0.8, -0.6, -0.4, -0.2, 0, .2, .4, .6, .8
12. x:  $-\frac{3}{4}$ ,  $-\frac{1}{2}$ ,  $-\frac{1}{4}$ , 0,  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$ , 1,  $\frac{5}{4}$ ,  $\frac{3}{2}$ ,  $\frac{7}{4}$ , 2  
y: -1,  $-\frac{2}{3}$ ,  $-\frac{1}{3}$ , 0,  $\frac{1}{3}$ ,  $\frac{2}{3}$ , 1,  $\frac{4}{3}$ ,  $\frac{5}{3}$ , 2,  $\frac{7}{3}$ ,  $\frac{8}{3}$

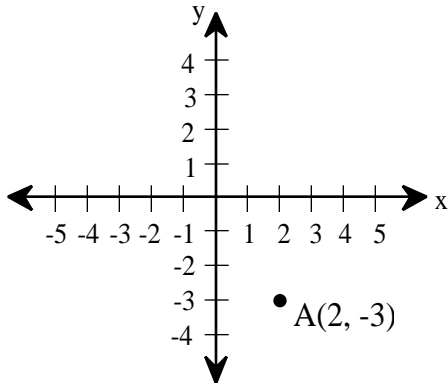
## GRAPHING POINTS

Points on a coordinate grid are written as ordered pairs, (x, y), where the first number is the x-coordinate, that is, the horizontal distance from the y-axis. The second number is the y-coordinate, that is, the vertical distance from the x-axis. Taken together, the two coordinates name exactly one point on the graph. The examples below show how to place a point on an xy-coordinate grid. For more information, see Year 1, Chapter 2, problems GS-24 through 27 or Year 2, Chapter 1, problem GO-66.

### Example 1

Graph point A(2, -3).

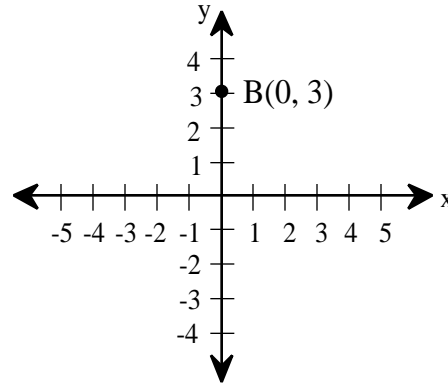
Go right 2 units from the origin (0, 0), then go down 3 units. Mark the point.



### Example 2

Graph the ordered pair B(0, 3).

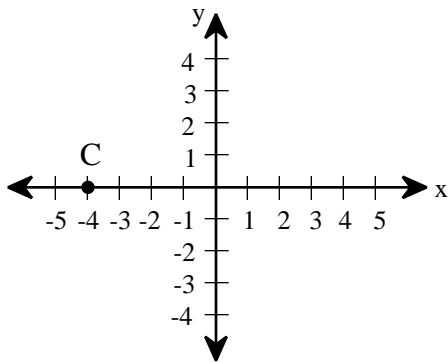
From the origin, do not go right or left; just go up 3 units. Mark the point.



### Example 3

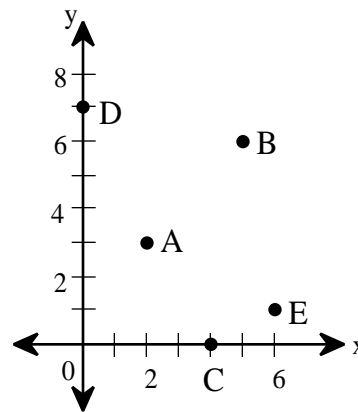
Plot the point C(-4, 0) on a coordinate grid.

Go to the left from the origin 4 units, but do not go up or down. Mark the point.



### Example 4

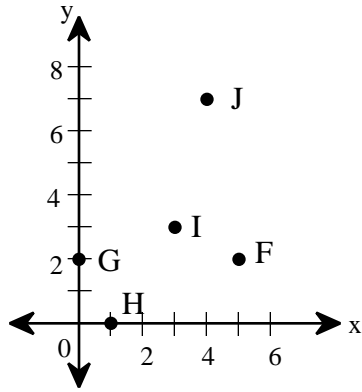
Name the coordinate pair for each point on the graph.



