

SV-6.

How to Graph a Quadratic Function

There are many possible relationships between x and y . During the year we will explore several types of equations and their graphs, in addition to linear and quadratic functions. For now, we will write all of the equations in y -form so that it is easy to substitute values for x (input) and find each paired y -value (output).

Graph $y = x^2$ by completing the following steps:

- a) On your paper, copy and complete the table for values of x and y .

x	-3	-2	-1.3	-1	0	0.8	1	2	3
y									

- b) On graph paper, carefully plot the points and draw a smooth curve through them.
- c) On the same set of axes, graph the following two quadratic functions. Use a table like the one above. Check with members of your team to be sure that everyone gets the same values. In the first equation, remember to square first, then change the sign of the result.

$$y = -x^2$$

$$y = (-x)^2$$

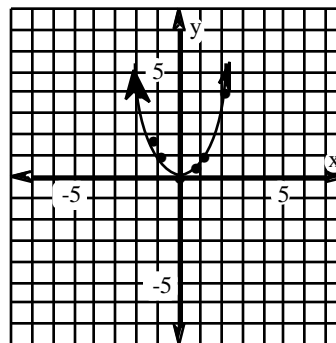


Solution.

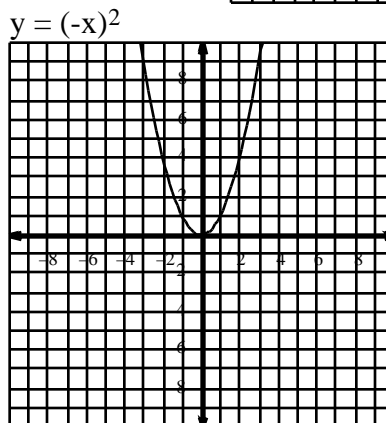
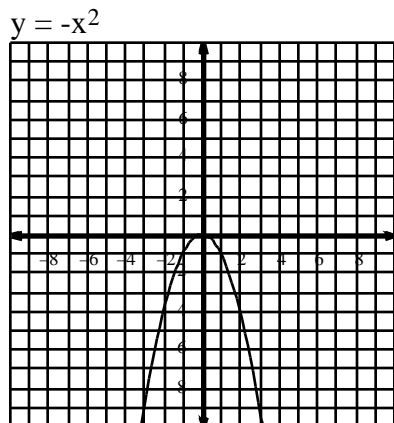
We can always graph any relation by making a table of values. The completed table is below.

x	-3	-2	-1.3	-1	0	0.8	1	2	3
y	9	4	1.69	1	0	0.64	1	4	9

We plot each (x, y) pair and connect the points with a smooth curve. Note: I had to estimate when placing some points. This "u-shaped" graph is a parabola.



For the other two graphs asked for in part (c), we make a table of values and plot the points. (You can use the same table as part (a) if you like. If you make your own table, just be sure to use several points.) The two graphs are shown below.



Try it!

3. Where do the graphs of each of the following quadrants cross the x-axis?

a) $y = (x - 5)(x + 2)$ b) $y = x(x - 8)$ c) $y = (2x + 1)(x - 3)$

Solutions.

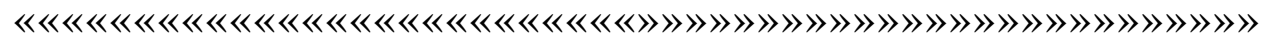
a) (5, 0), (-2, 0) b) (0, 0), (8, 0) c) (-0.5, 0), (3, 0)

In problem SV-18 we find out how to graph non-linear inequalities. Read this problem before continuing.



