# *Number Sense*

**Integers**
**Absolute Value**
**Ratio**
**Proportions**
**Percentage of Increase and Decrease**
**Simple and Compound Interest**
**Equivalent Fractions**
**Addition and Subtraction of Fractions**
**Multiplying and Dividing Fractions**
**Fraction, Decimal, and Percent Equivalents**
**Laws of Exponents**
**Scientific Notation**
**Diamond Problems**

# INTEGERS

## MODELING INTEGERS

Integers are positive and negative whole numbers and zero. A negative integer is written with a negative sign in front of the numeral, such as -6 and -11, and is to the left of zero on the number line. A positive integer may be written with or without a positive sign in front of the number. For example, 5 and +5 are both ways of indicating 5 units (or "steps") to the right of zero on the number line.

We use tiles to model integers. Positive integer tiles are represented with a positive sign (+). Negative integer tiles are represented with a negative sign ( – ). Zero is represented by an equal number of positive signs and negative signs and is referred to as a neutral field composed of zero pairs. Zero pairs are circled.

4 is modeled as ++++ and recorded as + 4.

-3 is modeled as – – – and recorded as -3.

0 is modeled as $\begin{matrix}+\\-\end{matrix}$ and recorded as 0. (A negative neutralizes a positive or makes it zero.)

$\begin{matrix}+ & + & + & +\\- & - & - & \end{matrix}$ This model represents +1 because any number of zeros does not change the value of the number being modeled.

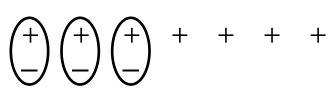
For additional information, see Year 1, Chapter 2, problems GS-3 and 4 or Year 2, Chapter 2, problems FT-27 and 28.

### Examples

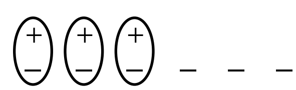
a) zero pairs in a neutral field

$\begin{matrix}+ & + & + & +\\- & - & - & -\end{matrix}$

b) zero pairs in the number 4



c) -3 with more than 6 tiles



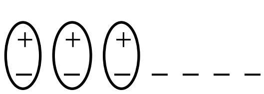
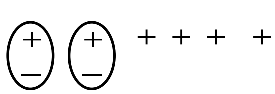
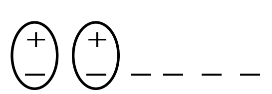
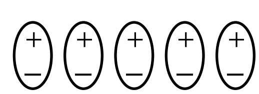
### Problems

Represent the following integers with at least 9 tiles.

1. -4
2. 5
3. -6
4. 0

### Answers

Note: In this section of the guide, answers are provided as examples of correct answers. There may be several acceptable models for some problems.

1. 
2. 
3. 
4. 

## ADDITION OF INTEGERS WITH TILES

Tiles can be used to model addition of integers.

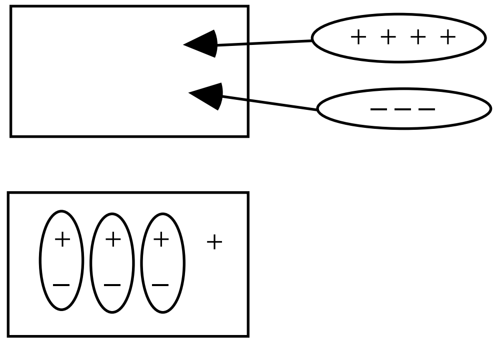
4 + (-3)

Draw a box.

Build the first number.
Put it in the box.

Build the second number.
Put it in the box.

Circle all the zero pairs.

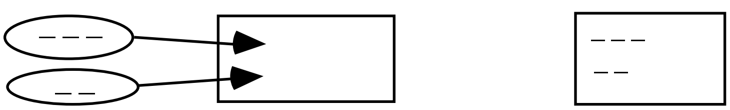


Count the uncircled tiles and record the equat

For more information on modeling addition of integers, see Year 1, Chapter 2, problems GS-5 and 14 or Year 2, Chapter 2, problems FT-29 and 30.

### Example 1

Draw tiles to compute -3 + (-2).

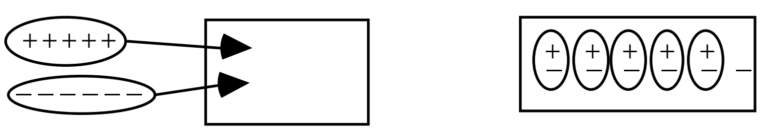


-3 + (-2) = -5

There are no zero pairs here to circle, so the answer is -5.

### Example 2

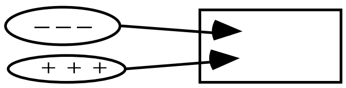
Draw tiles to add 5 + (-6). Remember to circle the zero pairs.

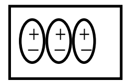


5 + (-6) = -1

## Example 3

Draw tiles to compute -3 + 3.





-3 + 3 = 0

## Problems

Draw tiles to compute the following problems.

1. -2 + (-3)
2. -2 + (-5)
3. 5 + 2
4. 4 + (-4)
5. 5 + (-3)
6. -5 + 3
7. -3 + 7
8. -5 + 6
9. -2 + 2
10. 1 + (-4) + (-1)

## Answers

1. -2 + (-3) = -5

   – – – – –

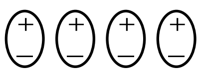
2. -2 + (-5) = -7

   – – – – – – –

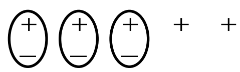
3. 5 + 2 = 7

   + + + + + + +

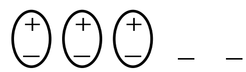
4. 4 + (-4) = 0

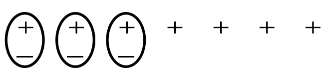
5. 5 + (-3) = 2

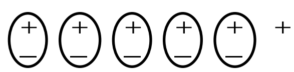
6. -5 + 3 = -2

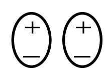
7. -3 + 7 = 4

8. -5 + 6 = 1

9. -2 + 2 = 0

10. 1 + (-4) + (-1)= -4

## ADDITION OF INTEGERS IN GENERAL

When you added integers using the tile model, zero pairs were only formed if the two numbers had different signs. After you circled the zero pairs, you counted the uncircled tiles to find the sum. If the signs were the same, no zero pairs were formed and you found the sum of the tiles. Integers can be added without building models using the rules below.

- If the signs are the same, add the numbers and keep the same sign.
- If the signs are different, ignore the signs (that is, use the absolute value of each number.) Subtract the number closest to zero from the number farthest from zero. The sign of the answer is the same as the number that is farthest from zero, that is, the number with the greater absolute value.

For more information on the rules for addition of integers, see Year 2, Chapter 2, problem FT-50.

### Example

-4 + 2 (-4) is farther from zero on the number line than 2, so |-4| - |2| = 4 - 2 = 2. The answer is -2, since the "4" is negative.

### Problems

Use either the Tile (Neutral Field) Model or the rules above to find these sums and/or differences.

1. 4 + (-2)
2. 6 + (-1)
3. 7 + (-7)
4. -10 + 6
5. -8 + 2
6. -12 + 7
7. -5 + (-8)
8. -10 + (-2)
9. -11 + (-16)
10. -8 + 10
11. -7 + 15
12. -26 + 12
13. -3 + 4 + 6
14. 56 + 17
15. 7 + (-10) + (-3)
16. -95 + 26
17. 35 + (-6) + 8
18. -113 + 274
19. 105 + (-65) + 20
20. -6 + 2 + (-4) + 3 + 5
21. 5 + (-3) + (-2) + (-8)
22. -6 + (-3) + (-2) + 9
23. -6 + (-3) + 9
24. 20 + (-70)
25. 12 + (-7) + (-8) + 4 + (-3)
26. -26 + (-13)
27. -16 + (-8) + 9
28. 12 + (-13) + 18 + (-16)
29. 50 + (-70) + 30
30. 19 + (-14) + (-5) + 20

### Answers

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 2 | 2. | 5 | 3. | 0 | 4. | -4 | 5. | -6 | 6. | -5 |
| 7. | -13 | 8. | -12 | 9. | -27 | 10. | 2 | 11. | 8 | 12. | -14 |
| 13. | 7 | 14. | 73 | 15. | -6 | 16. | -69 | 17. | 37 | 18. | 161 |
| 19. | 60 | 20. | 0 | 21. | -8 | 22. | -2 | 23. | 0 | 24. | -50 |
| 25. | -2 | 26. | -39 | 27. | -15 | 28. | 1 | 29. | 10 | 30. | 20 |

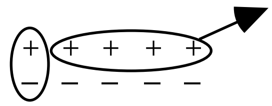
## SUBTRACTION OF INTEGERS

In class, students are introduced to subtraction of integers by using tiles. They build neutral fields of zero pairs and then remove some tiles. For example: 0 - 4

Start with a neutral field of zero pairs.

\+ + + + +
− − − − −

Remove 4 positive tiles. What is left?



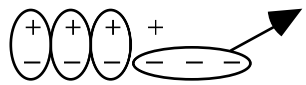
The zero pairs left are circled and the remaining tiles are counted. The answer is -4. They record the difference in an equation.

0 - 4 = -4

In -2 – (-3), students build the first integer with a neutral field.

\+ + + +
− − − − − −

Next, they physically remove the second integer.



They need to be careful here to circle all remaining zero pairs.

They record the difference in an equation.

-2 – (-3) = 1

Sometimes a neutral field is not needed. For example, -6 – (-3) does not need a neutral field.

Build the first integer.

− − − − − −

Ask yourself, “Are there enough negative tiles so that I can remove 3 negatives?” If the answer is “yes,” you do not need a neutral field. Remove the three negative tiles and record your answer in an equation.

-6 – (-3) = -3

Be careful with problems like -6 – 3. This problem means -6 minus a positive 3. You will need a neutral field because −−−−−− (-6) has no positive tiles to remove.

-6 – 3 = -9

For more information on modeling subtraction of integers, see Year 1, Chapter 3, problems PR-15 through 16, 41 through 43, and 50 or Year 2, Chapter 2, problems FT-55 through 58.

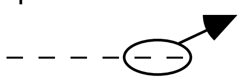
## Problems

Find each difference.  Use tiles for at least the first five differences.

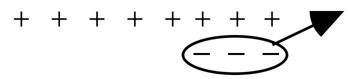
1. $-6 - (-2)$
2. $5 - (-3)$
3. $6 - (-3)$
4. $-7 - 3$
5. $7 - (-3)$
6. $7 - 3$
7. $5 - (3)$
8. $-12 - (-10)$
9. $-12 - 10$
10. $12 - (-10)$
11. $-6 - (-3) - 5$
12. $6 - (-3) - 5$
13. $8 - (-8)$
14. $-9 - 9$
15. $-9 - 9 - (-9)$

## Answers

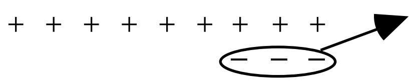
1. -4



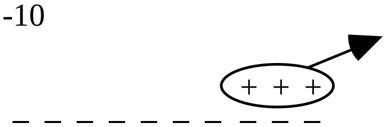
2. 8



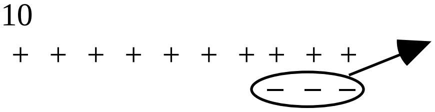
3. 9



4. -10



5. 10



6. 4
7. 2
8. -2
9. -22
10. 22
11. -8
12. 4
13. 16
14. -18
15. -9

In the next six examples, compare (a) to (b), (c) to (d), and (e) to (f). Notice that examples (a), (c), and (e) are subtraction problems and examples (b), (d), and (f) are addition problems. The answers to each pair of examples are the same. Also notice that the second integers in the pairs are opposites (that is, they are the same distance from zero on opposite sides of the number line) while the first integers in each pair are the same.

a) 2 – (-6) 2 – (-6) = 8

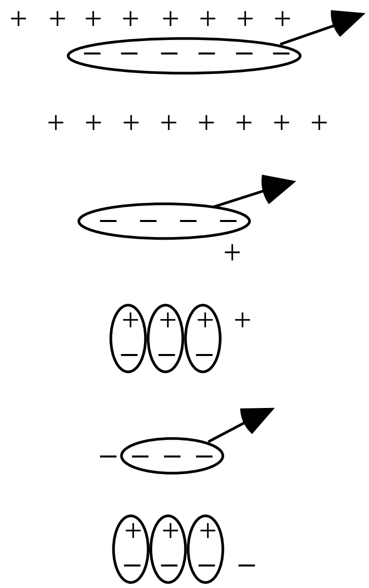
b) 2 + 6 2 + 6 = 8

c) -3 – (-4) -3 – (-4) = 1

d) -3 + 4 -3 + 4 = 1

e) -4 – (-3) -4 – (-3) = -1

f) -4 + 3 -4 + 3 = -1



You can conclude that subtracting an integer is the same as adding its opposite. This is summarized below.

