

Statistics, Data Analysis, and Probability

Measures of Central Tendency

Sampling Populations

Correlation

Probability

Tree Diagrams

MEASURES OF CENTRAL TENDENCY

The measures of central tendency are numbers that locate or approximate the “center” of a set of data. Mean, median, and mode are the most common measures of central tendency.

The mean is the arithmetic average of a data set. Add all the values in a set and divide this sum by the number of values in the set.

For additional information, see Year 1, Chapter 1, problems AR-26, 27, and 37 or Year 2, Chapter 1, problems GO-30, 64, and 65.

Example 1

Find the mean of this set of data: 34, 31, 37, 44, 38, 34, 42, 34, 43, and 41.

- $34 + 31 + 37 + 44 + 38 + 34 + 42 + 34 + 43 + 41 = 378$
- $378 \div 10 = 37.8$

The mean of this set of data is 37.8.

Example 2

Find the mean of this set of data: 92, 82, 80, 92, 78, 75, 95, and 77.

- $92 + 82 + 80 + 92 + 78 + 75 + 95 + 77 + 77 = 748$
- $748 \div 9 = 83.1$

The mean of this set of data is 83.1.

Problems

Find the mean of each set of data.

1. 29, 28, 34, 30, 33, 26, and 34.
2. 25, 34, 35, 27, 31, and 30.
3. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.
4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The mode is the value in a data set that occurs most often. Data sets may have more than one mode.

Example 3

Find the mode of this set of data: 34, 31, 37, 44, 34, 42, 34, 43, and 41.

- The mode of this data set is 34 since there are three 34s and only one of each of the other numbers.

Example 4

Find the mode of this set of data: 92, 82, 80, 92, 78, 75, 95, 77, and 77.

- The modes of this set of data are 77 and 92 since there are two of each of these numbers and only one of each of the other numbers. This data set is said to be bimodal since it has two modes.

Problems

Find the mode of each set of data.

5. 29, 28, 34, 30, 33, 26, and 34.
6. 25, 34, 35, 27, 25, 31, and 30.
7. 80, 89, 79, 84, 95, 79, 89, 76, 82, 76, 92, 89, 81, and 123.
8. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The median is the middle number in a set of data arranged in numerical order. If there are an even number of values, the median is the mean of the two middle numbers.

Example 5

Find the median of this set of data: 34, 31, 37, 44, 38, 34, 43, and 41.

- Arrange the data in order: 31, 34, 34, 34, 37, 38, 41, 43, 44.
- Find the middle value(s): 37 and 38.
- Since there are two middle values, find their mean: $37 + 38 = 75$, $75 \div 2 = 37.5$. Therefore, the median of this data set is 37.5.

Example 6

Find the median of this set of data: 92, 82, 80, 92, 78, 75, 95, 77, and 77.

- Arrange the data in order: 75, 77, 77, 78, 80, 82, 92, 92, and 95.
- Find the middle value(s): 80. Therefore, the median of this data set is 80.

Problems

Find median of each set of data.

9. 29, 28, 34, 30, 33, 26, and 34.
10. 25, 34, 27, 25, 31, and 30.
11. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.
12. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The range of a set of data is the difference between the highest value and the lowest value.

Example 7

Find the range of this set of data: 114, 109, 131, 96, 140, and 128.

- The highest value is 140.
- The lowest value is 96.
- $140 - 96 = 44$.
- The range of this set of data is 44.

Example 8

Find the range of this set of data: 37, 44, 36, 29, 78, 15, 57, 54, 63, 27, and 48.

- The highest value is 78.
- The lowest value is 27.
- $78 - 27 = 51$.
- The range of this set of data is 51.

Problems

Find the range of each set of data in problems 9 through 12.

Outliers are numbers in a data set that are either much higher or much lower than the other numbers in the set.

Example 9

Find the outlier of this set of data: 88, 90, 96, 93, 87, 12, 85, and 94.

- The outlier is 12.

Example 10

Find the outlier of this set of data: 67, 54, 49, 76, 64, 59, 60, 72, 123, 44, and 66.

- The outlier is 123.

Problems

Find the outlier for each set of data.

13. 70, 77, 75, 68, 98, 70, 72, and 71.

14. 14, 22, 17, 61, 20, 16, and 15.

15. 1376, 1645, 1783, 1455, 3754, 1790, 1384, 1643, 1492, and 1776.

16. 62, 65, 93, 51, 55, 14, 79, 85, 55, 72, 78, 83, 91, and 76.

A stem-and-leaf plot is a way to display data that shows the individual values from a set of data and how the values are distributed. This type of display clearly shows median, mode, range, and outliers. The “stem” part on the graph represents the leading digit(s) of the number. The “leaf” part of the graph represents the other digit(s).

For additional information, see Year 1, Chapter 1, problems AR-38, 39, and 44 or Year 2, Chapter 1, problem GO-45.