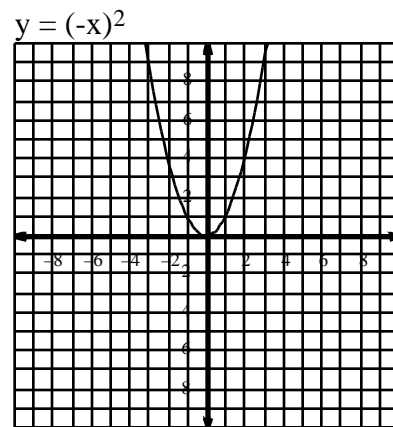
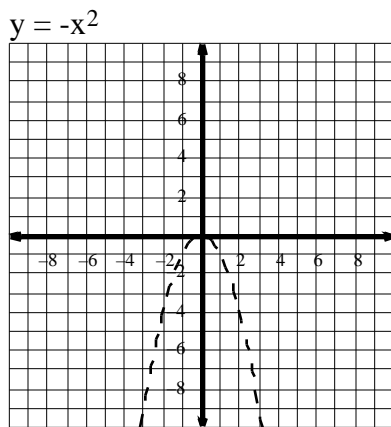
SV-19. Sketch (do not make a table and plot points) the graphs of the inequalities below on graph paper. You can do this quickly if you refer to your graphs in problem SV-6, part (c).

a) $y > -x^2$ b) $y < (-x)^2$



Solution.

As we learned in problem SV-18, to graph an inequality, we first graph as if it were an equality first. We already did this in problem SV-6. These graphs are shown below.



The first one is a dashed curve because the inequality is strictly greater than (rather than greater than or equal to). Next we choose a point not on the curve and test it in the inequality to see if it makes the inequality true or false. I will try the point (0, 1) in each inequality.

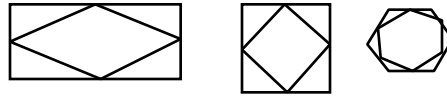
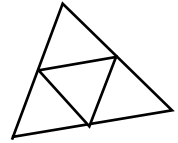
(0, 1):

$$\begin{array}{r} 1 > -0^2 \\ 1 > 0 \text{ True} \end{array}$$

(0, 1):

$$\begin{array}{r} 1 < (-0)^2 \\ 1 < 0 \text{ False} \end{array}$$

To answer the first question: we get another triangle when we connect the midpoints of the sides of a triangle. Sketches of what we get in the remaining situations are shown below. Connecting the midpoints of the sides of a rectangle creates a parallelogram. The midpoints of a square: another square. The midpoints of a hexagon: another hexagon.

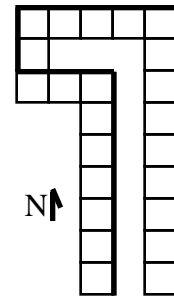


Try it!

- Imagine that you walk 7 blocks north, turn and walk 3 blocks west, turn north again and go 2 blocks north. There you turn and head east for 5 blocks, where you turn and walk 9 blocks south. Draw a sketch of your path. How far are you from where you started?

Solution.

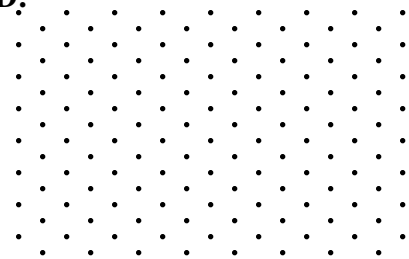
The sketch is shown at right. When the walk is finished, you are just two blocks east of where you started. The blocks are shown to help visualize the directions. The solid line is the path you followed.



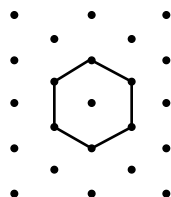
We begin isometric drawings in problem SV-54. The students are usually very interested in this topic, but some students have trouble seeing the drawings as three dimensional. Others can see them as 3-D, but have trouble drawing them. Note: we want the students to use the isometric paper to draw the figures; make sure your child keeps some on hand. To help visualize, have sugar cubes, or some other set of small cubes available for your child to build the object before drawing it or answering questions about it. This helps the students understand that sometimes there can be pieces of the building hidden by other cubes.

This is an **ISOMETRIC DOT PAPER GRID**.

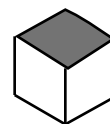
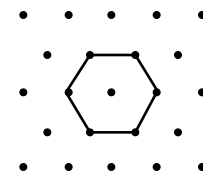
IMPORTANT DETAIL: Isometric dot paper is not the same if it is turned sideways. To be sure that your paper is correctly lined up, draw a hexagon like the one shown below. If the sides are vertical, then the paper is turned correctly; if the sides have points, then turn the paper 90°. Another way to check that your paper is turned correctly is to look at the left margin of the paper. The dots should make a straight, vertical column. If the dots zigzag in and out between rows, the paper is turned the wrong way.