**Subtraction of Integers in General:** To find the difference of two integers, change the subtraction sign to an addition sign. Next change the sign of the integer you are subtracting, then apply the rules for addition of integers.

For more information on the rules for subtracting integers, see Year 1, Chapter 3, problem PR-50 or Year 2, Chapter 2, problems FT-86 through 88.

## Examples

Use the rule for subtracting integers stated above to compute.

a) 9 – (-12) becomes 9 + (+12) = 21

b) -9 – (-12) becomes -9 + (+12) = 3

c) -9 – 12 becomes -9 + (-12) = -21

d) 9 – 12 becomes 9 + (-12) = -3

## Problems

Use the rule stated above to find each difference.

| | | | | | |
|---|---|---|---|---|---|
| 1. | 9 – (-3) | 2. | 9 – 3 | 3. | -9 – 3 |
| 4. | -9 – (-3) | 5. | -14 – 15 | 6. | -16 – (-15) |
| 7. | -40 – 62 | 8. | -40 – (-62) | 9. | 40 – 62 |
| 10. | 40 – (-62) | 11. | -5 – (-3) – 5 – 6 | 12. | -5 – 3 – (-5) – (-6) |
| 13. | 5 – 3 – (-5) – 6 | 14. | 5 – (-4) – 6 – (-7) | 15. | -125 – (-125) – 125 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 16. | 5 – (-6) | 17. | 12 – 14 | 18. | 20 – 25 | 19. | -3 – 2 |
| 20. | -7 – 3 | 21. | -10 – 5 | 22. | -30 – 7 | 23. | -3 – (-3) |
| 24. | -3 – (-4) | 25. | 10 – (-3) | 26. | 5 – (-9) | 27. | 27 – (-7) |
| 28. | 15 – 32 | 29. | -58 – 37 | 30. | -79 – (-32) | 31. | -62 – 81 |
| 32. | -106 – 242 | 33. | 47 – (-55) | 34. | 257 – 349 | 35. | -1010 – (-1010) |

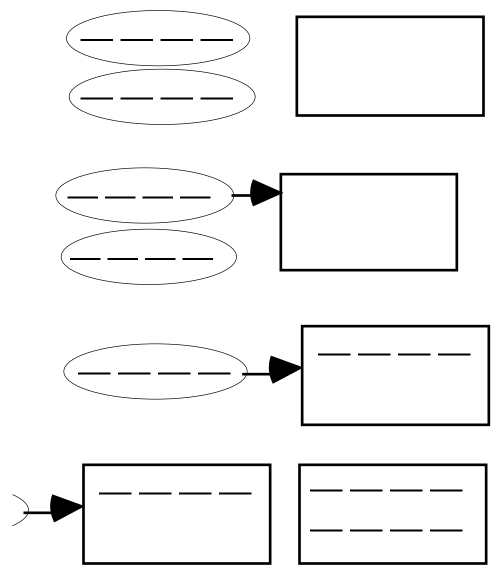
## Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 12 | 2. | 6 | 3. | -12 | 4. | -6 | 5. | -29 |
| 6. | -1 | 7. | -102 | 8. | 22 | 9. | -22 | 10. | 102 |
| 11. | -13 | 12. | 3 | 13. | 1 | 14. | 10 | 15. | -125 |
| 16. | 11 | 17. | -2 | 18. | -5 | 19. | -5 | 20. | -10 |
| 21. | -15 | 22. | -37 | 23. | 0 | 24. | 1 | 25. | 13 |
| 26. | 14 | 27. | 34 | 28. | -17 | 29. | -95 | 30. | -47 |
| 31. | -143 | 32. | -348 | 33. | 102 | 34. | -92 | 35. | 0 |

# MULTIPLICATION OF INTEGERS WITH TILES

Multiplication is repeated addition. 4 · 3 is the same as 3 + 3 + 3 + 3. Integer tiles can be used to model multiplication of integers.

**Modeling Multiplication With Tiles When the First Factor is Positive:** An example is 2(-4). The first factor (2) is a positive number. We will put 2 groups into a box. The second factor (-4) tells how many are in each group.



Step 1: Start with an empty box. Build two sets of four negative tiles.

Step 2: Physically push the tiles into the box, one group at a time, to show the repeated addition.

a) "One group of -4 ..."

b) "Two groups of -4."

Step 3: Count the tiles in the box to find the product.

Step 4: Write the multiplication equation.

2(-4) = -8

For more information on modeling multiplication of integers, see Year 1, Chapter 2, problem GS-48 and Chapter 3, problems PR-54 and 71, or Year 2, Chapter 2, problems FT-102 through 107.

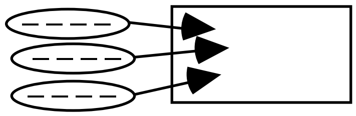
## Problems

Use the tile drawings to multiply in the first five problems below. Remember to write an equation to show your answer. Find the product for each of the remaining problems.

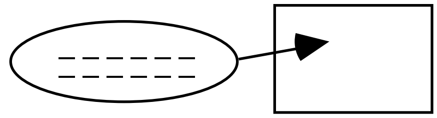
1. 3(-4)
2. 2(-6)
3. 5(-1)
4. 6(-2)
5. 7(-3)
6. (8)(-7)
7. 7(-12)
8. 13(-5)
9. (9)(-8)
10. 7(-5)
11. 8(-6)
12. 9(-7)
13. 10(8)
14. 22(-10)
15. 4(-30)

## Answers

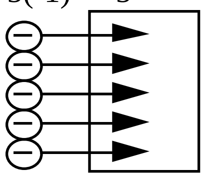
1. 3(-4) = -12



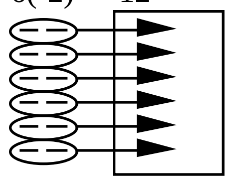
2. 2(-6) = -12



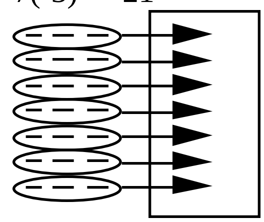
3. 5(-1) = -5



4. 6(-2) = -12



5. 7(-3) = -21



6. -56 7. -84 8. -65 9. -72 10. -48

11. -63 12. -80 13. -220 14. -120

**Modeling Multiplication With Tiles When the First Factor is Negative:** Students have seen the tile model for multiplying integers like 4(-3), which means adding in 4 groups of -3 to get a total of -12. When the first number is a negative, such as -4(3), the multiplication problem becomes repeated subtraction.

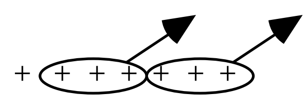
## Example 1

Use tile drawings to multiply (-2)(3).

Step 1: Build a large neutral field.

\+ + + + + + +
– – – – – – –

Step 2: Circle and remove one group of +3. Do it again. You removed two groups of +3.



Step 3: Circle any zero pairs. Count the remaining tiles.

\+
– – – – – – – -6

Step 4: Record the equation, showing the product.

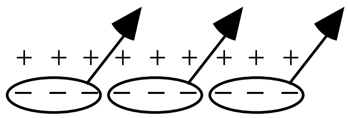
(-2)(3) = -6

### Example 2

Use tile drawings to multiply (-3)(-3).

Step 1: + + + + + + + + +
– – – – – – – – –

Step 2:



Step 3: + + + + + + + + + 9

Step 4: (-3)(-3) = 9

## MULTIPLICATION OF INTEGERS IN GENERAL

To multiply without tiles, use the absolute value of the product and use the rules below to determine the sign of the answer.

- When pairs of integers with the same sign are multiplied, the product is positive.
- When pairs of integers with different signs are multiplied, the product is negative.

For more information about the rules for multiplication of integers, see Year 1, Chapter 3, problems PR-66 and PR-71 or Year 2, Chapter 2, problems FT-106 through 109.

### Problems

Use either the Tile (Neutral Field) Model or the rules above to find each product.

| | | | |
|---|---|---|---|
| 1. (-3)(2) | 2. (-5)(4) | 3. (-11)(7) | 4. (-12)(5) |
| 5. (-23)(7) | 6. (4)(-6) | 7. (7)(-9) | 8. (15)(-8) |
| 9. (35)(-3) | 10. (115)(-7) | 11. (-5)(-6) | 12. (-7)(-8) |
| 13. (-24)(-8) | 14. (-137)(-4) | 15. (-231)(-17) | 16. (-3)(5)(-6) |
| 17. (-7)(-4)(3) | 18. (-5)(-5)(-5) | 19. (-3)(-2)(6)(4) | 20. (-5)(-4)(-2)(-3) |
| 21. -2(4) | 22. -5(1) | 23. 2(-3) | 24. -15(-10) |
| 25. -15(10) | 26. 12(-12) | 27. -11(-10) | 28. (-2)(-3)(-5) |
| 29. (-2)(-3)(4)(6) | 30. (-2)(-3)(-4)(-6) | 31. (-2)(-3)(4)(-6) | 32. 2(-3)(4)(-6) |
| 33. -2(3)(-5) | | | |

## Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | -6 | 2. | -20 | 3. | -77 | 4. | -60 | 5. | -161 |
| 6. | -24 | 7. | -63 | 8. | -120 | 9. | -105 | 10. | -805 |
| 11. | 30 | 12. | 56 | 13. | 192 | 14. | 548 | 15. | 3927 |
| 16. | 90 | 17. | 84 | 18. | -125 | 19. | 144 | 20. | 120 |
| 21. | -8 | 22. | -5 | 23. | -6 | 24. | 150 | 25. | -150 |
| 26. | -144 | 27. | 110 | 28. | -30 | 29. | 144 | 30. | 144 |
| 31. | -144 | 32. | 144 | 33. | 30 | | | | |

## DIVISION OF INTEGERS

When dividing integers, the rules for the sign of the product are the same as those for multiplication. Use the absolute values to divide and then determine the sign. When dividing two integers with the same sign, the result is positive. When dividing two integers with different signs, the result is negative.

Examples $14 \div (-7) = -2$ $-14 \div (-7) = 2$

For more information on division of integers, see Year 1, Chapter 6, problems MB-109 and 110 or Year 2, Chapter 2, problems FT-118 through 121.

### Problems

Divide.

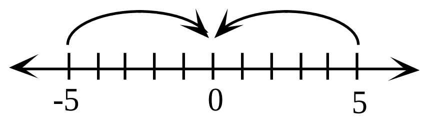
| | | | | | |
|---|---|---|---|---|---|
| 1. | $10 \div (-2)$ | 2. | $15 \div (-3)$ | 3. | $108 \div (-3)$ |
| 4. | $258 \div (-6)$ | 5. | $-14 \div 7$ | 6. | $-56 \div 4$ |
| 7. | $-110 \div 11$ | 8. | $-95 \div 95$ | 9. | $-68 \div (-4)$ |
| 10. | $-125 \div (-25)$ | 11. | $-96 \div (-12)$ | 12. | $-115 \div 23$ |
| 13. | $342 \div (-6)$ | 14. | $- 217 \div (-217)$ | 15. | $-2088 \div (-24)$ |

### Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | -5 | 2. | -5 | 3. | -36 | 4. | -43 | 5. | -2 |
| 6. | -14 | 7. | -10 | 8. | -1 | 9. | 17 | 10. | 5 |
| 11. | 8 | 12. | -5 | 13. | -57 | 14. | 1 | 15. | 87 |

## ABSOLUTE VALUE

**Absolute value** is the distance a point is from zero on the number line. It is always a positive number since it measures the physical distance from zero.




The symbol for absolute value is | | . On the number line above, both 5 and -5 are 5 units from zero. This distance is displayed as |-5| = 5 and is read, "The absolute value of negative five equals five." Similarly, |5| = 5 means, "The absolute value of five is five."

|x| = 5 means that x could be <u>either</u> 5 or -5 because both of those points are five units from zero.

The problem |x| = -5 has no solution because the absolute value of a number has to be positive. The only exception to this is when a negative sign appears outside the absolute value symbol.

For additional information, see Year 1, Chapter 2, problem GS-76 or Year 2, Chapter 2, problem FT-44.