Example 12

Make a stem-and-leaf plot of this set of data:
34, 31, 37, 44, 38, 29, 34, 42, 43, 34, 52, and 41.

$$\begin{array}{r|l} 2 & 9 \\ 3 & 1\ 4\ 4\ 4\ 7\ 8 \\ 4 & 1\ 2\ 3\ 4 \\ 5 & 2 \end{array}$$

Example 13

Make a stem-and-leaf plot of this set of data:
92, 82, 80, 92, 78, 75, 95, 77, and 77.

$$\begin{array}{r|l} 7 & 5\ 7\ 7\ 8 \\ 8 & 0\ 2 \\ 9 & 2\ 2\ 5 \end{array}$$

Problems

Make a stem-and-leaf plot of each set of data.

17. 29, 28, 34, 30, 33, 26, 18, and 34.

18. 25, 34, 27, 25, 19, 31, 42, and 30.

19. 80, 89, 79, 84, 95, 79, 89, 67, 82, 76, 92, 89, 81, and 123.

20. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

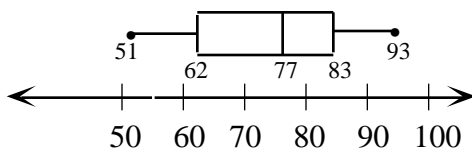
A way to display data that shows how the data is grouped or clustered is a box-and-whisker plot. The box-and-whisker plot displays the data using quartiles. This type of display clearly shows the median, range, and outliers of a data set. It is a very useful display for comparing sets of data.

For additional information, see Year 2, Chapter 1, problem GO-65.

Example 14

Display this data in a box-and-whisker plot:
51, 55, 55, 62, 65, 72, 76, 78, 79, 82, 83, 85, 91,
and 93.

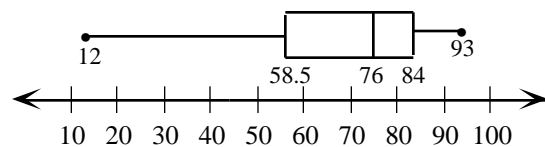
- Since this data is already in order from least to greatest, it can be seen that the range is $93 - 51 = 42$. Thus you start with a number line with equal intervals from 50 to 95.
- The median of the set of data is 77. A line is drawn at this value above the number line.
- The median of the lower half of the data (the lower quartile) is 62. A line is drawn at this value above the number line.
- The median of the upper half of the data (the upper quartile) is 83. A line is drawn at this value above the number line.
- A box is drawn between the upper and lower quartiles.
- Place a dot at the minimum value (51) and a dot at the maximum value (93). The lines which connect these dots to the box are called the whiskers.



Example 15

Display this data in a box-and-whisker plot:
62, 65, 93, 51, 12, 79, 85, 55, 72, 78, 83, 91,
and 76.

- Place the data in order from least to greatest: 12, 51, 55, 62, 65, 72, 76, 78, 79, 83, 85, 91, 93. The range is $93 - 12 = 81$. Thus you want a number line with equal intervals from 10 to 100.
- Find the median of the set of data: 76. Draw the line.
- Find the lower quartile: $55 + 62 = 117$; $117 \div 2 = 58.5$. Draw the line.
- Find the upper quartile: $83 + 85 = 168$; $168 \div 2 = 84$. Draw the line.
- Draw the box connecting the upper and lower quartiles. Place a dot at the minimum value (12) and a dot at the maximum value (93). Draw the whiskers.



Problems

Create a stem-and-leaf plot and a box-and-whisker plot for each set of data in problems 21 through 24.

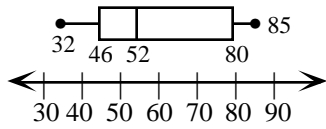
21. 45, 47, 52, 85, 46, 32, 83, 80, and 75. 22. 75, 62, 56, 80, 72, 55, 54, and 80.
23. 49, 54, 52, 58, 61, 72, 73, 78, 73, 82, 24. 65, 35, 48, 29, 57, 87, 94, 68, 86, 73,
83, 73, 61, 67, and 68. 58, 74, 85, 91, 88, and 97.
25. Given a set of data: 265, 263, 269, 259, 267, 264, 253, 275, 264, 260, 273, 257, and 291.
- Make a stem-and-leaf plot of this data.
 - Find the mean, median, and mode of this data.
 - Find the range of this data.
 - Make a box-and-whisker plot for this data.
26. Given a set of data: 48, 42, 37, 29, 49, 46, 38, 28, 45, 45, 35, 46.25, 34, 46, 46.5, 43, 46.5, 48, 41.25, 29, and 47.75.
- Make a stem-and-leaf plot of this data.
 - Find the mean, median, and mode of this data.
 - Find the range of the data.
 - Make a box-and-whisker plot for this data.