This is turned correctly:

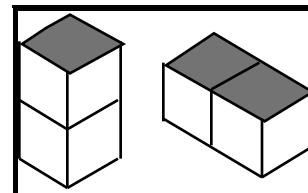


this is not:



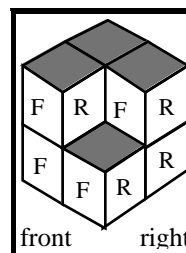
This is a **3-D ISOMETRIC** drawing of a cube.

By stacking and pushing cubes together, we can create other solids. Two cubes can be joined to form two isometric solids as shown at right:



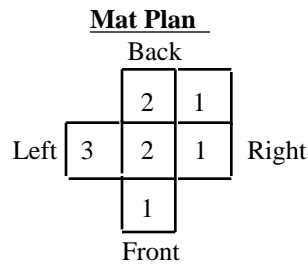
Keep real cubes handy as you are working on these problems. Setting up what is pictured using real cubes can be very helpful when interpreting the drawings.

When we look at an isometric drawing we will **ALWAYS** view it from the front, right, and top parts of the solid as shown at right. Building models on a 3" x 5" index card marked "front," "back," "left," and "right" is helpful, because you can turn the model and view it from several directions without it falling apart or having to rebuild it.



Do not forget to make four copies of a cube.

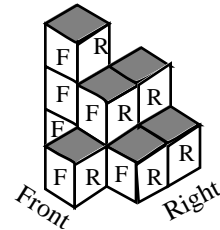
DIRECTIONS: The easiest way to represent multiple cube shapes on square grid paper is with a Mat Plan. A **MAT PLAN** is a top (or bottom) view of a multiple cube solid. The following is an example of a Mat Plan:



The number in each square represents the number of cubes in that stack. Enough information is given for you to be able to reconstruct the multiple cube shape.

- a) How many cubes will it take to build this solid?

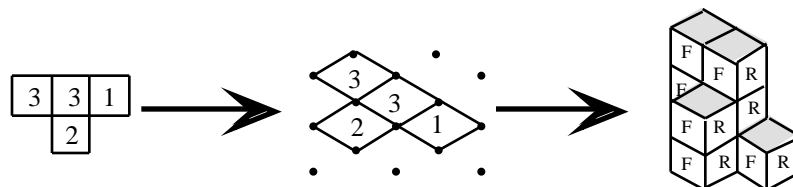
This is an **ISOMETRIC VIEW** of the solid represented by this Mat Plan. The shaded areas are the tops of each stack.



- b) If you did not already know the Mat Plan for this isometric view, could this picture be hiding a cube?
- c) What would be the Mat Plan if there were a hidden cube?

REMEMBER: So that we can always recognize what each of us draws, that is, so that we draw figures in a consistent orientation (position), we will always draw isometric views so that the right face, the front, and the top are the three represented views. Another way to remember this is to think of the base of the isometric view as the Mat Plan rotated 45° clockwise (see the figures in the next problem).

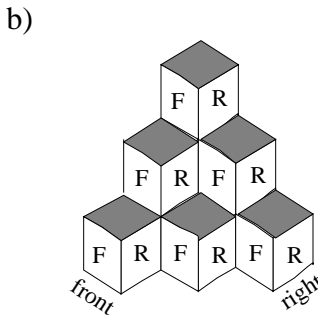
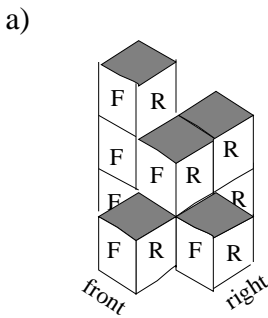
A helpful technique for isometric drawing is to first transform the outline of the Mat Plan to isometric paper. The middle figure shows the Mat Plan rotated 45° clockwise. This view can now act as a floor plan and you can build the picture next to it as shown.



VOLUME is the number of 1 x 1 x 1 cubes, or parts of cubes, that fit inside a three dimensional figure or object. For example, the solid in the box above has a volume of nine cubic units.