### Examples

a) |-6| = 6 b) |7| = 7 c) |x| = 9 x = -9 or 9

d) |x| = -3 e) - |x| = -3 -3 or 3 f) |3 - 8| = |-5| = 5

Part (d) has no solution, since any absolute value is positive.

Notice the negative sign outside the absolute value symbol in the last example. This sign means "the opposite of the absolute value."

### Problems

Determine the absolute value or the values of x.

| | | | | |
|---|---|---|---|---|
| 1. \|-11\| | 2. \|12\| | 3. \|x\| = 4 | 4. \|x\| = 16 | 5. \|x\| = 24 |
| 6. \|x\| = 13 | 7. \|-9\| | 8. \|x\| = -13 | 9. - \|x\| = -13 | 10. - \|7\| |
| 11. \|x\| = 7 | 12. \|-7\| | 13. \|5 – 8\| | 14. \|-6 – 3\| | 15. \|-6 + 3\| |

### Answers

| | | | | |
|---|---|---|---|---|
| 1. 11 | 2. 12 | 3. 4, -4 | 4. 16, -16 | 5. 24, -24 |
| 6. 13, -13 | 7. 9 | 8. no solution | 9. 13, -13 | 10. -7 |
| 11. 7, -7 | 12. 7 | 13. 3 | 14. 9 | 15. 3 |

# RATIO

A **ratio** is a comparison of two quantities by division. It can be written in several ways:

$\frac{\text{65 miles}}{\text{1 hour}}$ , 65 miles: 1 hour, or 65 miles to 1 hour.

Both quantities of a ratio can be multiplied by the same number. We can use a **ratio table** to organize the multiples. Each ratio in the table will be equivalent to the others. Patterns in the ratio table can be used in problem solving.

For additional information, see Year 1, Chapter 5, problem GO-1 or Year 2, Chapter 6, problem RS-3.

## Example 1

120 cups of coffee can be made from one pound of coffee beans. Doubling the amount of coffee beans will double the number of cups of coffee that can be made. Use the doubling pattern to complete the ratio table for different weights of coffee beans.

| Pounds of coffee beans | 1 | 2 | 4 | 8 |
|---|---|---|---|---|
| Cups of coffee | 120 | 240 | | |

Doubling two pounds of beans doubles the number of cups of coffee made, so for 4 pounds of beans, $2 \cdot 240 = 480$ cups of coffee are made. Since 4 pounds of beans make 480 cups of coffee, 8 pounds of beans make $2 \cdot 480 = 960$ cups of coffee.

## Example 2

You can use the ratio table from Example 1 to determine how many cups of coffee you could make from 6 pounds of beans. Add another column to your ratio table.

| Pounds of coffee beans | 1 | 2 | 4 | 8 | 6 |
|---|---|---|---|---|---|
| Cups of coffee | 120 | 240 | 480 | 960 | |

You know that the value of 6 is halfway between the values of 4 and 8. The value halfway between 480 and 960 is 720, so 6 pounds of beans should make 720 cups of coffee.

## Example 3

Jane's cookie recipe uses $2\frac{1}{2}$ cups of flour and 2 eggs. She needs to know how much flour she will need if she uses 9 eggs. Jane started a ratio table but discovered that she could not simply keep doubling because 9 is not a multiple of 2. She used other patterns to find her answer. Study the top row of Jane's table to find the patterns she used, then complete the table.

| Number of eggs | 2 | 4 | 12 | 36 | 9 |
|---|---|---|---|---|---|
| Cups of flour | $2\frac{1}{2}$ | | | | |

Jane doubled to get 4, tripled 4 to get 12, and tripled again to get 36. Then she divided 36 by four to get to 9. She followed the same steps to complete the "cups of flour" row. Jane will need $11\frac{1}{4}$ cups of flour.

| Number of eggs | 2 | 4 | 12 | 36 | 9 |
|---|---|---|---|---|---|
| Cups of flour | $2\frac{1}{2}$ | 5 | 15 | 45 | $11\frac{1}{4}$ |

> In the examples above, the ratio tables have the ratios listed in the order in which they were calculated. When a row of a ratio table is provided, it is acceptable to complete a ratio table in the order that fits the patterns you see and use. In most cases it is easier to see patterns by listing the values in the first (or top) row in order.

## Example 4

Peter and Mandy each completed this ratio table correctly, but they did it in a different order using different patterns.

| 2 | 4 | 6 | 8 |
|---|---|---|---|
| 17 | | | |

Peter tripled $\frac{2}{17}$ to get $\frac{6}{51}$. Then he doubled $\frac{2}{17}$ to get $\frac{4}{34}$ and doubled that to get $\frac{8}{68}$. Mandy, however, doubled $\frac{2}{17}$ to get $\frac{4}{34}$ and doubled that to get $\frac{8}{68}$. Then she used the halfway values to get $\frac{6}{51}$.

## Problems

Complete each ratio table.

1.

| 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 |
|---|---|---|---|---|---|---|---|
| 5 | 10 | | | | | | |

2.

| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 30 |
|---|---|---|---|---|---|---|---|
| 7 | 14 | | | | | | |

3.

| 5 | | | 20 | 30 | 45 | 60 | 100 |
|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | | | | | |

4.

| 7 | 14 | 35 | | 49 | | 700 | |
|---|---|---|---|---|---|---|---|
| 4 | | | 40 | | 100 | | 1000 |

5.

| 6 | 12 | 24 | 18 | | 48 | | 120 |
|---|---|---|---|---|---|---|---|
| 11 | | | | 66 | | 99 | |

6.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 12 | 18 | 30 | | | 360 | |
| 5.7 | | | | 57 | 114 | | 684 |

7.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 9 | 36 | 45 | | | 180 | 900 | 300 |
| 48 | | | 120 | 480 | | | |

8.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10 | 15 | 20 | 25 | | 35 | | 4000 |
| 8.5 | | | | 34 | | 340 | |

## Answers

1.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 |
| 5 | 10 | 15 | 20 | 25 | 30 | 40 | 50 |

2.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 30 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 70 |

3.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5 | 10 | 15 | 20 | 30 | 45 | 60 | 100 |
| 2 | 4 | 6 | 8 | 12 | 18 | 24 | 40 |

4.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7 | 14 | 35 | 70 | 49 | 175 | 700 | 1750 |
| 4 | 8 | 20 | 40 | 28 | 100 | 400 | 1000 |

5.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 12 | 24 | 18 | 36 | 48 | 54 | 120 |
| 11 | 22 | 44 | 33 | 66 | 88 | 99 | 220 |

6.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 12 | 18 | 30 | 60 | 120 | 360 | 720 |
| 5.7 | 11.4 | 17.1 | 28.5 | 57 | 114 | 342 | 684 |

7.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 9 | 36 | 45 | 22.5 | 90 | 180 | 900 | 300 |
| 48 | 192 | 240 | 120 | 480 | 960 | 4800 | 1600 |

8.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10 | 15 | 20 | 25 | 40 | 35 | 400 | 4000 |
| 8.5 | 12.75 | 17 | 21.25 | 34 | 29.75 | 340 | 3400 |

## PROPORTIONS

### SOLVING PROPORTIONS

A **proportion** is an equation that states that two ratios are equal.

For example: $\frac{3}{8} = \frac{6}{16}$ , $\frac{4}{12} = \frac{6}{18}$ , and $\frac{2}{5} = \frac{x}{40}$ .

Some proportions have an unknown value such as the x in the last example. Because the unknown value is part of two equivalent fractions, you can use various methods for finding equivalent fractions to solve a proportion like $\frac{2}{5} = \frac{x}{40}$ .

One method of solving a proportion is to use a ratio table like the one at right. From the ratio table you can determine that x = 16.

| | | | |
|---|---|---|---|
| 2 | 4 | 8 | 16 |
| 5 | 10 | 20 | 40 |

Another method is to use the Giant **1**, as shown at right.

$\frac{2}{5} = \frac{x}{40}$ $\qquad \frac{2}{5} \cdot \frac{8}{8} = \frac{16}{40}$ $\qquad x = 16$

For additional information, see Year 1, Chapter 5, probler MB-8 or Year 2, Chapter 6, problems RS-17 and 23.

### Problems

Use a ratio table to solve each proportion, then use the Giant **1** to solve them again.

1. $\frac{3}{7} = \frac{x}{112}$
2. $\frac{4}{9} = \frac{x}{216}$
3. $\frac{2}{5} = \frac{x}{95}$
4. $\frac{2}{11} = \frac{x}{198}$
5. $\frac{14}{17} = \frac{x}{238}$
6. $\frac{12}{19} = \frac{324}{x}$

### Answers

1. x = 48 2. x = 96 3. x = 38 4. x = 36 5. x = 196 6. x = 513

# APPLICATIONS USING PROPORTIONS

Proportions can be used to solve various types of problems.

To solve a problem using a proportion, write the information you know in one ratio and the information you need to know in another ratio with a variable to represent what is unknown. Keep the units in each ratio in the same order, such as $\frac{\text{65 miles}}{\text{1 hour}} = \frac{\text{x miles}}{\text{4 hours}}$.
Solve the proportion using the method you prefer.

## Example 1

Balvina knows that 6 cups of rice will make enough Spanish rice to feed 15 people. She needs to know how many cups of rice are needed to feed 135 people. Write a proportion to solve the problem.

What we know is $\frac{\text{6 cups}}{\text{15 people}}$, and what we want to know is $\frac{\text{x cups}}{\text{135 people}}$, so $\frac{6}{15} = \frac{x}{135}$.

We can solve the proportion using a ratio table.

| Cups of Rice | 6 | 12 | 18 | 24 | 48 | 54 |
|---|---|---|---|---|---|---|
| Servings | 15 | 30 | 45 | 60 | 120 | 135 |

Balvina needs 54 cups of rice to feed 135 people.

We can also solve the proportion using the Giant **1**.

$\frac{6}{15}\left(\frac{\ }{\ }\right) = \frac{x}{135}$ Since 135 ÷ 15 = 9, use $\frac{9}{9}$ in the Giant **1**.

$\frac{6}{15}\left(\frac{9}{9}\right) = \frac{54}{135}$ Again, Balvina needs 54 cups of rice.

## Example 2

Ivanna needs to buy 360 cherries for a large salad. She can buy 9 cherries for $0.57. How much will 360 cherries cost Ivanna? Write a proportion to solve the problem.

What we know is $\frac{\text{9 cherries}}{\$0.57}$, and what we want to know is $\frac{\text{360 cherries}}{\$x}$, so $\frac{9}{0.57} = \frac{360}{x}$.

We can solve the proportion using a ratio table. To get from 9 cherries to 360 cherries, we multiply the known ratio by 10, then by 4.

| Number of Cherries | 9 | 90 | 360 |
|---|---|---|---|
| Cost | $0.57 | $5.70 | $22.80 |

The 360 cherries will cost Ivanna $22.80.

We can also solve the proportion using the Giant **1**.

$\frac{9}{0.57}\ \boxed{-} = \frac{360}{x}$ Since 360 ÷ 9 = 40, use $\frac{40}{40}$ in the Giant **1**.

$\frac{9}{0.57}\ \boxed{\frac{40}{40}} = \frac{360}{22.80}$ Again, the 360 cherries will cost Ivanna \$22.80.

The problems below involve rate. For additional information, see Year 2, Chapter 9, problems CB-14 and 17.

## Example 3

Elaine can plant 6 flowers in 15 minutes. How long will it take her to plant 30 flowers at the same rate? Write a proportion to solve the problem.

What we know is $\frac{\text{6 flowers}}{\text{15 minutes}}$, and what we want to know is $\frac{\text{30 flowers}}{\text{x minutes}}$, so $\frac{6}{15} = \frac{30}{x}$.

We can solve the proportion using a ratio table.

| Number of Flowers | 6 | 12 | 18 | 24 | 30 |
|---|---|---|---|---|---|
| Number of Minutes | 15 | 30 | 45 | 60 | 75 |

It will take Elaine 75 minutes (or 1 hour and 15 minutes) to plant 30 flowers.

We can also solve the proportion using the Giant **1**.

$\frac{6}{15}\ \boxed{-} = \frac{30}{x}$ Since 30 ÷ 6 = 5, use $\frac{5}{5}$ in the Giant **1**.

$\frac{6}{15}\ \boxed{\frac{5}{5}} = \frac{30}{75}$ Again, it will take Elaine 75 minutes to plant 30 flowers.

## Problems

Solve the following proportions involving rate.

1. A plane travels 3400 miles in 8 hours. How far would it travel in 6 hours at this rate?

2. Shane rode his bike for 2 hours and traveled 12 miles. At this rate, how long would it take him to travel 22 miles?

3. Selina’s car used 15.6 gallons of gas to go 234 miles. At this rate, how many gallons would it take her to go 480 miles?