Answers

1. 30.57 2. $30.\overline{33}$ 3. $86.\overline{13}$ 4. $106.\overline{6}$
5. 34 6. 25 7. 89 8. 101 and 113
9. median 30;
range 8 10. median 28.5;
range 9 11. median 82;
range 47 12. median 107;
range 27
13. 98 14. 61 15. 3754 16. 14
17. 18. 19. 20.
- | | | | | | | | |
|---|---------|---|---------|----|---------------|----|-------------------|
| 1 | 8 | 1 | 9 | 6 | 7 | 9 | 1 |
| 2 | 6 8 9 | 2 | 5 5 7 | 7 | 6 9 9 | 10 | 0 1 1 3 4 5 7 8 9 |
| 3 | 0 3 4 4 | 3 | 0 1 4 5 | 8 | 0 1 2 4 9 9 9 | 11 | 1 3 3 6 8 |
| | | 4 | 2 | 9 | 2 5 | | |
| | | | | 10 | | | |
| | | | | 11 | | | |
| | | | | 12 | 3 | | |

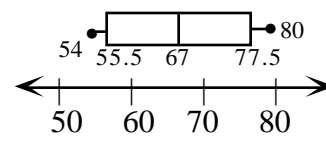
21.

3	2
4	5 6 7
5	2
7	5
8	0 3 5



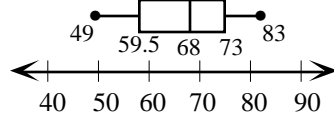
22.

5	4 5 6
6	2
7	2 5
8	0 0



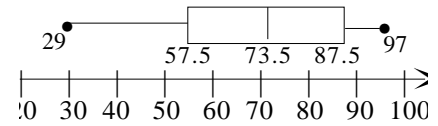
23.

4	9
5	2 4 8
6	1 1 7 8
7	2 3 3 3 8
8	2 3



24.

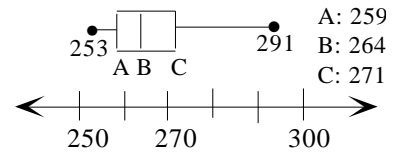
2	9
3	5
4	8
5	7 8
6	5 8
7	3 4
8	5 6 7 8
9	1 4 7



25.

25	3 7 9
26	0 3 4 4 5 7 9
27	3 5
28	
29	1

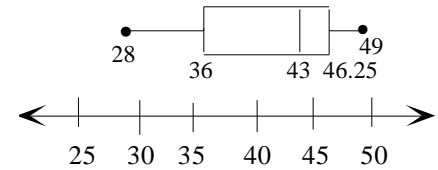
Mean: 266.15
 Median: 264
 Mode: 264
 Range: 38



26.

2	8 9 9
3	4 5 7 8
4	1.25 2 3 5 5 6 6 6.25 6.5 6.5 7.75 8 8 9

Mean: 41.4405
 Median: 43
 Mode: 29, 45, 46.5,
 46, and 48
 Range: 21



SAMPLING POPULATIONS

In order to have students conduct a representative survey, they are introduced to different types of surveys and methods for selecting survey samples. The vocabulary is introduced in Year 1, Chapter 10, problem JM-90.

A population is a collection of objects or a group of people about whom information is gathered.

A sample is a subgroup of the population. For example, if you want to conduct a survey at your school about what foods to serve in the cafeteria, the population would be the entire student body.

A representative sample is a subgroup of the population that matches the general characteristics of the entire population. If you choose to sample 10% of the students, you need to include the correct fraction of students from each grade and an equal number of male and female students.

Problems

Answer the following questions.

1. If survey results were published in an advertisement that 3 out of 4 people surveyed said they preferred the advertised product, what questions would you ask about the people surveyed?
2. What can people making surveys do to ensure that the survey is truly random?

Answers

1. How many people were surveyed? Who did the people surveyed represent? Could all the people surveyed be employees of the advertiser? Who conducted the survey? What questions were asked?
2. Determine the general characteristics of the total population being surveyed and make sure a subgroup representing each characteristic is surveyed.

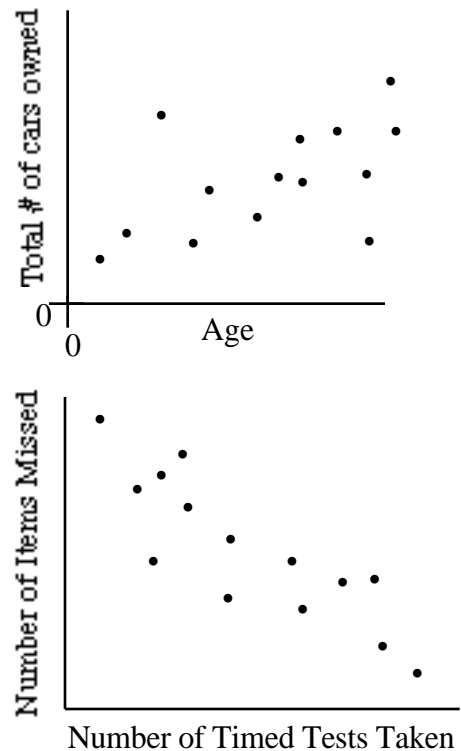
CORRELATION

A scatter plot like the first one at right, appears to have a positive correlation, since people seem to own more cars as they increase in age. The second scatter plot shows a negative correlation, where the number of incorrect items on tests decreases as more tests are taken.