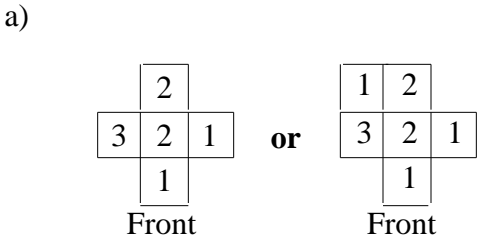
SV-68. Create Mat Plans from the following isometric views and find the volume of each.



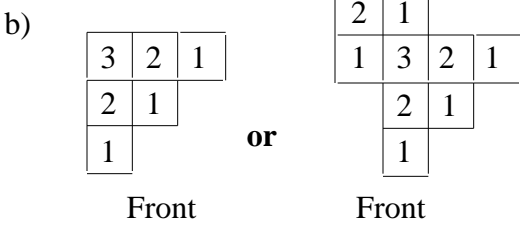
- c) Did you have to make any assumptions about hidden cubes when you drew the Mat Plans? If so, what assumptions did you make in each case and why?
- d) What other view would you need to see to be sure how many cubes there are? Explain.



Solutions.



Volume = 9 or 10



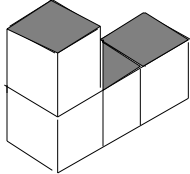
Volume = 10, 11, 12, 13, or 14

- c) Each of these could have hidden cubes. Rather than make an assumption, I drew the two possibilities.
- d) In each case, if we could see the left, bottom or top view we could decide whether or not there are hidden cubes when viewing from the standard position.

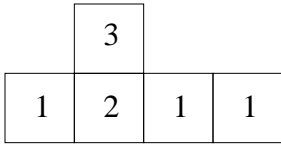
Try it!

6.

a) Create a Mat plan of this isometric drawing.



b) Create the isometric drawing of this Mat plan.

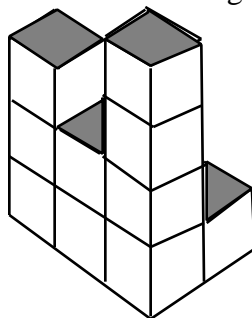


7. Determine the volume of each of the three dimensional figures shown below.

a) Mat Plan

3	4	2
2	1	1

b) Isometric drawing



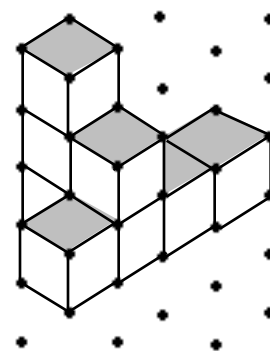
Solutions.

6.

a)

2	1	1

b)



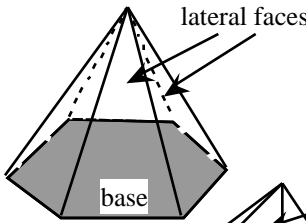
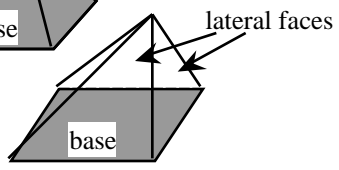
7. a) The volume is the number of cubes in the figure. To find the volume of this solid we sum the number of cubes shown in the Mat plan. $V = 3 + 4 + 2 + 2 + 1 + 1 = 13$ cubic units.
- b) The volume is ambiguous because there might be cubes hidden in the second and third row back. There are 10 cubes visible so the volume is at least 10 cubic units. There could be as many as 10 hidden cubes. The Mat plan at right shows the maximum number of hidden cubes with italics. The volume could be 10, 11, 12, . . . , 20 cubic units.

	2	<i>1</i>
3	2	2
2	3	
4	1	

We move to calculated surface area and volume next. Read the boxes below and problem SV-101 before doing the problem below.

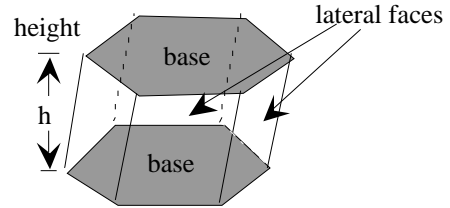
A PYRAMID is

- 1) a polyhedron
- 2) with a base that is a polygon and
- 3) lateral faces that are formed by connecting each vertex of the base to a single given point (the vertex of the pyramid) that is above or below the surface that contains the base. The lateral faces do NOT need to be congruent.