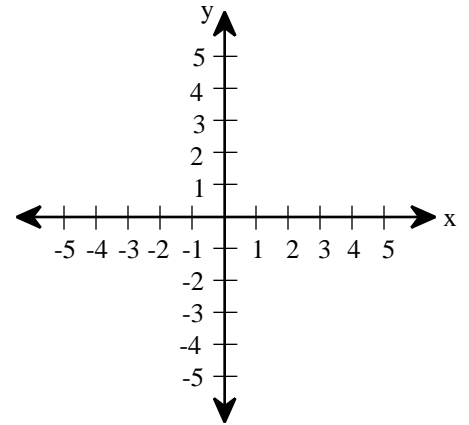
A(2, 3) B(5, 6) C(4, 0) D(0, 7) E(6, 1)

### Problems

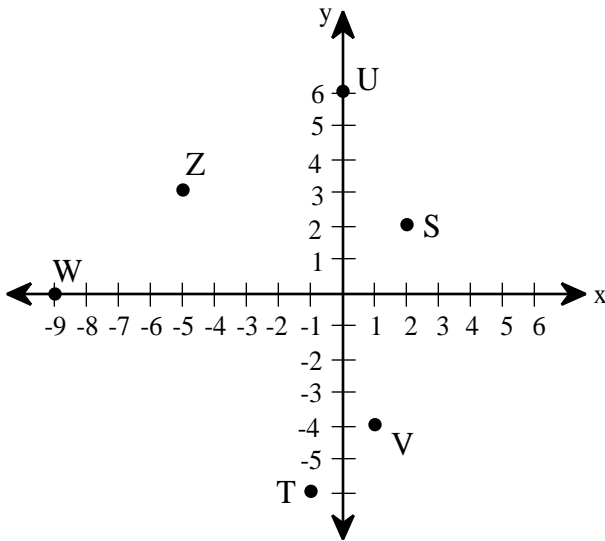
1. Name the coordinate pair for each point shown on the grid below.
2. Use the ordered pair to locate each point on a coordinate grid.



- K(0, -4)
- L(-5, 0)
- M(-2, -3)
- N(-2, 3)
- O(2, -3)
- P(-4, -6)
- Q(4, -5)
- R(-5, -4)
- T(-1, -6)



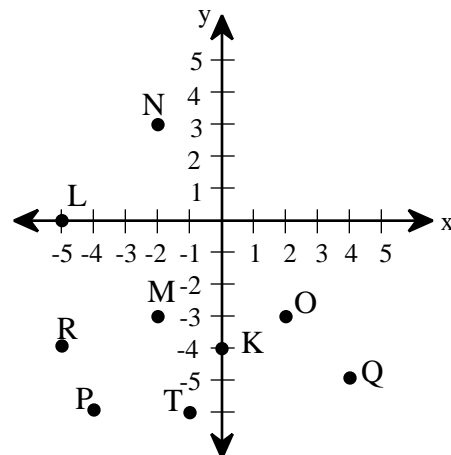
3. Name the coordinates of the points on the coordinate grid below.



### Answers

1. F(5, 2)  
 G(0, 2)  
 H(1, 0)  
 I(3, 3)  
 J(4, 7)

2.



3. S(2, 2)  
 T(-1, -6)  
 U(0, 6)  
 V(1, -4)  
 W(-9, 0)  
 Z(-5, 3)



## GRAPHING ALGEBRAIC RELATIONSHIPS

An algebraic relationship is a rule that describes how two numbers are related. In the rule the numbers are usually represented by the variables  $x$  and  $y$ . More than one ordered pair  $(x, y)$  is described by each rule. To organize several ordered pairs for each rule, list them in a row or column or in a vertical or horizontal table. These organized tables are sometimes referred to as  $xy$ -tables or input/output tables. The ordered pairs are then used to make a graph that illustrates the relationship. For additional information, see Year 1, Chapter 2, problems GS-35, 65, and 67 or Year 2, Chapter 2, problems FT-19 and 45.

### Example 1

Graph the algebraic relationship  $y = x + (-2)$ .