## Answers

1. 2550 miles
2. $3\frac{2}{3}$ hours
3. 32 gallons

Proportions can also be used to solve percent problems by writing $\frac{\%}{100} = \frac{\text{part}}{\text{whole}}$. For more information, refer to Year 1, Chapter 6, problem MB-64 or Year 2, Chapter 6, problem RS-37.

## Example 4

32 is what percent of 40?

$\frac{32}{40} = \frac{x}{100}$

x = 80, so 32 is 80% of 40.

## Example 5

15% of 80 is what number?

$\frac{15}{100} = \frac{x}{80}$

x = 12

## Problems

Solve the following percent problems using proportions.

1. What is 8% tax on an $80 purchase?
2. Bev scored 33 points on a 40-point quiz. What was her percent correct?
3. Elizabeth answered 114 questions correctly on her science test. Her score was 95%. How many questions were on the test?

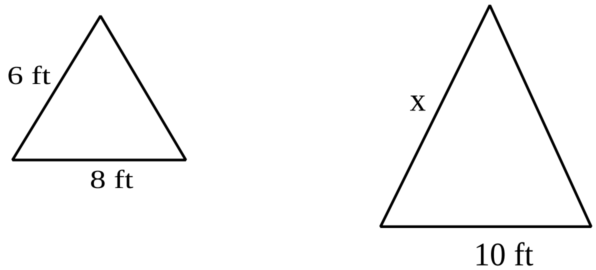
## Answers

1. $6.40
2. 82.5%
3. 120 questions

The ratio of pairs of corresponding sides of similar figures are equal. When given a pair of similar figures, you can use a proportion to find the length of an unknown side. For more information, refer to Year 1, Chapter 6, problem MB-87 or Year 2, Chapter 6, problems RS-75 through RS-77.

## Example 6

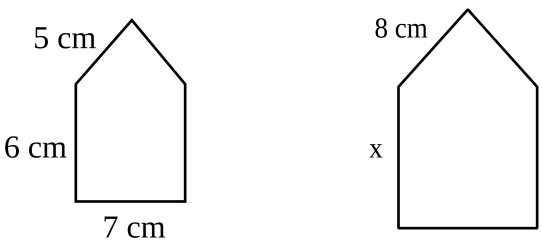
Two similar triangles.




$$\frac{x\ \text{ft}}{6\ \text{ft}} = \frac{10\ \text{ft}}{8\ \text{ft}} \qquad x = 7.5\ \text{ft}$$

## Example 7

Two similar pentagons.

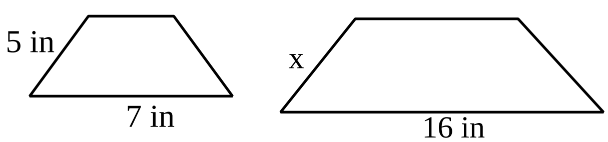



$$\frac{5\ \text{cm}}{8\ \text{cm}} = \frac{6\ \text{cm}}{x\ \text{cm}} \qquad x = 9.6\ \text{cm}$$

## Problems

Solve for the unknown side in these similar figures.

1.

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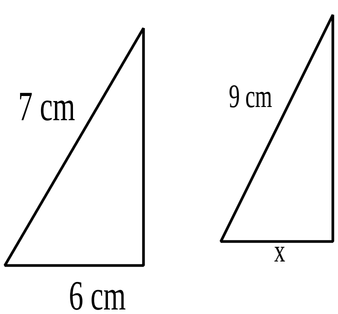


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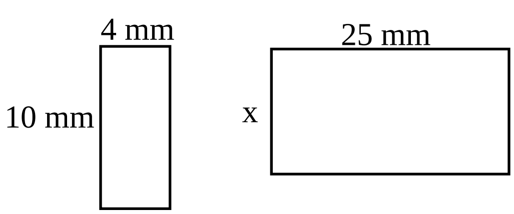


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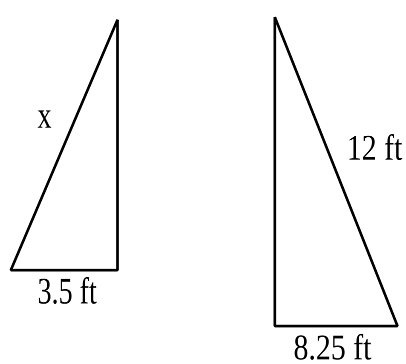



## Answers

1. 11.4 in
2. 7.7 cm
3. 10 mm
4. 5.09 ft

## PERCENTAGE OF INCREASE AND DECREASE

Another kind of problem that can be solved using proportions involves finding the percentage of increase or decrease. Students have been taught to use the following formula to find percent:

$$\frac{\%}{100} = \frac{\text{part}}{\text{whole}}$$

When change in percent is found, the same concept is used. The formula becomes:

$$\frac{\%}{100} = \frac{\text{change (increase or decrease)}}{\text{original amount}}$$

For additional information, see Year 2, Chapter 7, problem CT-105.

### Example 1

A town's population grew from 1,879 to 7,426 over five years. What was the percentage of increase?

- Subtract to find the change:

$$7426 - 1879 = 5547$$

- Put the known numbers in the proportion:

$$\frac{\%}{100} = \frac{5547}{1879} \; \frac{\text{increase}}{\text{original}}$$

- The percentage becomes x, the unknown:

$$\frac{x}{100} = \frac{5547}{1879}$$

- Cross multiply:

$$1879x = 554{,}700$$

- Divide each side by 1879:

$$x = 295.2\% \text{ increase}$$

The population increased by 295.2%

### Example 2

A sumo wrestler retired from sumo wrestling and went on a diet. When he retired he weighed 385 pounds. After two years he weighed 238 pounds. What was the percentage of decrease in his weight?

- Subtract to find the change:

$$385 - 238 = 147$$

- Put the known numbers in the proportion:

$$\frac{\%}{100} = \frac{147}{385} \; \frac{\text{decrease}}{\text{original}}$$

- The percentage becomes x, the unknown:

$$\frac{x}{100} = \frac{147}{385}$$

- Cross multiply:

$$385x = 14{,}700$$

- Divide each side by 385:

$$x = 38.\overline{18}\%.$$

His weight decreased by $38.\overline{18}\%$.

## Problems

Solve the following problems.

1. Thirty years ago gasoline cost $0.30 per gallon on average. Today gasoline averages about $1.50 per gallon. What is the percentage of increase in the cost of gasoline?

2. When Spencer was 5, he was 28 inches tall. Today he is 5 feet 3 inches tall. What is the percentage of increase in Spencer's height?

3. The cars of the early 1900s cost $500. Today a new car costs an average of $27,000. What is the percentage of increase of the cost of an automobile?

4. The population of the U.S. at the first census in 1790 was 3,929 people. By 2000 the population had increased to 284,000,000! What is the percentage of increase?

5. In 2000 the rate for a first class U.S. postage stamp increased to $0.34. This represents a $0.31 increase since 1917. What is the percentage of increase since 1917?

6. In 1906 Americans consumed an average of 26.85 gallons of whole milk per year. By 1998 the average consumption was 8.32 gallons. What is the percentage of decrease in consumption of whole milk?

7. In 1984 there were 125 students for each computer in U.S. public schools. By 1998 there were 6.1 students for each computer. What is the percentage of decrease in the ratio of students to computers?

8. Sara bought a dress on sale for $30. She saved 45%. What was the original cost?

9. Pat was shopping and found a jacket with the original price of $120 on sale for $9.99. What was the percentage of decrease?

10. The price of a pair of pants decreased from $49.99 to $19.95. What was the percentage of decrease?

## Answers

1. $1.50 - 0.30 = 1.20$; $\frac{x}{100} = \frac{1.20}{0.30}$ ; $x = 400\%$

2. $63 - 28 = 35$; $\frac{x}{100} = \frac{35}{28}$ ; $x = 125\%$

3. $27{,}000 - 500 = 26{,}500$; $\frac{x}{100} = \frac{26\,500}{500}$ ; $x = 5300\%$

4. $284{,}000{,}000 - 3929 = 283{,}996{,}071$; $\frac{x}{100} = \frac{283\,996\,071}{3929}$ ; $x = 7{,}228{,}202.4\%$

5. $\frac{x}{100} = \frac{0.31}{0.34}$ ; $x = 91.2\%$

6. $26.85 - 8.32 = 18.53$; $\frac{x}{100} = \frac{18.53}{26.85}$ ; $x = 69.01\%$

7. $125 - 6.1 = 118.9$; $\frac{x}{100} = \frac{118.9}{125}$ ; $x = 95.12\%$

8. $100 - 45 = 55\%$ decrease; $\frac{55}{100} = \frac{30}{x}$ ; $x = \$55$

9. $120 - 9.99 = 110.01$; $\frac{x}{100} = \frac{110.01}{120}$ ; $x = 91.7\%$

10. $49.99 - 19.95 = 30.04$; $\frac{x}{100} = \frac{30.04}{49.99}$ ; $x = 60.1\%$

# SIMPLE AND COMPOUND INTEREST

Students are introduced to the idea of earning interest in Chapter 7 of Year 2. First they find simple interest using the formula $I = Prt$. For additional information, see Year 2, Chapter 7, problem CT-57.

## Example 1

Wayne earns 5.3% simple interest for 5 years on \$3000. How much interest does he earn?

| | | |
|---|---|---|
| Put the numbers in the formula $I = Prt$. | $I = 3000(5.3\%)5$ | |
| Change the percent to a decimal. | $= 3000(0.053)5$ | |
| Multiply. | $= 795$ | Wayne would earn \$795 interest. |

Students are introduced to compound interest using the formula $A = P(1 + r)^t$. For additional information, see Year 2, Chapter 7, problem CT-58.

## Example 2

Use the numbers in Example 1 to find how much money Wayne would have if he earned 5.3% interest compounded annually.

| | |
|---|---|
| Put the numbers in the formula $A = P(1 + r)^t$. | $A = 3000(1 + 5.3\%)^5$ |
| Change the percent to a decimal. | $= 3000(1 + 0.053)^5$ |
| Multiply. | $= 3883.86$ |

Wayne would have \$3883.86.

Students are asked to compare the difference in earnings when an amount is earning simple or compound interest. In these examples, Wayne would have \$88.86 more with compound interest than he would have with simple interest: \$3883.86 – \$3795 = \$88.86.

## Problems

Solve the following problems.

1. Tong loaned Jody \$50 for a month. He charged 5% simple interest for the month. How much did Jody have to pay Tong?

2. Jessica's grandparents gave her \$2000 for college to put in a savings account until she starts college in four years. Her grandparents agreed to pay her an additional 7.5% simple interest on the \$2000 for every year. How much extra money will her grandparents give her at the end of four years?

3. David read an ad offering $8\frac{3}{4}$ % simple interest on accounts over $500 left for a minimum of 5 years. He has $500 and thinks this sounds like a great deal. How much money will he earn in the 5 years?

4. Javier's parents set an amount of money aside when he was born. They earned 4.5% simple interest on that money each year. When Javier was 15, the account had a total of $1012.50 interest paid on it. How much did Javier's parents set aside when he was born?

5. Kristina received $125 for her birthday. Her parents offered to pay her 3.5% simple interest per year if she would save it for at least one year. How much interest could Kristina earn?

6. Kristina decided she would do better if she put her money in the bank, which paid 2.8% interest compounded annually. Was she right?

7. Suppose Jessica (from problem 2) had put her $2000 in the bank at 3.25% interest compounded annually. How much money would she have earned there at the end of 4 years?

8. Mai put $4250 in the bank at 4.4% interest compounded annually. How much was in her account after 7 years?

9. What is the difference in the amount of money in the bank after five years if $2500 is invested at 3.2% interest compounded annually or at 2.9% interest compounded annually?

10. Ronna was listening to her parents talking about what a good deal compounded interest was for a retirement account. She wondered how much money she would have if she invested $2000 at age 20 at 2.8% interest compounded quarterly and left it until she reached age 65. Determine what the value of the $2000 would become.

## Answers

1. I = 50(0.05)1 = $2.50; Jody paid back $52.50.

2. I = 2000(0.075)4 = $600

3. I = $500(0.0875)5 = $218.75

4. $1012.50 = x(0.045)15; x = $1500

5. I = 125(0.035)1 = $4.38

6. $A = 125(1 + 0.028)^1$ = $128.50; no, for one year she needs to take the higher interest rate if the compounding is done annually Only after one year will compounding outstrip simple interest.