A **PRISM** is:

- 1) a polyhedron
- 2) with two **congruent** (same size and shape) parallel bases that are polygons, and
- 3) lateral faces (faces on the sides) that are parallelograms formed by connecting the corresponding vertices of the two bases. Lateral faces may also be rectangles, rhombi, or squares.

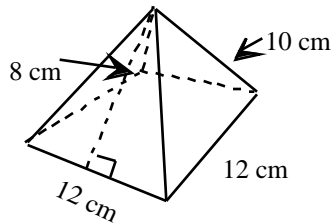


The **VOLUME** of a prism is the product of the **base area** (B) and **height** (h) (perpendicular distance between the bases): $V = Bh$. Volume is expressed in **cubic** units.

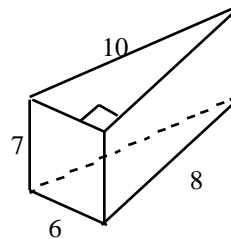


SV-103. Calculate the total surface area of the following figures by using subproblems to first draw a picture of what each face looks like as a single polygon. Part (a) has been done as an example for you. Don't forget the bases!

- a) The base of the pyramid is a square. b) The figure is a prism.



$$\text{S.A.} = \square + 4(\triangle)$$

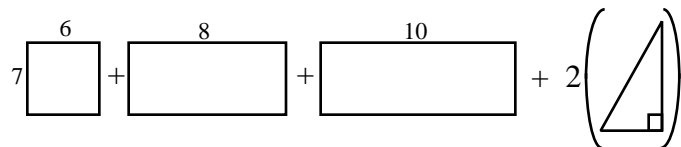


Solutions.

To calculate the surface area we must dissect the solid into its parts or faces. Part (a) shows the dissection of this square-based pyramid: it is made up of one square (the base) and four triangles (the faces). To find the area of the square base we multiply the length times the width: $12(12) = 144$. The faces of the pyramid are triangles, four of them each, with a base of 12 cm and a height of 8 cm. The area of one such triangle is $\frac{1}{2}(12)(8) = 48$. Four of these triangles have an area of $4(48) = 192$. We add the area of the base to the area of the faces to get the total surface area: $144 + 192 = 336$ square cm. For part (b) the dissection looks like the figure below:

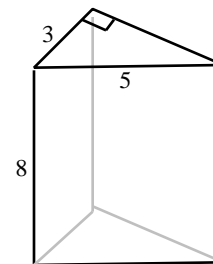
The areas are easily calculated:

$$7(6) + 7(8) + 7(10) + 2\left(\frac{1}{2}\right)(6)(8) = 42 + 56 + 70 + 48 = 216 \text{ sq. units.}$$



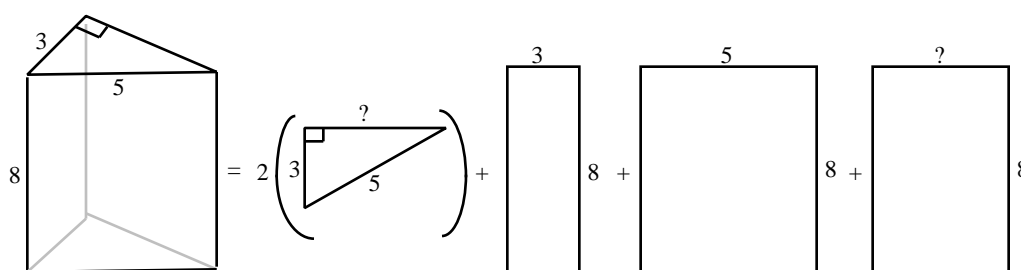
Try it!

8. Find the surface area of the figure at right. Show the area dissection of the figure.

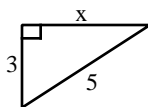


Solution.