Students should begin to understand the concept that a positive correlation exists if the items on both axes tend to increase and that a negative correlation exists if the items on the y-axis decrease as the items on the x-axis increase. The closer the points come to approaching a straight line, the stronger the correlation.

One caution: It is easy to jump to the conclusion that if there is a strong correlation, one factor causes the other. This is not necessarily true. For example, a scatter plot showing the relationship of height to reading ability might show a very strong correlation but neither increased height increases reading ability, nor does increased reading ability cause growth. In this case both height and reading ability are age related.

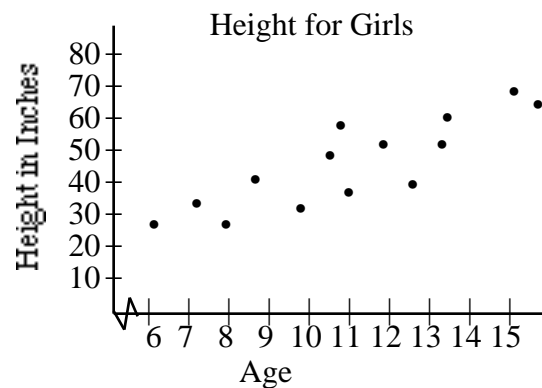
For additional information, see Year 1, Chapter 10, problems JM-116 through 119 or see Year 2, Chapter 1, problems GO-71 and GO-84 through 86.



Problems

Use the scatter plot at right to answer the following questions:

1. Is there a correlation between age and height in girls?
2. If there is a correlation, is it positive or negative?
3. What does your answer to problem 2 mean?
4. What will happen to the scatter plot if the x-axis is increased to age 40?



Answers

1. yes
2. positive
3. Girls increase in height as they age.
4. The correlation would continue until it leveled off somewhere between ages 15 and 20. From that point, height should stay the same since growth has ended.

PROBABILITY

Probability: A number between zero and one that states the likelihood of an event occurring. It is the ratio of specified outcomes to all possible outcomes (the sample space).

Outcome: Any possible or actual result or consequence of the action(s) considered, such as rolling a five on a die or getting tails when flipping a coin.

Event: An outcome or group of outcomes from an experiment, such as rolling an even number on a die.

Probability is the likelihood that a specific outcome will occur. If all the outcomes of an event are equally likely to occur, then the probability that a specified outcome occurs is:

$$P(\text{outcome}) = \frac{\text{number of ways that the specified outcome occurs}}{\text{total number of possible outcomes}}$$

Fractions are used to express the probability that certain events will (or will not) happen. The denominator of the fraction shows the total number of outcomes for an event. The numerator of the fraction indicates the number of times an event could happen.

$$\text{Theoretical probability} = \frac{\text{number of event outcomes}}{\text{total number of possible outcomes}}$$

Experimental probability refers to the occurrence of an event when the activity is actually done.

Two events are dependent if the outcome of the first event affects the outcome of the second event. For example, if you draw a card from a deck and do not replace it for the next draw, the two events – drawing one card without replacing it, then drawing a second card – are dependent.

Two events are independent if the outcome of the first event does not affect the outcome of the second event. For example, if you draw a card from a deck but replace it before you draw again, the two events are independent.

For additional information, see Year 1, Chapter 10, problems JM-7, 13, and 17 or Year 2, Chapter 3, problem MD-5.

Example 1

If you roll a fair, 6-sided die, what is $P(3)$, that is, the probability that you will roll a 3?

Because the six sides are equally likely to come up, and there is only one 3, $P(3) = \frac{1}{6}$.

Example 2

There are 5 marbles in a bag: 2 clear, 1 green, 1 yellow, and 1 blue.

If one marble is chosen randomly from the bag, what is the probability that it will be yellow?

$$P(\text{yellow}) = \frac{1 \text{ (yellow)}}{5 \text{ (outcomes)}} = \frac{1}{5}$$

Problems

Answer these questions.

1. There are six crayons in a box: 1 black, 1 white, 1 red, 1 yellow, 1 blue, and 1 green. What is the probability of randomly choosing a green?
2. A spinner is divided into four equal sections numbered 2, 4, 6, and 8. What is the probability of spinning an 8?
3. A fair die numbered 1, 2, 3, 4, 5, and 6 is rolled. What is the chance that an even number will be rolled?
4. The light is out in Sara's closet, so she cannot see the clothes that are hanging there. She has three t-shirts hanging in front of her: 1 brown, 1 black, and 1 navy blue. What is the probability that she chooses the black one?