7. $A = 2000(1 + 0.0325)^4$ = $2272.95

8. $A = 4250(1 + 0.044)^7$ = $5745.03

9. $A = 2500(1 + 0.032)^5 – 2500(1 + 0.029)^5$ = $2926.43 – $2884.14 = $42.29

10. $A = 2000(1 + 0.028)^{180}$ (because 45 · 4 = 180 quarters) = $288,264.15

## EQUIVALENT FRACTIONS

Fractions that name the same value are called **equivalent fractions**, such as $\frac{2}{3} = \frac{6}{9}$ .
Three methods for finding equivalent fractions are using a ratio table, the Identity Property of Multiplication (the Giant **1**), and a rectangular area model. The ratio table method is discussed in this guide in the "Ratio" section.

For additional information, see Year 1, Chapter 5, problems GO-14 and GO-80 or Year 2, Chapter 3, problem MD-28.

## THE IDENTITY PROPERTY OF MULTIPLICATION or THE GIANT 1

Multiplying by 1 does not change the value of a number. The Giant **1** uses a fraction that has the same numerator and denominator, such as $\frac{2}{2}$ , to find an equivalent fraction.

### Example 1

Find three equivalent fractions for $\frac{1}{2}$ .

$\frac{1}{2} \cdot \frac{2}{2} = \frac{2}{4}$ $\qquad$ $\frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6}$ $\qquad$ $\frac{1}{2} \cdot \frac{4}{4} = \frac{4}{8}$

### Example 2

Use the Giant **1** to find an equivalent fraction to $\frac{7}{12}$ using 96ths: $\frac{7}{12} \cdot 1 = \frac{?}{96}$

Which Giant 1 do you use?

Since 96 ÷ 12 = 8, the Giant **1** is $\frac{8}{8}$ : $\frac{7}{12} \cdot \frac{8}{8} = \frac{56}{96}$

### Problems

Use the Giant **1** to find the specified equivalent fraction. Your answer should include the Giant **1** you use and the equivalent numerator.

1. $\frac{4}{3} \cdot 1 = \frac{?}{15}$
2. $\frac{5}{9} \cdot 1 = \frac{?}{36}$
3. $\frac{9}{2} \cdot 1 = \frac{?}{38}$
4. $\frac{3}{7} \cdot 1 = \frac{?}{28}$
5. $\frac{5}{3} \cdot 1 = \frac{?}{18}$
6. $\frac{6}{5} \cdot 1 = \frac{?}{15}$

## Answers

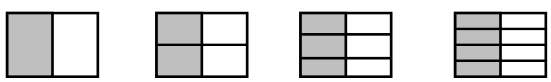
1. $\frac{5}{5}$, 20 2. $\frac{4}{4}$, 20 3. $\frac{19}{19}$, 171 4. $\frac{4}{4}$, 12 5. $\frac{6}{6}$, 30 6. $\frac{3}{3}$, 18

## RECTANGULAR AREA MODEL

This method for finding equivalent fractions is based on the area of a rectangle. Draw and shade a rectangle to represent the original fraction. Next, add horizontal lines to the rectangle to divide the area equally so that the rectangle has the same number of equal pieces as the number in the denominator of the second fraction. Note that each rectangle has the same amount of shaded area. Renaming the shaded area in terms of the new, smaller pieces gives the equivalent fraction.

### Example 1

Use the rectangle area model to find three equivalent fractions for $\frac{1}{2}$.



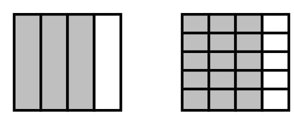
$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

The number of horizontal rows in the rectangle model corresponds to a Giant **1**, $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$.

### Example 2

Use the area model to find the specified equivalent fraction.

$$\frac{3}{4} \cdot 1 = \frac{?}{20}$$



$$\frac{3}{4} \cdot \frac{5}{5} = \frac{15}{20}$$

After drawing the fraction $\frac{3}{4}$, the diagram is divided into five horizontal rows because 20 ÷ 4 equals 5. The diagram now shows 15 shaded parts out of 20 total parts. This area model shows the equivalent fractions: $\frac{3}{4} = \frac{15}{20}$.

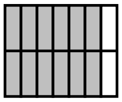
## Problems

Draw rectangular models to find the specified equivalent fraction.

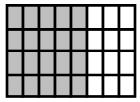
1. $\frac{6}{7} = \frac{?}{14}$ 2. $\frac{5}{8} = \frac{?}{32}$ 3. $\frac{2}{9} = \frac{?}{18}$ 4. $\frac{4}{3} = \frac{?}{9}$

## Answers

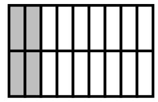
1. $\frac{12}{14}$



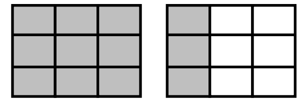
2. $\frac{20}{32}$



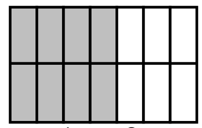
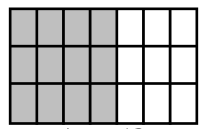
3. $\frac{4}{18}$



4. $\frac{12}{9}$



The following table summarizes the three methods for finding equivalent fractions.

| Fraction | Ratio Table | Giant 1 | Rectangular Model |
|---|---|---|---|
| $\frac{4}{7}$ | 4 \| 8 \| 12<br>7 \| 14 \| 21 | $\frac{4}{7}$ $\frac{2}{2}$ = $\frac{8}{14}$<br>$\frac{4}{7}$ $\frac{3}{3}$ = $\frac{12}{21}$ | <br>$\frac{4}{7} = \frac{8}{14}$<br><br>$\frac{4}{7} = \frac{12}{21}$ |

## ADDITION AND SUBTRACTION OF FRACTIONS

Before fractions can be added or subtracted, the fractions must have the same denominator, that is, a common denominator. We will present three methods for adding or subtracting fractions.

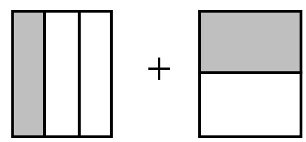
## AREA MODEL METHOD

For additional information, see Year 1, Chapter 7, problems GH-3, 4, 5 or Year 2, Chapter 3, problem MD-56.

Step 1: Copy the problem.

$\frac{1}{3} + \frac{1}{2}$

Step 2: Draw and divide equal-sized rectangles for each fraction. One rectangle is cut horizontally. The other is cut vertically. Label each rectangle, with the fraction it represents.

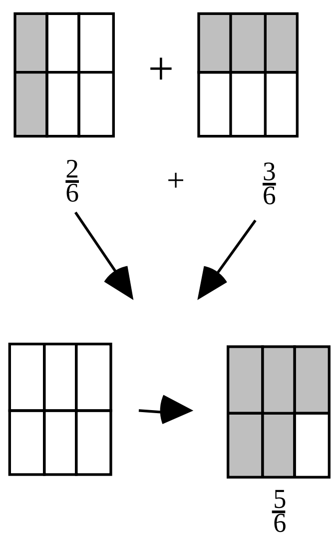


$\frac{1}{3} \quad + \quad \frac{1}{2}$

Step 3: Superimpose the lines from each rectangle onto the other rectangle, as if one rectangle is placed on top of the other one.

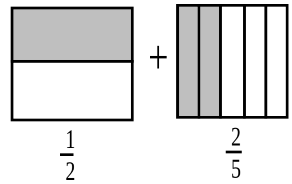
Step 4: Rename the fractions as sixths, because the new rectangles are divided into six equal parts. Change the numerators to match the number of sixths in each figure.

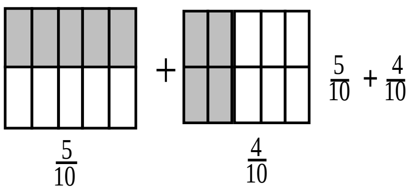
Step 5: Draw an empty rectangle with sixths, then combine all sixths by shading the same number of sixths in the new rectangle as the total that were shaded in both rectangles from the previous step.



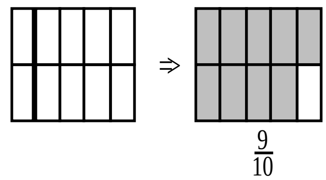
### Example 1

$\frac{1}{2} + \frac{2}{5}$ can be modeled as:





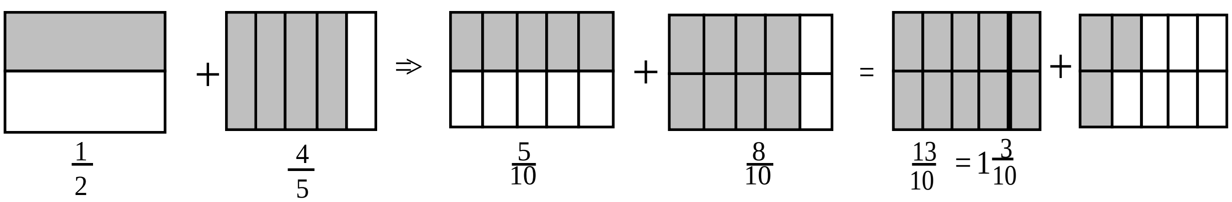
so



Thus, $\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$.

### Example 2

$\frac{1}{2} + \frac{4}{5}$ would be:

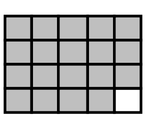
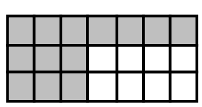
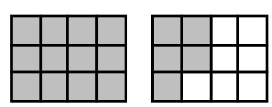


### Problems

Use the area model method to add the following fractions.

1. $\frac{3}{4} + \frac{1}{5}$
2. $\frac{1}{3} + \frac{2}{7}$
3. $\frac{2}{3} + \frac{3}{4}$

### Answers

1. $\frac{19}{20}$ 
2. $\frac{13}{21}$ 
3. $\frac{17}{12} = 1\frac{5}{12}$ 

## IDENTITY PROPERTY OF MULTIPLICATION (Giant **1**) METHOD

The Giant **1**, known in mathematics as the Identity Property of Multiplication, uses a fraction with the same numerator and denominator ( $\frac{3}{3}$ , for example) to write an equivalent fraction that helps to create common denominators. (Refer to the ratio and fractions sections of this guide for more information.) For additional information, see Year 1, Chapter 7, problems GH-19, 30, and 33 or Year 2, Chapter 3, problem MD-68.

### Example

Add $\frac{1}{3} + \frac{1}{4}$ using the Giant **1**.

Step 1: Multiply both $\frac{1}{3}$ and $\frac{1}{4}$ by Giant **1**s to get a common denominator.

$$\frac{1}{3}\left(\frac{4}{4}\right) + \frac{1}{4}\left(\frac{3}{3}\right) = \frac{4}{12} + \frac{3}{12}$$

Step 2: Add the numerators of both fractions to get the answer.

$$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

## RATIO TABLE METHOD

In Year 1, the least common multiple is found by using ratio tables. The least common multiple is used as the common denominator of the fractions. The Giant **1** or another ratio table can be used to find the new numerators. For additional information, see Year 1, Chapter 6, problem MB-22.

### Example

Solve $\frac{3}{4} - \frac{2}{7}$ using a ratio table to find the least common denominator of the fractions.