Students need to find and organize some ordered pairs for the graph. Start with a table of  $x$ -values. Then take the  $x$ -values (or the input values) in the table and add  $-2$  to each of them to get the corresponding  $y$ -value (output).

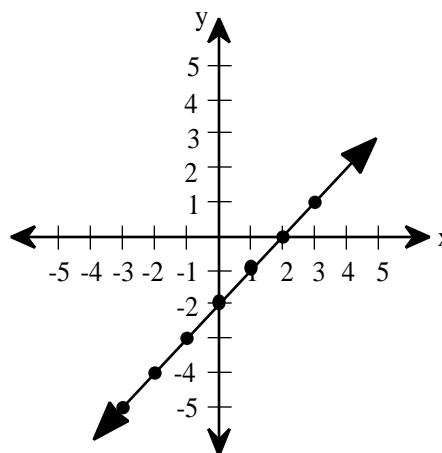
$x$	-3	-2	-1	0	1	2	3
$y$	-5			-2		0	1

$-2 + (-2) = -4$   
 $-1 + (-2) = -3$   
 $1 + (-2) = -1$

Once we have the  $(x, y)$  values completed in the table, we place those points on the coordinate grid. Remember that we start at the origin  $(0,0)$  and move horizontally right or left before we go up or down for the second number (coordinate).

Those points are written  $(x, y)$ :  $(-3, -5)$ ,  $(-2, -4)$ ,  $(-1, -3)$

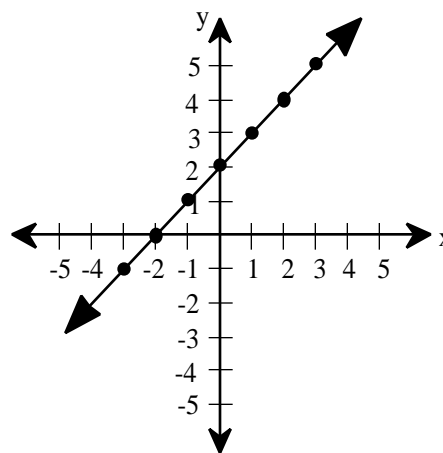
Finally, draw the line or curve through the points when the input values may be any number, including fractions, decimals, and irrational numbers.



## Example 2

For  $y = x + 2$ , add 2 to each  $x$ -value to get the corresponding  $y$ -value.

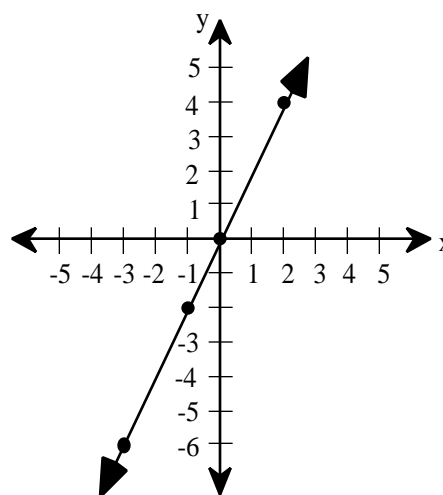
$x$	-3	-2	-1	0	1	2	3
$y$	-1	0	1	2	3	4	5



## Example 3

For  $y = 2x$ , multiply each  $x$ -value to get its corresponding  $y$ -value.

$x$	-3	-2	-1	0	1	2	3
$y$	-6	-4	-2	0	2	4	6



You may want to use the same  $x$  values for each problem. However, some rules are easier to graph if you select  $x$ -values that make the arithmetic simpler. For example, if you use even numbers in problem 5 below, all  $y$ -values will be integers.

$x$	-3	-2	-1	0	1	2	3
$y$							

## Problems

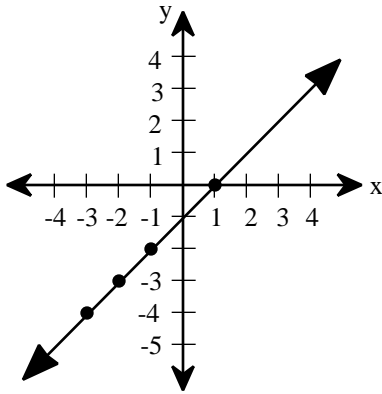
For each rule, make a table to organize the ordered pairs, then graph the relationship on a coordinate grid.