Answers

1. $\frac{1}{6}$ 2. $\frac{1}{4}$ 3. $\frac{3}{6}$ or $\frac{1}{2}$ 4. $\frac{1}{3}$

Example 3

Joe flipped a coin 50 times. When he recorded his tosses, his result was 30 heads and 20 tails. Joe's activity provided data to calculate experimental probability for flipping a coin.

- a) What is the theoretical probability of Joe flipping heads?
The theoretical probability is 50% or $\frac{1}{2}$, because there are only two possibilities (heads and tails), and each is equally likely to occur.

- b) What was the experimental probability of flipping a coin and getting heads based on Joe's activity?

The experimental probability is $\frac{30}{50}$, $\frac{3}{5}$, or 60%. These are the results Joe actually got when he flipped the coin.

- c) Are these dependent or independent events?

These are independent events. Each flip still has the same possible outcomes (heads or tails), and you can only get one result at a time.

Example 4

Decide whether these statements are theoretical or experimental.

- a) The chance of rolling a 6 on a fair die is $\frac{1}{6}$.
This statement is theoretical.
- b) I rolled the die 12 times and 5 came up three times.
This statement is experimental.
- c) There are 15 marbles in a bag; 5 blue, 6 yellow, and 4 green. The probability of getting a blue marble is $\frac{1}{3}$. This statement is theoretical.
- d) When Veronika pulled three marbles out of the bag she got 2 yellow and 1 blue, or $\frac{2}{3}$ yellow, $\frac{1}{3}$ blue. This statement is experimental.

Example 5

Juan pulled a red card from the deck of regular playing cards. This probability is $\frac{26}{52}$ or $\frac{1}{2}$. He puts the card back into the deck. Will his chance of pulling a red card next time change?

No, his chance of pulling a red card next time will not change, because he replaced the card. There are still 26 red cards out of 52. This is an example of an independent event; his pulling out and replacing a red card does not affect any subsequent selections from the deck.

Example 6

Brett has a bag of 30 multi-colored candies. 15 are red, 6 are blue, 5 are green, 2 are yellow, and 2 are brown. If he pulls out a yellow candy and eats it, does this change his probability of pulling any other candy from the bag?

Yes, this changes the probability, because he now has only 29 candies in the bag and only 1 yellow candy. Originally, his probability of yellow was $\frac{2}{30}$ or $\frac{1}{15}$; it is now $\frac{1}{29}$. Similarly, red was $\frac{15}{30}$ or $\frac{1}{2}$ and now is $\frac{15}{29}$, better than $\frac{1}{2}$. This is an example of a dependent event.

Problems

Decide whether these are independent or dependent events.

1. Flipping two coins.
2. Taking a black 7 out of a deck of cards and not returning it.
3. Taking a red licorice from a bag and eating it.

Answers

1. independent 2. dependent 3. dependent

PROBABILITY FOR TWO OR MORE EVENTS

Addition and multiplication are used to determine the likelihood of an event occurring in more complex situations.

Addition of fractions is used in the following example when we are trying to determine the probability of two or more events.

Example 7

A spinner is divided into five equal sections numbered 1, 2, 3, 4, and 5. What is the probability of spinning a 2 or a 5?

Step 1: Determine both probabilities: $P(2) = \frac{1}{5}$ and $P(5) = \frac{1}{5}$

Step 2: Add the fractions describing each probability: $\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

The probability of spinning a 2 or a 5 is $\frac{2}{5}$: $P(2 \text{ or } 5) = \frac{2}{5}$

Problems

Answer the following questions.

1. One die, numbered 1, 2, 3, 4, 5, and 6, is rolled. What is the probability of rolling a 1 or a 6?
2. A spinner is divided into eight equal sections. The sections are numbered 1, 2, 3, 4, 5, 6, 7, and 8. What is the probability of spinning a 2, 3, or a 4?
3. Patty has a box of 12 colored pencils. There are 2 blue, 1 black, 1 gray, 3 red, 2 green, 1 orange, 1 purple, and 1 yellow. Patty closes her eyes and chooses one pencil. She is hoping to choose a green or a red. What is the probability she will get her wish?
4. John has a bag of jelly beans. There are 100 beans in the bag. $\frac{1}{4}$ of the beans are cherry, $\frac{1}{4}$ of the beans are orange, $\frac{1}{4}$ of the beans are licorice, and $\frac{1}{4}$ of the beans are lemon. What is the probability that John will chose one of his favorite flavors, orange or cherry?

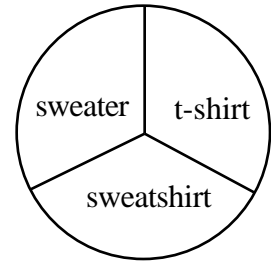
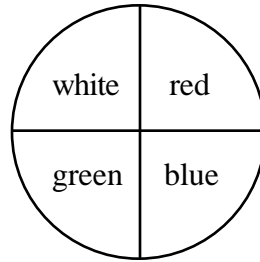
Answers

1. $\frac{2}{6}$ or $\frac{1}{3}$
2. $\frac{3}{8}$
3. $\frac{5}{12}$
4. $\frac{2}{4}$ or $\frac{1}{2}$

Multiplication of fractions is used in the following example where the desired outcome is the product of two possibilities.