Use a ratio table to find the least common denominator of the fractions. (This is the same as finding the least common multiple of the denominators, 4 and 7.)

| | | | | | | |
|---|---|---|---|---|---|---|
| 4 | 8 | 12 | 16 | 20 | 24 | (28) |
| 7 | 14 | 21 | (28) | 35 | 42 | 49 |

You then use the Giant **1** to find the new numerator.

$$\frac{3}{4} - \frac{2}{7} \Rightarrow \frac{3}{4}\cdot\left(\frac{7}{7}\right) - \frac{2}{7}\cdot\left(\frac{4}{4}\right) \Rightarrow \frac{21}{28} - \frac{8}{28} \Rightarrow \frac{13}{28}$$

## Problems

Find each sum or difference. Use the method of your choice.

| | | | |
|---|---|---|---|
| 1. $\frac{1}{3} + \frac{2}{5}$ | 2. $\frac{1}{6} + \frac{2}{3}$ | 3. $\frac{3}{8} + \frac{2}{5}$ | 4. $\frac{1}{4} + \frac{3}{7}$ |
| 5. $\frac{2}{9} + \frac{3}{4}$ | 6. $\frac{5}{12} + \frac{1}{3}$ | 7. $\frac{4}{5} - \frac{1}{3}$ | 8. $\frac{3}{4} - \frac{1}{5}$ |
| 9. $\frac{7}{9} - \frac{2}{3}$ | 10. $\frac{3}{4} + \frac{1}{3}$ | 11. $\frac{5}{6} + \frac{2}{3}$ | 12. $\frac{7}{8} + \frac{1}{4}$ |
| 13. $\frac{6}{7} - \frac{2}{3}$ | 14. $\frac{1}{4} - \frac{1}{3}$ | 15. $\frac{3}{5} + \frac{3}{4}$ | 16. $\frac{5}{7} - \frac{3}{4}$ |
| 17. $\frac{1}{3} - \frac{3}{4}$ | 18. $\frac{2}{5} + \frac{9}{15}$ | 19. $\frac{3}{5} - \frac{2}{3}$ | 20. $\frac{5}{6} - \frac{11}{12}$ |

## Answers

| | | | | |
|---|---|---|---|---|
| 1. $\frac{11}{15}$ | 2. $\frac{5}{6}$ | 3. $\frac{31}{40}$ | 4. $\frac{19}{28}$ | 5. $\frac{35}{36}$ |
| 6. $\frac{3}{4}$ | 7. $\frac{7}{15}$ | 8. $\frac{11}{20}$ | 9. $\frac{1}{9}$ | 10. $\frac{13}{12} = 1\frac{1}{12}$ |
| 11. $\frac{3}{2} = 1\frac{1}{2}$ | 12. $\frac{9}{8} = 1\frac{1}{8}$ | 13. $\frac{4}{21}$ | 14. $-\frac{1}{12}$ | 15. $\frac{27}{20} = 1\frac{7}{20}$ |
| 16. $-\frac{1}{28}$ | 17. $-\frac{5}{12}$ | 18. 1 | 19. $-\frac{1}{15}$ | 20. $-\frac{1}{12}$ |

To summarize addition and subtraction of fractions:

1. Rename each fraction with equivalents that have a common denominator.
2. Add or subtract only the numerators, keeping the common denominator.
3. Simplify if possible.

## SUBTRACTING MIXED NUMBERS

To subtract mixed numbers, change the mixed numbers into fractions greater than one, find a common denominator, then subtract. For additional information, see Year 1, Chapter 7, problems GH-31 and GH-33.

### Example

Find the difference: $3\frac{1}{5} - 1\frac{2}{3}$ .

$$3\frac{1}{5} = \frac{16}{5} \cdot \frac{3}{3} = \frac{48}{15}$$

$$-1\frac{2}{3} \quad \frac{5}{3} \cdot \frac{5}{5} = \frac{25}{15}$$

$$\frac{23}{15} \text{ or } 1\frac{8}{15}$$

### Problems

Find each difference.

1. $2\frac{1}{2} - 1\frac{3}{4}$
2. $4\frac{1}{3} - 3\frac{5}{6}$
3. $1\frac{1}{6} - \frac{3}{4}$
4. $5\frac{2}{5} - 3\frac{2}{3}$
5. $7 - 1\frac{2}{3}$
6. $5\frac{3}{8} - 2\frac{2}{3}$

### Answers

1. $\frac{5}{2} - \frac{7}{4}$ $\quad \frac{10}{4} - \frac{7}{4}$ $\quad \frac{3}{4}$
2. $\frac{13}{3} - \frac{23}{6}$ $\quad \frac{26}{6} - \frac{23}{6}$ $\quad \frac{3}{6}$ or $\frac{1}{2}$
3. $\frac{7}{6} - \frac{3}{4}$ $\quad \frac{14}{12} - \frac{9}{12}$ $\quad \frac{5}{12}$
4. $\frac{27}{5} - \frac{11}{3}$ $\quad \frac{81}{15} - \frac{55}{15}$ $\quad \frac{26}{15}$ or $1\frac{11}{15}$
5. $\frac{7}{1} - \frac{5}{3}$ $\quad \frac{21}{3} - \frac{5}{3}$ $\quad \frac{16}{3}$ or $5\frac{1}{3}$
6. $\frac{43}{8} - \frac{8}{3}$ $\quad \frac{129}{24} - \frac{64}{24}$ $\quad \frac{65}{24}$ or $2\frac{17}{24}$

## MULTIPLYING AND DIVIDING FRACTIONS

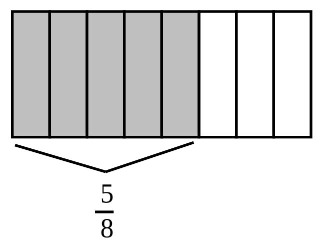
## MULTIPLYING FRACTIONS WITH AN AREA MODEL

Multiplication of fractions is taught using a rectangular area model. Lines that divide the rectangle to represent one fraction are drawn vertically, and the correct number of parts are shaded. Then lines that divide the rectangle to represent the second fraction are drawn horizontally and part of the shaded region is darkened to represent the product of the two fractions. For additional information, see Year 1, Chapter 7, problems GH-43, 46, 47 and GH-58 or Year 2, Chapter 3, problems MD-40, 41, and 46.

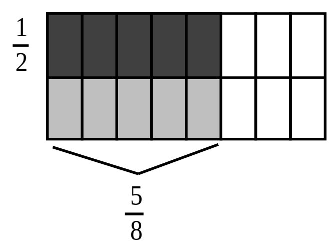
### Example 1

$\frac{1}{2} \cdot \frac{5}{8}$ (that is, $\frac{1}{2}$ of $\frac{5}{8}$)

Step 1: Draw a unit rectangle and divide it into 8 pieces vertically. Lightly shade 5 of those pieces. Label it $\frac{5}{8}$.




Step 2: Use a horizontal line and divide the unit rectangle in half. Darkly shade $\frac{1}{2}$ of $\frac{5}{8}$ and label it.




Step 3: Write an equation. $\frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$

The rule for multiplying fractions derived from the models above is to multiply the numerators, then multiply the denominators. Simplify the product when possible.

### Example 2

a) $\frac{2}{3} \cdot \frac{2}{7}$ $\quad \frac{2 \cdot 2}{3 \cdot 7}$ $\quad \frac{4}{21}$

b) $\frac{3}{4} \cdot \frac{6}{7}$ $\quad \frac{3 \cdot 6}{4 \cdot 7}$ $\quad \frac{18}{28}$ $\quad \frac{9}{14}$

## Problems

Draw an area model for each of the following multiplication problems and write the answer.

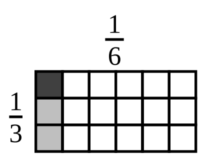
1. $\frac{1}{3} \cdot \frac{1}{6}$
2. $\frac{1}{4} \cdot \frac{3}{5}$
3. $\frac{2}{3} \cdot \frac{5}{9}$

Use the rule for multiplying fractions to find the answer for the following problems. Simplify when possible.

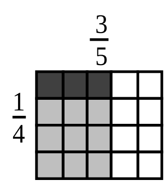
4. $\frac{1}{3} \cdot \frac{2}{5}$
5. $\frac{2}{3} \cdot \frac{2}{7}$
6. $\frac{3}{4} \cdot \frac{1}{5}$
7. $\frac{2}{5} \cdot \frac{2}{3}$
8. $\frac{2}{3} \cdot \frac{1}{4}$
9. $\frac{5}{6} \cdot \frac{2}{3}$
10. $\frac{4}{5} \cdot \frac{3}{4}$
11. $\frac{2}{15} \cdot \frac{1}{2}$
12. $\frac{3}{7} \cdot \frac{1}{2}$
13. $\frac{3}{8} \cdot \frac{4}{5}$
14. $\frac{2}{9} \cdot \frac{3}{5}$
15. $\frac{3}{10} \cdot \frac{5}{7}$
16. $\frac{5}{11} \cdot \frac{6}{7}$
17. $\frac{5}{6} \cdot \frac{3}{10}$
18. $\frac{10}{11} \cdot \frac{3}{5}$
19. $\frac{5}{12} \cdot \frac{3}{5}$
20. $\frac{7}{9} \cdot \frac{5}{14}$

## Answers

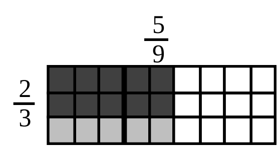
1. $\frac{1}{18}$




2. $\frac{3}{20}$




3. $\frac{10}{27}$




4. $\frac{2}{15}$
5. $\frac{4}{21}$
6. $\frac{3}{20}$
7. $\frac{4}{15}$
8. $\frac{2}{12} = \frac{1}{6}$
9. $\frac{10}{18} = \frac{5}{9}$
10. $\frac{12}{20} = \frac{3}{5}$
11. $\frac{2}{30} = \frac{1}{15}$
12. $\frac{3}{14}$
13. $\frac{12}{40} = \frac{3}{10}$
14. $\frac{6}{45} = \frac{2}{15}$
15. $\frac{15}{70} = \frac{3}{14}$
16. $\frac{30}{77}$
17. $\frac{15}{60} = \frac{1}{4}$
18. $\frac{30}{55} = \frac{6}{11}$
19. $\frac{15}{60} = \frac{1}{4}$
20. $\frac{35}{96} = \frac{5}{18}$

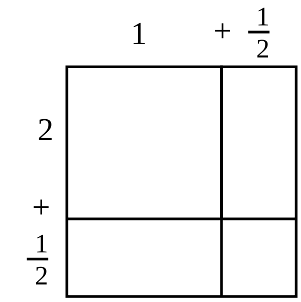
## MULTIPLYING MIXED NUMBERS

There are two ways to multiply mixed numbers. One is with generic rectangles. For additional information, see Year 1, Chapter 7, problems GH-60, 61, and 62.

### Example 1

Find the product: $2\frac{1}{2} \cdot 1\frac{1}{2}$.

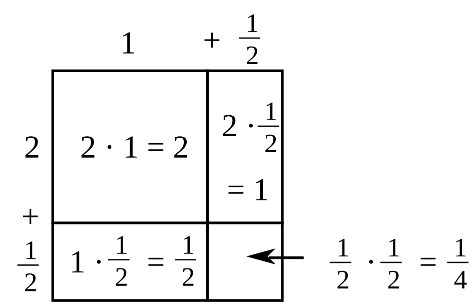
Step 1: Draw the generic rectangle. Label the top 1 plus $\frac{1}{2}$. Label the side 2 plus $\frac{1}{2}$.




Step 2: Write the area of each smaller rectangle in each of the four parts of the drawing.

Find the total area:

$2 + 1 + \frac{1}{2} + \frac{1}{4} = 3\frac{3}{4}$



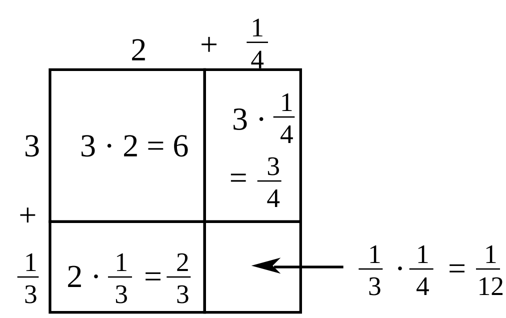

Step 3: Write an equation: $2\frac{1}{2} \cdot 1\frac{1}{2} = 3\frac{3}{4}$

### Example 2

Find the product: $3\frac{1}{3} \cdot 2\frac{1}{4}$.

$6 + \frac{3}{4} + \frac{2}{3} + \frac{1}{12}$ $\quad$ $6 + \frac{9}{12} + \frac{8}{12} + \frac{1}{12}$

$6\frac{18}{12}$ $\quad$ $7\frac{1}{2}$



## Problems

Use a generic rectangle to find each product.

1. $1\frac{1}{4} \cdot 1\frac{1}{2}$ 2. $3\frac{1}{6} \cdot 2\frac{1}{2}$ 3. $2\frac{1}{4} \cdot 1\frac{1}{2}$ 4. $1\frac{1}{3} \cdot 1\frac{1}{6}$ 5. $1\frac{1}{2} \cdot 1\frac{1}{3}$

## Answers

1. $1\frac{7}{8}$ 2. $7\frac{11}{12}$ 3. $3\frac{3}{8}$ 4. $1\frac{5}{9}$ 5. 2

1.

| | $1$ | $+\ \frac{1}{2}$ |
|---|---|---|
| $1$ | $1$ | $\frac{1}{2}$ |
| $+\ \frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |

2.

| | $2$ | $+\ \frac{1}{2}$ |
|---|---|---|
| $3$ | $6$ | $\frac{3}{2}$ |
| $+\ \frac{1}{6}$ | $\frac{2}{6}$ | $\frac{1}{12}$ |

3.

| | $1$ | $+\ \frac{1}{2}$ |
|---|---|---|
| $2$ | $2$ | $1$ |
| $+\ \frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |

4.

| | $1$ | $+\ \frac{1}{6}$ |
|---|---|---|
| $1$ | $1$ | $\frac{1}{6}$ |
| $+\ \frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{18}$ |

5.

| | $1$ | $+\ \frac{1}{3}$ |
|---|---|---|
| $1$ | $1$ | $\frac{1}{3}$ |
| $+\ \frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{6}$ |

You can also multiply mixed numbers by changing them to fractions greater than 1, then multiplying the numerators and multiplying the denominators. Simplify if possible. For additional information, see Year 1, Chapter 7, problem GH-65.