- $y = x + (-1)$
- $y = x + (-4)$
- $y = 2x + 2$
- $y = 3x + 1$
- $y = \frac{x}{2}$
- $y = \frac{x}{2} + 3$
- $y = \frac{x}{3}$
- $y = 3x - 3$

## Answers

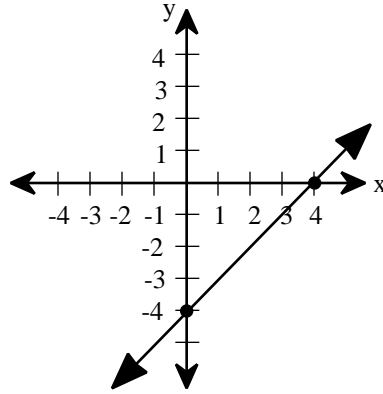
1.  $y = x + (-1)$

x	-3	-2	-1	0	1	2	3
y	-4	-3	-2	-1	0	1	2



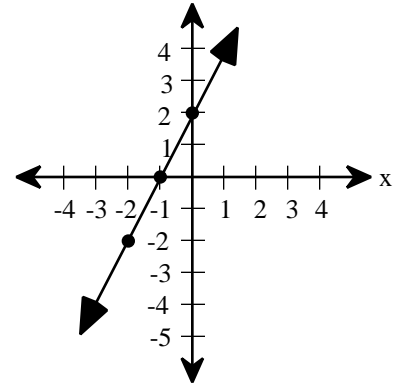
2.  $y = x + (-4)$

x	-3	-2	-1	0	1	2	3
y	-7	-6	-5	-4	-3	-2	-1



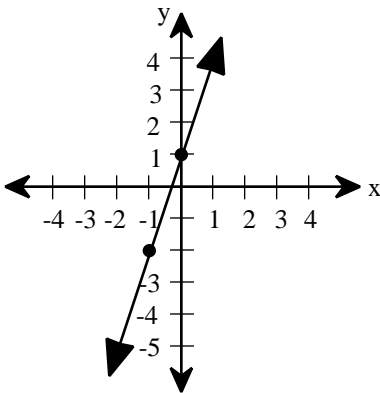
3.  $y = 2x + 2$

x	-3	-2	-1	0	1	2	3
y	-4	-2	0	2	4	6	8



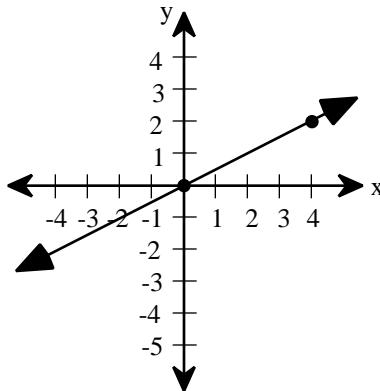
4.  $y = 3x + 1$

x	-4	-2	-1	0	4
y	-11	-5	-2	1	13



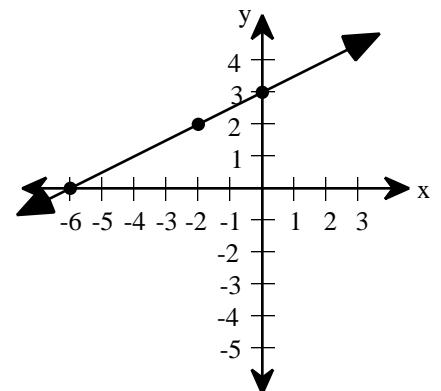
5.  $y = \frac{x}{2}$

x	-4	-2	-1	0	4
y	-2	-1	-0.5	0	2



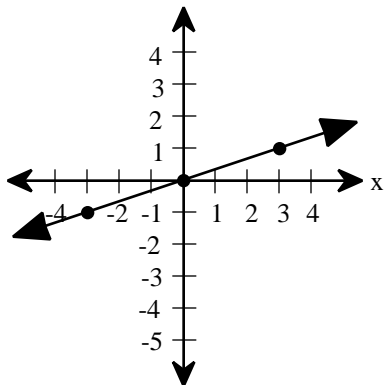
6.  $y = \frac{x}{2} + 3$

x	-4	-2	-1	0	4
y	1	2	2.5	3	5



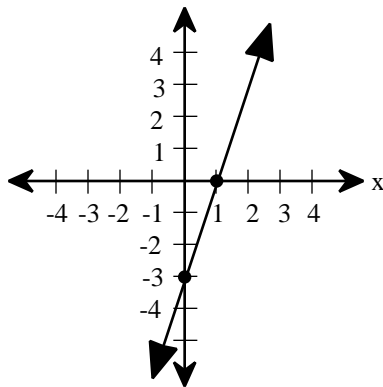
7.  $y = \frac{x}{3}$

x	-3	-1	0	1	3
y	-1	$-\frac{1}{3}$	0	$\frac{1}{3}$	1



8.  $y = 3x - 3$

x	-4	-2	-1	0	4
y	-15	-9	-6	-3	9



The relationship described by each of these rules is a line. Other algebraic rules may produce graphs which follow patterns that are not lines but curves.

# LINEAR FUNCTIONS

## WRITING AND GRAPHING LINEAR EQUATIONS ON A FLAT SURFACE

Slope is a number that indicates the steepness (or flatness) of a line, as well as its direction (up or down) left to right.

Slope is determined by the ratio  $\frac{\text{vertical change}}{\text{horizontal change}}$  between any two points on a line.

For lines that go up (from left to right), the sign of the slope is positive. For lines that go down (left to right), the sign of the slope is negative.