Example 8

If each of the regions in each spinner at right is the same size, what is the probability of spinning each spinner and getting a green t-shirt?



Step 1: Determine both possibilities:

$$P(\text{green}) = \frac{1}{4} \quad \text{and} \quad P(\text{t-shirt}) = \frac{1}{3}$$

Step 2: Multiply both probabilities: $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

The probability of spinning a green t-shirt is $\frac{1}{12}$: $P(\text{green t-shirt}) = \frac{1}{12}$

You can also use a probability rectangle to organize the information from this problem. For more information, refer to Year 2, Chapter 3, problems MD-15 and 16.

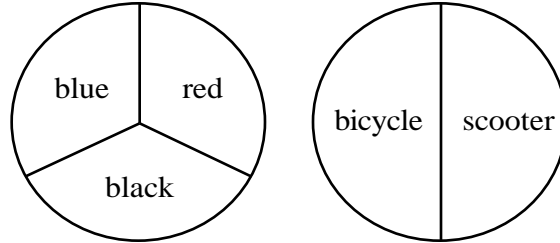
		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
		white	red	blue	green
$\frac{1}{3}$	sweater				
$\frac{1}{3}$	sweatshirt				
$\frac{1}{3}$	t-shirt				

Each box in the rectangle represents a combination of a color and a sweater, sweatshirt, or a t-shirt. The area of each box represents the probability of getting each combination. The shaded region represents the probability of getting a green t-shirt: $\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$.

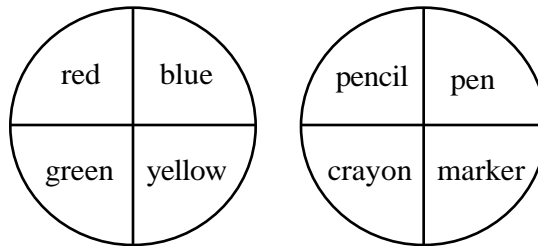
Problems

Determine the following probabilities. A probability rectangle may be useful in some of the problems below.

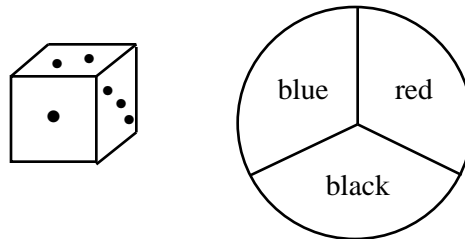
1. If each section in each spinner is the same size, what is the probability of getting a blue bicycle?



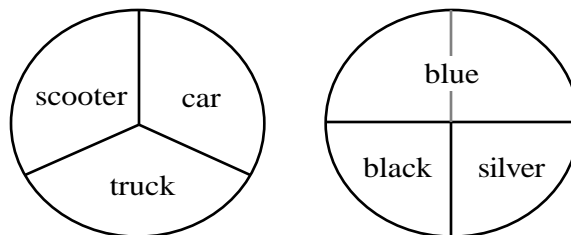
2. What is the probability of spinning each spinner below and getting a red pencil?



3. Mary is playing a game in which she rolls one die and spins a spinner. What is the probability she will get the 3 and black she needs to win the game?



4. Use the spinners below to tell Paul what his chances are of getting the silver truck he wants.



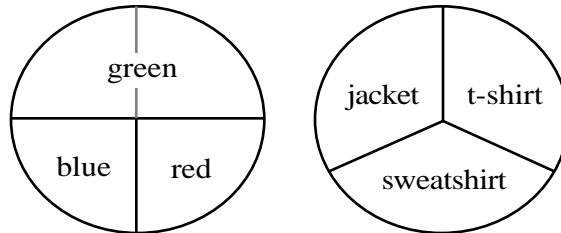
Answers

1. $\frac{1}{6}$ 2. $\frac{1}{16}$ 3. $\frac{1}{18}$ 4. $\frac{1}{12}$

Sometimes addition and multiplication of fractions are both used to determine a desired outcome.

Example 9

What is the probability of spinning a blue t-shirt or a green sweatshirt using the spinners below?