## Example 3

$2\frac{1}{2} \cdot 1\frac{1}{4}$ $\quad \frac{5}{2} \cdot \frac{5}{4}$ $\quad \frac{5 \cdot 5}{2 \cdot 4}$ $\quad \frac{25}{8}$ $\quad 3\frac{1}{8}$

## Problems

Find each product, using the method of your choice. Simplify when possible.

1. $2\frac{1}{4} \cdot 1\frac{3}{8}$ 2. $3\frac{3}{5} \cdot 2\frac{4}{7}$ 3. $2\frac{3}{8} \cdot 1\frac{1}{6}$ 4. $3\frac{7}{9} \cdot 2\frac{5}{8}$

5. $1\frac{2}{9} \cdot 2\frac{3}{7}$ 6. $3\frac{4}{7} \cdot 5\frac{8}{11}$ 7. $2\frac{3}{8} \cdot 1\frac{1}{16}$ 8. $2\frac{8}{9} \cdot 2\frac{5}{8}$

9. $1\frac{1}{3} \cdot 1\frac{4}{7}$ 10. $2\frac{1}{7} \cdot 2\frac{7}{10}$

## Answers

1. $3\frac{3}{32}$ 2. $9\frac{9}{35}$ 3. $2\frac{37}{48}$ 4. $9\frac{11}{12}$ 5. $2\frac{61}{63}$

6. $20\frac{5}{11}$ 7. $2\frac{67}{18}$ 8. $7\frac{7}{12}$ 9. $2\frac{2}{21}$ 10. $5\frac{11}{14}$

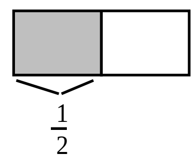
# DIVIDING FRACTIONS USING AN AREA MODEL

Division of fractions is introduced with a rectangular area model. The division problem $8 \div 2$ means, "In 8, how many groups of 2 are there?" Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, "In $\frac{1}{2}$, how many fourths are there?" For additional information, see Year 1, Chapter 7, problems GH-89 and 91 or Year 2, Chapter 7, problem CT-32.

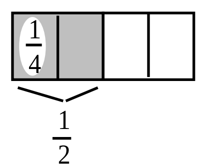
## Example 1

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{4}$.

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$. Shade $\frac{1}{2}$ of it.




Step 2: Then divide the <u>original</u> rectangle into four equal pieces. Each section represents $\frac{1}{4}$. In the shaded section, $\frac{1}{2}$, there are 2 fourths.



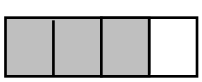

Step 3: Write the equation.

$$\frac{1}{2} \div \frac{1}{4} = 2$$

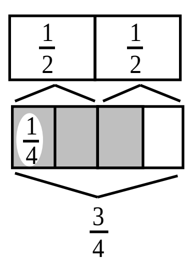
## Example 2

In $\frac{3}{4}$, how many $\frac{1}{2}$ s are there?

That is, $\frac{3}{4} \div \frac{1}{2} = ?$



Start with $\frac{3}{4}$.




In $\frac{3}{4}$ there is one full $\frac{1}{2}$ shaded and half of another one (that is half of one-half).

So: $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$
( one and one-half halves)

## Problems

Use the rectangular model to divide.

1. $1\frac{1}{3} \div \frac{1}{6}$  2. $\frac{3}{2} \div \frac{3}{4}$  3. $1 \div \frac{1}{4}$  4. $1\frac{1}{4} \div \frac{1}{2}$  5. $2\frac{2}{3} \div \frac{1}{6}$

## Answers

1. 8  2. 2  3. 4  4. $2\frac{1}{2}$  5. 16

## DIVIDING FRACTIONS USING RECIPROCALS

Two numbers that have a product of 1 are reciprocals. For example, $\frac{1}{4} \cdot \frac{4}{1} = 1$, $\frac{1}{2} \cdot \frac{2}{1} = 1$, and $\frac{1}{6} \cdot \frac{6}{1} = 1$, so $\frac{1}{4}$ and $\frac{4}{1}$, $\frac{1}{2}$ and $\frac{2}{1}$, and $\frac{1}{6}$ and $\frac{6}{1}$ are all reciprocals. For additional information, see Year 1, Chapter 7, problem GH-94 or Year 2, Chapter 7, problem CT-22.

There is another way to divide fractions: invert the divisor, that is, write its reciprocal. (The divisor is the number after the division sign.) After inverting the divisor, change the division sign to a multiplication sign and multiply. Simplify if possible.

### Example 1

$\frac{3}{4} \div \frac{1}{2}$  $\frac{3}{4}\ \frac{2}{1}$  $\frac{6}{4}$ or $1\frac{1}{2}$

### Example 2

$1\frac{1}{3} \div \frac{1}{6}$  $\frac{4}{3}\ \frac{6}{1}$  $\frac{24}{3}$ or 8

The examples above were written horizontally, but division of fractions problems can also be written in the vertical form such as $\frac{\frac{1}{2}}{\frac{1}{4}}$, $\frac{\frac{3}{4}}{\frac{1}{2}}$, and $\frac{1\frac{1}{3}}{\frac{1}{6}}$. They still mean the same thing:

$\frac{\frac{1}{2}}{\frac{1}{4}}$ means, "In $\frac{1}{2}$, how many $\frac{1}{4}$ s are there?" $\frac{\frac{3}{4}}{\frac{1}{2}}$ means, "In $\frac{3}{4}$, how many $\frac{1}{2}$ s are there?"

$\frac{1\frac{1}{3}}{\frac{1}{6}}$ means, "In $1\frac{1}{3}$, how many $\frac{1}{6}$ s are there?"

You can use a Super Giant **1** to solve these vertical division problems. This Super Giant **1** uses the reciprocal of the divisor. For additional information, see Year 1, Chapter 7, problems GH-116 and 120 or Year 2, Chapter 7, problems CT-36 and 48.

### Example 3

$$\frac{\frac{1}{2}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1} = \frac{4}{2} = 2$$

### Example 4

$$\frac{\frac{3}{4}}{\frac{1}{2}} \cdot \frac{\frac{2}{1}}{\frac{2}{1}} = \frac{\frac{6}{4}}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

### Example 5

$$\frac{1\frac{1}{3}}{\frac{1}{6}} = \frac{\frac{4}{3}}{\frac{1}{6}} \cdot \frac{\frac{6}{1}}{\frac{6}{1}} = \frac{\frac{24}{3}}{1} = \frac{24}{3} = 8$$

### Example 6

$$\frac{2}{3} \div \frac{1}{5} = \frac{2}{3} \cdot \frac{5}{1} = \frac{10}{3} = 3\frac{1}{3}$$

Compared to:

$$\frac{\frac{2}{3}}{\frac{1}{5}} \cdot \frac{\frac{5}{1}}{\frac{5}{1}} = \frac{\frac{10}{3}}{1} = 3\frac{1}{3}$$

> Examples 1 and 4 and Examples 2 and 5 result in the same answer, because dividing by a number is the same as multiplying by its reciprocal.

## Problems

Solve these division problems. Use any method.

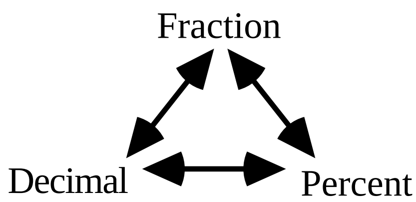
| | | | | |
|---|---|---|---|---|
| 1. $\frac{3}{7} \div \frac{1}{8}$ | 2. $1\frac{3}{7} \div \frac{1}{2}$ | 3. $\frac{4}{7} \div \frac{1}{3}$ | 4. $1\frac{4}{7} \div \frac{1}{3}$ | 5. $\frac{6}{7} \div \frac{5}{8}$ |
| 6. $\frac{3}{10} \div \frac{5}{7}$ | 7. $2\frac{1}{3} \div \frac{5}{8}$ | 8. $7 \div \frac{1}{3}$ | 9. $1\frac{1}{3} \div \frac{2}{5}$ | 10. $2\frac{2}{3} \div \frac{3}{4}$ |
| 11. $3\frac{1}{3} \div \frac{5}{6}$ | 12. $1\frac{1}{2} \div \frac{1}{2}$ | 13. $\frac{5}{8} \div 1\frac{1}{4}$ | 14. $10\frac{1}{3} \div \frac{1}{6}$ | 15. $\frac{3}{5} \div 6$ |

## Answers

| | | | | |
|---|---|---|---|---|
| 1. $3\frac{3}{7}$ | 2. $2\frac{6}{7}$ | 3. $1\frac{5}{7}$ | 4. $4\frac{5}{7}$ | 5. $1\frac{13}{35}$ |
| 6. $\frac{21}{50}$ | 7. $3\frac{11}{15}$ | 8. 21 | 9. $3\frac{1}{3}$ | 10. $3\frac{5}{9}$ |
| 11. $1\frac{9}{11}$ | 12. $\frac{1}{3}$ | 13. $\frac{1}{2}$ | 14. 62 | 15. $\frac{1}{10}$ |

## FRACTION, DECIMAL, AND PERCENT EQUIVALENTS

Fractions, decimals, and percents are different ways to represent the same number.




For additional information, see Year 1, Chapter 3, problems PR-25 and PR-26, Chapter 6, problem MB-23, 83 or Year 2, Chapter 3, problem MD-109.

### Examples

*Decimal to percent:*

Multiply the decimal by 100.

$(0.34)(100) = 34\%$

*Percent to decimal:*

Divide the percent by 100.

$78.6\% \div 100 = 0.786$

*Fraction to percent:*

Write a proportion to find an equivalent fraction using 100 as the denominator. The numerator is the percent.

$\frac{4}{5} = \frac{x}{100}$ so $\frac{4}{5} = \frac{80}{100} = 80\%$

*Percent to fraction:*

Use 100 as the denominator. Use the percent as the numerator. Simplify as needed.

$22\% = \frac{22}{100} = \frac{11}{50}$

*Decimal to fraction:*

Use the decimal as the numerator. Use the decimal place value name as the denominator. Simplify as needed.

a) $0.2 = \frac{2}{10} = \frac{1}{5}$ b) $0.17 = \frac{17}{100}$

*Fraction to decimal:*

Divide the numerator by the denominator.

$\frac{3}{8} = 3 \div 8 = 0.375$

## Problems

Convert the fraction, decimal, or percent as indicated.

1. Change $\frac{1}{4}$ to a decimal.

2. Change 50% into a fraction in lowest terms.

3. Change 0.75 to a fraction in lowest terms.

4. Change 75% to a decimal.

5. Change 0.38 to a percent.

6. Change $\frac{1}{5}$ to a percent.

7. Change 0.3 to a fraction.

8. Change $\frac{1}{8}$ to a decimal.

9. Change $\frac{1}{3}$ to a decimal.

10. Change 0.08 to a percent.

11. Change 87% to a decimal.

12. Change $\frac{3}{5}$ to a percent.

13. Change 0.4 to a fraction in lowest terms.

14. Change 65% to a fraction in lowest terms.

15. Change $\frac{1}{9}$ to a decimal.

16. Change 125% to a fraction in lowest terms.

17. Change $\frac{8}{5}$ to a decimal.

18. Change 3.25 to a percent.

19. Change $\frac{1}{16}$ to a decimal.
    Change the decimal to a percent.

20. Change $\frac{1}{7}$ to a decimal.

21. Change 43% to a fraction.
    Change the fraction to a decimal.

22. Change 0.375 to a percent.
    Change the percent to a fraction.

23. Change $\frac{7}{8}$ to a decimal.
    Change the decimal to a percent.

## Answers

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | 0.25 | 2. | $\frac{1}{2}$ | 3. | $\frac{3}{4}$ | 4. | 0.75 |
| 5. | 38% | 6. | 20% | 7. | $\frac{3}{10}$ | 8. | 0.125 |
| 9. | $0.3\bar{3}$ | 10. | 8% | 11. | 0.87 | 12. | 60% |
| 13. | $\frac{2}{5}$ | 14. | $\frac{13}{20}$ | 15. | $0.1\bar{1}$ | 16. | $1\frac{1}{4}$ |
| 17. | 1.6 | 18. | 325% | 19. | 0.0625; 6.25% | 20. | ≈ 0.143 |
| 21. | $\frac{43}{100}$ ; 0.43 | 22. | $37\frac{1}{2}$%; $\frac{3}{8}$ | 23. | 0.875; 87.5% | | |

## LAWS OF EXPONENTS

In the expression $5^2$, 5 is the **base** and 2 is the **exponent**. For $x^a$, x is the base and a is the exponent. $5^2$ means $5 \cdot 5$. $5^3$ means $5 \cdot 5 \cdot 5$, so you can write $\frac{5^5}{5^2}$ (which means $5^5 \div 5^2$) or you can write it like this: $\frac{5\ 5\ 5\ 5\ 5}{5\ 5}$.