Any linear equation written as  $y = mx + b$ , where  $m$  and  $b$  are any real numbers, is said to be in slope-intercept form. The slope of the line is  $m$ . The  $y$ -intercept is  $b$ , that is, the point  $(0, b)$  where the line intersects (crosses) the  $y$ -axis.

For additional information, see Year 2, Chapter 9, problem CB-39.

### Example 1

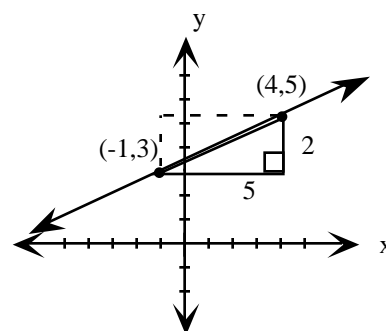
Write the slope of the line containing the points  $(-1, 3)$  and  $(4, 5)$ .

First graph the two points and draw the line through them.

Look for and draw a slope triangle using the two given points.

Write the ratio  $\frac{\text{vertical change in } y}{\text{horizontal change in } x}$  using the legs of the right

triangle:  $\frac{2}{5}$ .



Assign a positive or negative value to the slope (this one is positive) depending on whether the line goes up (+) or down (-) from left to right.

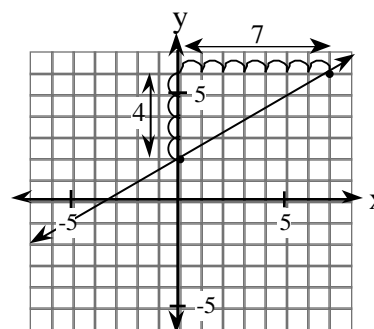
If the points are inconvenient to graph, use a “Generic Slope Triangle,” visualizing where the points lie with respect to each other.

### Example 2

Graph the linear equation  $y = \frac{4}{7}x + 2$  without making a table.

Using  $y = mx + b$ , the slope in  $y = \frac{4}{7}x + 2$  is  $\frac{4}{7}$  and the  $y$ -intercept is the point  $(0, 2)$ . To graph, begin at the

$y$ -intercept  $(0, 2)$ . Remember that slope is  $\frac{\text{vertical change}}{\text{horizontal change}}$  so go up 4 units (since 4 is positive) from  $(0, 2)$  and then move right 7 units. This gives a second point on the graph. To create the graph, draw a straight line through the two points.



## Problems

Write the slope of the line containing each pair of points.