Step 1: Determine the probabilities of each item:

$$P(\text{blue t-shirt}) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$$

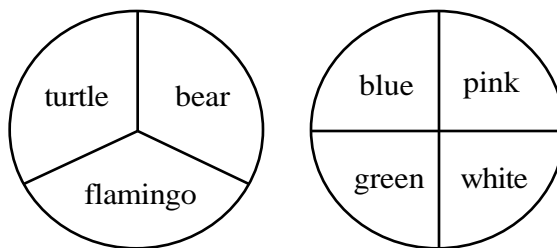
$$P(\text{green sweatshirt}) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

Step 2: Add the probabilities since either outcome is desired: $\frac{1}{12} + \frac{1}{6} = \frac{3}{12}$

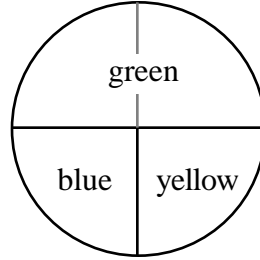
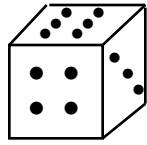
The probability of getting either the blue t-shirt or the green sweatshirt is $\frac{3}{12}$ or $\frac{1}{4}$.

Problems

1. Martha is spinning for her new prize. She wants either the blue stuffed bear or the stuffed pink flamingo. What is the probability she will get the prize she wants?



2. Carlos is playing a game with his friends. He can win either by rolling a 4 on a die and spinning blue or by rolling a 3 and spinning green. What is the chance that he will win the game?



Answers

1. $\frac{2}{12} = \frac{1}{6}$

2. $P(4, \text{blue}) = \frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24}$

$$P(3, \text{green}) = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$

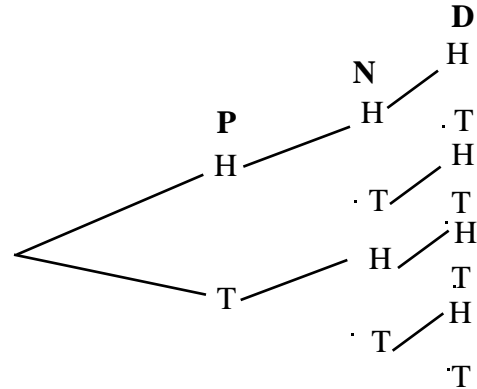
$$P(4, \text{blue or } 3, \text{green}) = \frac{1}{12} + \frac{1}{24} = \frac{3}{24} = \frac{1}{8}$$

TREE DIAGRAMS

A tree diagram is a visual method to find possible outcomes or combinations (called “permutations” in mathematics). It is useful when other methods, such as an organized list, are cumbersome or there are more than two outcomes and a two-dimensional probability rectangle is not feasible. Students are introduced to tree diagrams in Year 1, Chapter 10, problem JM-62 in a problem involving tossing three coins.

Tree diagrams are usually drawn horizontally. That is, each stage of the experiment is shown from left to right. Tree diagrams can be done with 2, 3, 4, or more possibilities.

For example, suppose you are flipping a penny, a nickel, and a dime. Start with the penny and show the two possible outcomes (shown under the “P” at right). Since there were two possible outcomes for the penny, there are two possible outcomes for the nickel for each of the penny’s outcomes. At this point, the tree diagram shows four possible outcomes for the flip of a penny followed by the flip of a nickel (under the “N”).



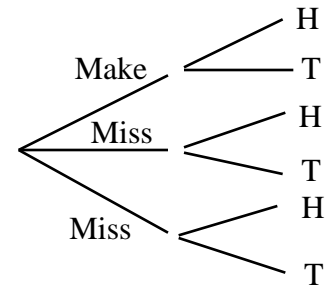
Next flip the dime. Each flip is independent of the other coin flips. If you flip heads on the penny and heads on the nickel, you could get heads or tails on the dime. By following each branch in the diagram starting from the penny, you can see that there are eight possible outcomes when you flip three coins (shown under the “D”). A way to calculate that there are eight possible outcomes is to note that there are two outcomes for the penny, two for the nickel, and two for the dime, so $2(2)(2) = 8$.

By following the branches from the start, far left, to the end of the branch, far right, you see all the possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT.

For additional information, see Year 2, Chapter 10, problem JM-62.

Example 1

At a class picnic Will and Jeff were playing a game where they would shoot a free throw and then flip a coin. Each boy only makes one free throw out of three attempts. Use a tree diagram and a rectangular grid to complete parts (a) through (d) below.



- $P(\text{miss, head}) = ?$
- $P(\text{miss, tail}) = ?$
- $P(\text{make, H}) = ?$
- $P(\text{make, T}) = ?$

	H	T
Make		
Miss		
Miss		

By tracing the branches or counting the small rectangles, the probabilities are:

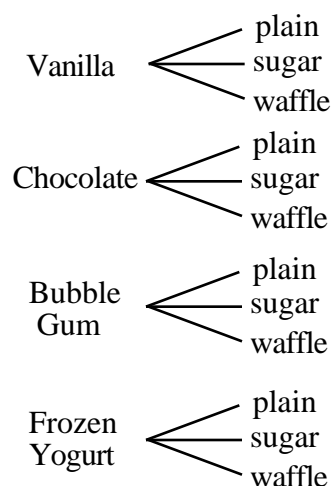
$$P(\text{make, heads}) = \frac{1}{6}, \quad P(\text{make, tails}) = \frac{1}{6}, \quad P(\text{miss, head}) = \frac{2}{6}, \quad \text{and} \quad P(\text{miss, tails}) = \frac{2}{6}$$

Note that the sum of the probabilities in parts (a) through (d) is one.