You can use the Giant **1** to find the numbers in common. There are two Giant **1**s, namely, $\frac{5}{5}$ twice so $\frac{5\ 5\ 5\ 5\ 5}{5\ 5} = 5^3$ or 125. Writing $5^3$ is usually sufficient.

When there is a variable, it is treated the same way. $\frac{x^7}{x^3}$ means $\frac{x\ x\ x\ x\ x\ x\ x}{x\ x\ x}$. The Giant **1** here is $\frac{x}{x}$ (three of them) . The answers is $x^4$.

$5^2 \cdot 5^3$ means $(5 \cdot 5)(5 \cdot 5 \cdot 5)$ which is $5^5$. $(5^2)^3$ means $(5^2)(5^2)(5^2)$ or $(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$ which is $5^6$.

When the problems have variables such as $x^4 \cdot x^5$, you only need to add the exponents. The answer is $x^9$. If the problem is $(x^4)^5$ ($x^4$ to the fifth power) it means $x^4 \cdot x^4 \cdot x^4 \cdot x^4 \cdot x^4$. The answer is $x^{20}$. You multiply exponents in this case.

If the problem is $\frac{x^{10}}{x^4}$, you subtract the bottom exponent from the top exponent $(10 - 4)$. The answer is $x^6$. You can also have problems like $\frac{x^{10}}{x^{-4}}$. You still subtract, $10 - (-4)$ is 14, and the answer is $x^{14}$.

You need to be sure the bases are the same to use these laws. $x^5 \cdot y^6$ cannot be further simplified.

In general the laws of exponents are:

$$x^a \cdot x^b = x^{(a+b)} \qquad (x^a)^b = x^{ab} \qquad \frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1 \qquad (x^a y^b)^c = x^{ac}y^{bc}$$

These rules hold if $x \neq 0$ and $y \neq 0$.

For additional information, see Year 2, Chapter 8, problem GS-26 and Chapter 10, problem MG-35.

## Examples

a) $x^8 \quad x^7 = x^{15}$ b) $\frac{x^{19}}{x^{13}} = x^6$ c) $(z^8)^3 = z^{24}$

d) $(x^2y^3)^4 = x^8y^{12}$ e) $\frac{x^4}{x^{-3}} = x^7$ f) $(2x^2y^3)^2 = 4x^4y^6$

g) $(3x^2y^{-2})^3 = 27x^6y^{-6}$ or $\frac{27x^6}{y^6}$ h) $\frac{x^8y^5z^2}{x^3y^6z^{-2}} = \frac{x^5z^4}{y}$ or $x^5y^{-1}z^4$

## Problems

Simplify each expression.

1. $5^2 \cdot 5^4$ 2. $x^3 \cdot x^4$ 3. $\frac{5^{16}}{5^{14}}$ 4. $\frac{x^{10}}{x^6}$ 5. $(5^3)^3$

6. $(x^4)^3$ 7. $(4x^2\, y^3)^4$ 8. $\frac{5^2}{5^{-3}}$ 9. $5^5 \cdot 5^{-2}$ 10. $(y^2)^{-3}$

11. $(4a^2b^{-2})^3$ 12. $\frac{x^5y^4z^2}{x^4y^3z^2}$ 13. $\frac{x^6y^2z^3}{x^{-2}y^3z^{-1}}$ 14. $4x^2 \cdot 2x^3$

## Answers

1. $5^6$ 2. $x^7$ 3. $5^2$ 4. $x^4$ 5. $5^9$

6. $x^{12}$ 7. $256x^8y^{12}$ 8. $5^5$ 9. $5^3$ 10. $y^{-6}$ or $\frac{1}{y^6}$

11. $64a^6b^{-6}$ or $\frac{64a^6}{b^6}$ 12. $xy$ 13. $\frac{x^8z^4}{y}$ or $x^8y^{-1}z^4$ 14. $8x^5$

# SCIENTIFIC NOTATION

Scientific notation is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when it is written as the product of two factors as described below.

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent (power of 10).
- The factors are separated by a multiplication sign.
- A positive exponent indicates a number whose absolute value is greater than one.
- A negative exponent indicates a number whose absolute value is less than one.

| Scientific Notation | Standard Form |
|---|---|
| $5.32 \cdot 10^{11}$ | 532,000,000,000 |
| $2.61 \cdot 10^{-15}$ | 0.00000000000000261 |

It is important to note that the exponent does not necessarily mean to use that number of zeros.

The number $5.32 \cdot 10^{11}$ means $5.32 \cdot 100{,}000{,}000{,}000$. Thus, two of the 11 places in the standard form of the number are the 3 and the 2 in 5.32. Standard form in this case is 532,000,000,000. In this example you are moving the decimal point to the right 11 places to find standard form.

The number $2.61 \cdot 10^{-15}$ means $2.61 \cdot 0.000000000000001$.
You are moving the decimal point to the left 15 places to find standard form.
Here the standard form is 0.00000000000000261.

For additional information, see Year 2, Chapter 10, problem MG-65.

## Example 1

Write each number in standard form.

$7.84 \cdot 10^{8}$ 784,000,000 and $3.72 \cdot 10^{-3}$ 0.00372

When taking a number in standard form and writing it in scientific notation, remember there is only one digit before the decimal point, that is, the number must be between 1 and 9, inclusive.

**Example 2** 52,050,000 $5.205 \cdot 10^{7}$ and 0.000372 $3.72 \cdot 10^{-4}$

The exponent denotes the number of places you had to move the decimal point in the standard form. In the first example above, the decimal point is at the end of the number and it was moved 7 places. In the second example above, the exponent is negative because the original number is very small, that is, less than one.

## Problems

Write each number in standard form.

1. $7.85 \cdot 10^{11}$  2. $1.235 \cdot 10^{9}$  3. $1.2305 \cdot 10^{3}$  4. $3.89 \cdot 10^{-7}$  5. $5.28 \cdot 10^{-4}$

Write each number in scientific notation.

6. 391,000,000,000  7. 0.0000842  8. 123056.7  9. 0.000000502

10. 25.7  11. 0.035  12. 5,600,000  13. 1346.8

14. 0.000000000006  15. 634,700,000,000,000

> Note:
> On your scientific calculator, displays like $4.357^{12}$ and $3.65^{-03}$ are numbers expressed in scientific notation. The first number means $4.357 \cdot 10^{12}$ and the second means $3.65 \cdot 10^{-3}$. The calculator does this because there is not enough room on its display window to show the entire number.

## Answers

1. 785,000,000,000  2. 1,235,000,000  3. 1230.5

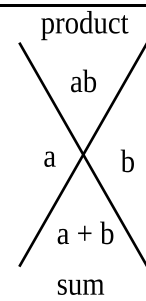
4. 0.000000389  5. 0.000528  6. $3.91 \cdot 10^{11}$

7. $8.42 \cdot 10^{-5}$  8. $1.230567 \cdot 10^{5}$  9. $5.02 \cdot 10^{-7}$

10. $2.57 \cdot 10^{1}$  11. $3.5 \cdot 10^{-2}$  12. $5.6 \cdot 10^{6}$

13. $1.3468 \cdot 10^{3}$  14. $6.0 \cdot 10^{-12}$  15. $6.347 \cdot 10^{14}$

# DIAMOND PROBLEMS

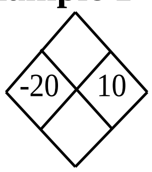



In every Diamond Problem, the product of the two side numbers (left and right) is the top number and their sum is the bottom number.

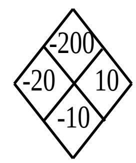
Diamond Problems are an excellent way of practicing addition, subtraction, multiplication, and division of both positive and negative integers, decimals and fractions. They have the added benefit of preparing students for factoring binomials in Algebra.

For more information, see Year 1, Chapter 5, problem GO-12 or Year 2, Chapter 1, problem GO-63.

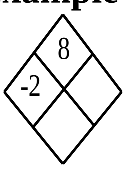
## Example 1



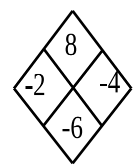

The top number is the product of -20 and 10, or -200. The bottom number is the sum of -20 and 10, or -20 + 10 = -10.



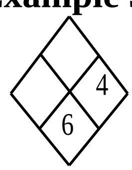

## Example 2



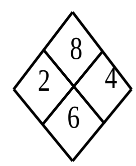

The product of the right number and -2 is 8. Thus, if you divide 8 by - 2 you get - 4, the right number.
The sum of -2 and - 4 is - 6, the bottom number.



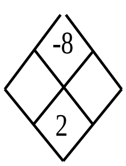

## Example 3




To get the left number, subtract 4 from 6,
6 – 4 = 2. The product of 2 and 4 is 8, the top number.



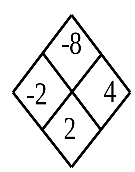

## Example 4




The easiest way to find the side numbers in a situation like this one is to look at all the pairs of factors of - 8.
They are:

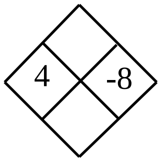
- 1, and 8, -2, and 4, -4, and 2 and -8 and 1.

Only one of these pairs has a sum of 2: - 2 and 4. Thus, the side numbers are - 2 and 4.



## Problems

Complete each of the following diamond problems.

1.
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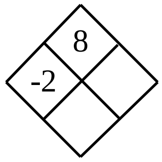


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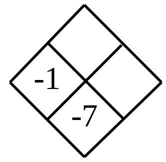


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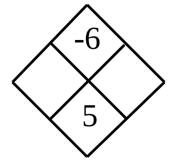


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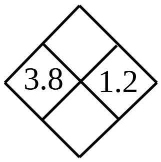


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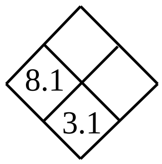


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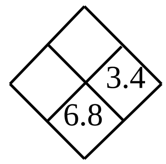


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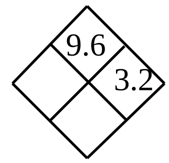


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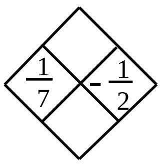


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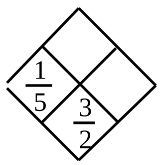


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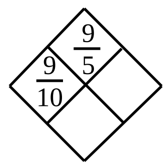


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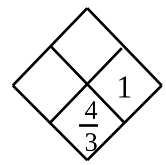


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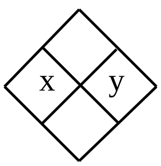


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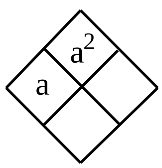


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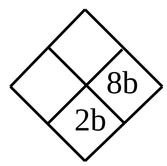


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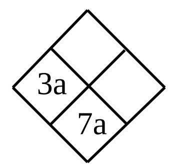


## Answers

1. -32 and -4
2. -4 and -6
3. -6 and 6
4. 6 and -1
5. 4.56 and 5
6. 5 and 40.5
7. 3.4 and 11.56
8. 3 and 6.2
9. $-\frac{1}{14}$ and $-\frac{5}{14}$
10. $\frac{13}{10}$ and $\frac{13}{50}$
11. $\frac{1}{2}$ and $\frac{14}{10}$ or $\frac{7}{5}$
12. $\frac{1}{3}$ and $\frac{1}{3}$
13. $xy$ and $x + y$
14. $a$ and $2a$
15. $-6b$ and $-48b^2$
16. $4a$ and $12a^2$