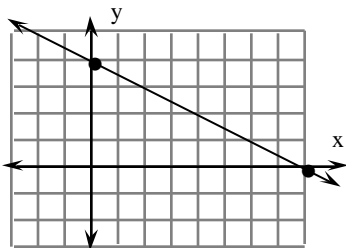
- (3, 4) and (5, 7)
- (5, 2) and (9, 4)
- (1, -3) and (-4, 7)
- (-2, 1) and (2, -2)
- (-2, 3) and (4, 3)
- (8, 3) and (3, 5)

Identify the slope and y-intercept in each equation.

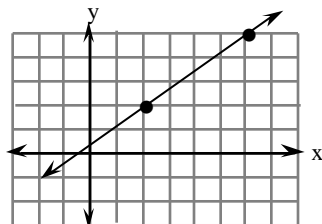
- $y = \frac{1}{2}x - 2$
- $y = -\frac{3}{5}x - \frac{5}{3}$
- $y = 2x + 4$
- $y = \frac{2}{3}x - 5$
- $y = -3x - \frac{1}{2}$
- $\frac{1}{2}x + 3$

Determine the slope of each line using the highlighted points.

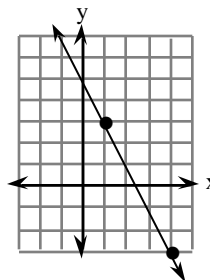
13.



14.



15.



Graph the following linear equations on graph paper without making a table.

- $y = \frac{1}{2}x + 3$
- $y = -\frac{3}{5}x - 1$
- $y = 4x$
- $y = -6x + \frac{1}{2}$
- $3x + 2y = 12$

## Answers

- $\frac{3}{2}$
- $\frac{1}{2}$
- 2
- $-\frac{3}{4}$
- 0
- $-\frac{2}{5}$
- $\frac{1}{2}, (0, -2)$
- $-\frac{3}{5}, (0, -\frac{5}{3})$
- 2, (0, 4)
- $\frac{2}{3}, (0, -5)$
- $-3, (0, -\frac{1}{2})$
- $\frac{1}{2}, (0, 3)$
- $-\frac{1}{2}$
- $\frac{3}{4}$
- 2
- line with slope  $\frac{1}{2}$  and y-intercept (0, 3)

17. line with slope  $-\frac{3}{5}$  and y-intercept  $(0, -1)$     18. line with slope 4 and y-intercept  $(0, 0)$

19. line with slope -6 and y-intercept  $(0, \frac{1}{2})$     20. line with slope  $-\frac{3}{2}$  and y-intercept  $(0, 6)$

Note: The topic of rate of change is addressed in the Proportions section of this guide (found in “Number Sense,” p. 21).

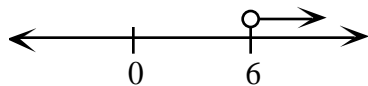
## INEQUALITIES

### GRAPHING INEQUALITIES

The solutions to an equation can be represented as a point (or points) on the number line. The solutions to inequalities are represented by rays or segments with solid or open endpoints. Solid endpoints indicate that the endpoint is included in the solution ( $\leq$  or  $\geq$ ), while the open dot indicates that it is not part of the solution ( $<$  or  $>$ ). For additional information, see year 2, Chapter 9, problems CB-79, 80, and 92.

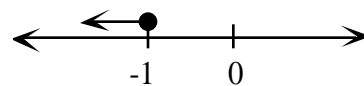
#### Example 1

$$x > 6$$



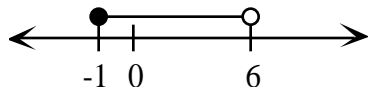
#### Example 2

$$x \leq -1$$



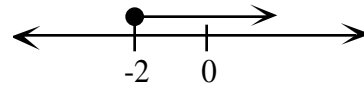
#### Example 3

$$-1 \leq y < 6$$



#### Example 4

$$y \geq -2$$



### Problems

Graph each inequality on a number line.

1.  $m < 2$

2.  $x \leq -1$

3.  $y \geq 3$

4.  $-1 \leq x \leq 3$

5.  $-6 < x < -2$

6.  $-1 < x < 2$

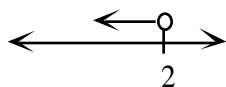
7.  $m > -9$

8.  $x \geq 1$

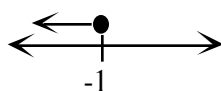
9.  $x \leq 3$

## Answers

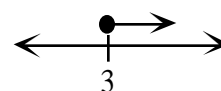
1.