Example 2

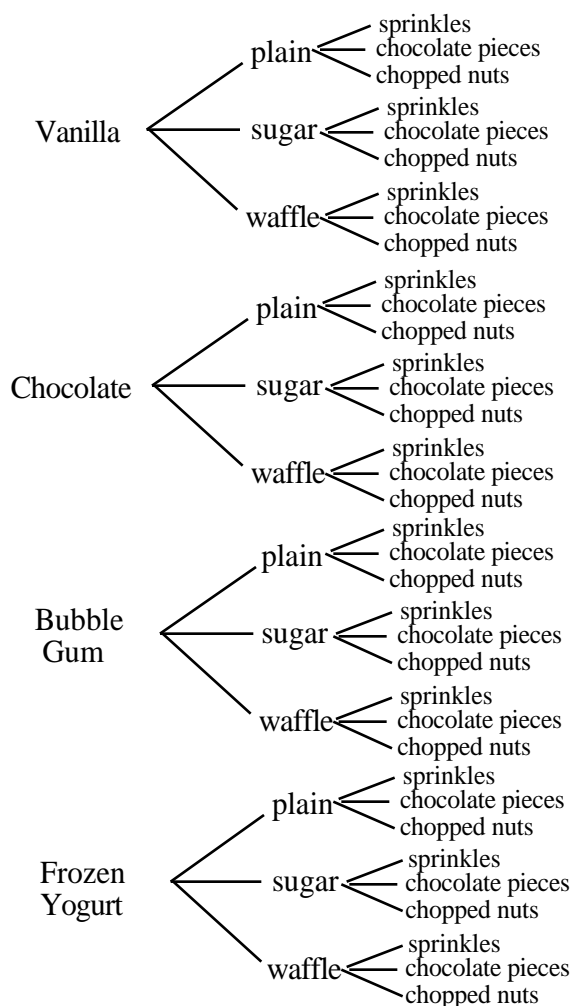
The local ice cream store has choices of plain, sugar, or waffle cones. Their ice cream choices are vanilla, chocolate, bubble gum, or frozen strawberry yogurt. Find all possible ice cream cone combinations.

There are 12 possible combinations.



Example 3

Suppose the following toppings are available for the ice cream cones in example 2: sprinkles, chocolate pieces, and chopped nuts. There are now three more possible outcomes for each of the 12 outcomes in example 2, so $3(12) = 36$ possible combinations.



Problems

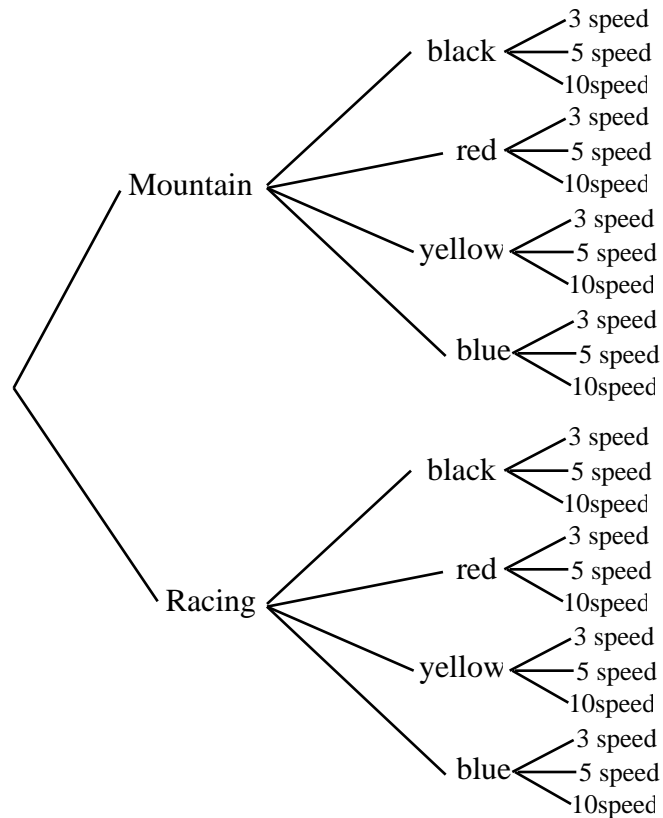
Draw tree diagrams to solve these problems.

1. How many different combinations are possible when buying a new bike if the following options are available:

- mountain bike or racing bike
- black, red, yellow, or blue paint
- 3-speed, 5-speed, or 10-speed

ANSWER

There are 24 possible combinations as shown at right.



2. A new truck is available with:
- standard or automatic transmission
 - 2-wheel or 4-wheel drive
 - regular or king cab
 - long or short bed

How many combinations are possible?

ANSWER

There are 16 possible combinations as shown at right.