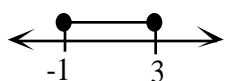
2.



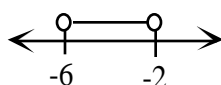
3.



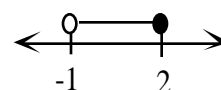
4.



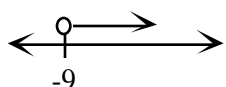
5.



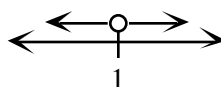
6.



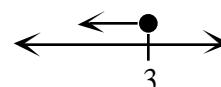
7.



8.



9.



## SOLVING INEQUALITIES

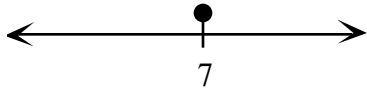
To solve an inequality, first solve it as you would an equation. Use the solution as a dividing point of the line. Then test a value from each side of the dividing point on the number line. If the test number is true, then that part of the number line is part of the solution. In addition, if the inequality is  $\geq$  or  $\leq$ , then the dividing point is part of the solution and is indicated by a solid dot. If the inequality is  $>$  or  $<$ , then the dividing point is not part of the solution, indicated by an open dot.

### Example 1

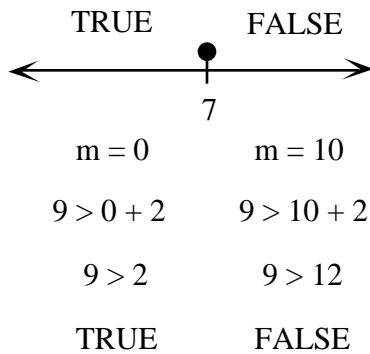
$$9 = m + 2$$

Solve the equation:  $9 = m + 2$   
 $7 = m$

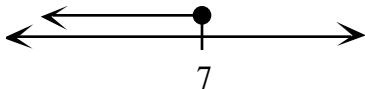
Draw a number line. Put a solid dot at 7.



Test a number on each side of 7 in the original inequality. We use 10 and 0.



The solution is  $m > 7$ .

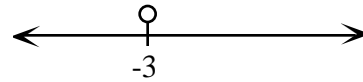


### Example 2

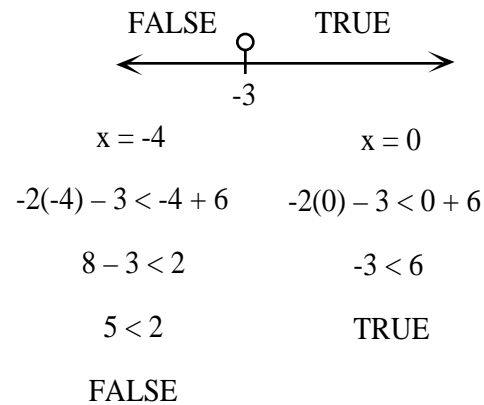
$$-2x - 3 < x + 6$$

Solve the equation:  $-2x - 3 = x + 6$   
 $-2x = x + 9$   
 $-3x = 9$   
 $x = -3$

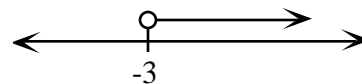
Draw a number line. Put an open dot at -3.



Test 0 and -4 in the original inequality.



The solution is  $x > -3$ .



### Problems

Solve each inequality.

1.  $x + 3 > -1$

2.  $y - 3 \leq 5$

3.  $-3x \leq -6$

4.  $2m + 1 \leq -7$

5.  $-7 < -2y + 3$

6.  $8 \leq -2m + 2$

7.  $2x - 1 < -x + 8$

8.  $2(m + 1) \leq m - 3$

9.  $3m + 1 \leq m + 7$

### Answers

1.  $x > -4$

2.  $y \leq 8$

3.  $x \geq 2$

4.  $m \leq -4$

5.  $y < 5$

6.  $m \leq -3$

7.  $x < 3$

8.  $m \leq -5$

9.  $m \leq 3$