Just as we can "dissect" complicated two-dimensional figures into smaller parts to find the area, we can break apart the three-dimensional figure to find the surface area. We can think of the surface area as what we would need to cut out of cardboard to make this figure. In this case,



the figure is made up of two triangles (the top and bottom) and three rectangles. Each rectangle has a height of 8 units. Now we must find the area of each of these shapes. We can easily find the area of the first two rectangles. To find the area of the triangle and the area of the last rectangle, we must find the length of the unknown side. We can use the Pythagorean Theorem to do this.



$$\begin{aligned} 3^2 + x^2 &= 5^2 \\ 9 + x^2 &= 25 \\ x^2 &= 25 - 9 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

Therefore, the sides marked with question marks are each 4 units in length. Now we can calculate all the areas.

$$\begin{aligned} SA &= 2 \left(\frac{1}{2} (3)(4) \right) + 3(8) + 5(8) + 4(8) \\ &= 12 + 24 + 40 + 32 \\ &= 108 \text{ square units} \end{aligned}$$

The calculation of the missing length and each area calculation were necessary subproblems in the solution of the original problem.

Again, whenever possible, have materials available so that your child can build or reenact the problems. This will help your child visualize figures and draw diagrams as well as make the problems real for them. Additionally, the added experience of handling manipulatives while exploring the mathematics will enable the students to understand the ideas and help them to remember what it is they are studying.

MORE TO TRY

Identify which of the following is a quadratic function:

1. $y = x^2 + 5x - 7$

2. $y = x^3 + 3x + 2$

3. $y = x^3 - 3$

4. $y = x^4 + 2$

105. $y = x^2 - 3x^4 - 2$

6. $y = x^2 - 7$

7. $y = -x - 5 - x^2$

8. $y = 5 - x^3$

9. $y = 7 + x^2$

10. $y = x^3 - x^2$

Make a table and graph the equation. Find the x-intercepts (zeros) accurate to the nearest tenth

11. $y = x^2 - 3x - 5$ 12. $y = x^2 - x - 12$ 13. $y = x^2 - 2x - 10$ 14. $y = x^2 - 10x + 9$

15. $y = x^2 + 2x - 7$ 16. $y = x^2 - 4x - 2$ 17. $y = x^2 - 6x + 5$ 18. $y = x^2 - 9x - 15$

19. $y = x^2 - 16$ 20. $y = x^2 - 49$ 21. $y = x^2 - 121$

Graph each parabola. Factor the equation and explain how the factored form relates to the graph.

22. $y = x^2 - 3x + 2$

23. $y = x^2 - 10x + 25$

24. $y = x^2 - x - 12$

25. $y = x^2 - 4x - 5$

26. $y = x^2 + 2x - 8$

27. $y = x^2 + 6x + 9$

28. $y = x^2 - 8x + 16$

29. $y = x^2 - 9$

Solve the following using the Zero Product Property.

30. $(x - 2)(x + 3) = 0$

31. $2x(x + 5)(x + 6) = 0$

32. $(x - 18)(x - 3) = 0$

33. $4x^2 - 5x - 6 = 0$

34. $(2x - 1)(x + 2) = 0$

35. $2x(x - 3)(x + 4) = 0$

36. $3x^2 - 13x - 10 = 0$

37. $2x^2 - x = 15$

Graph the following inequalities. Be sure to use a test point to determine which side to shade. Your graphs to the previous problems might be helpful.

38. $y < x^2 - 3x + 2$

39. $y > x^2 - 10x + 25$

40. $y < x^2 - x - 12$

41. $y < x^2 - 4x - 5$

42. $y > x^2 + 2x - 8$

43. $y < x^2 + 6x + 9$

44. $y < x^2 - 8x + 16$

45. $y < x^2 - 9$

Graph the following equations on the same set of axes. Write a conjecture about the pattern(s) you notice.

46. $y = x^2$
 $y = x^2 + 1$
 $y = x^2 + 3$
 $y = x^2 - 2$

47. $y = x^2$
 $y = 2x^2$
 $y = \frac{1}{2} x^2$
 $y = 3x^2$

48. $y = x^2$
 $y = (x + 5)^2$
 $y = (x - 3)^2$
 $y = (x + 1)^2$

Create isometric drawings from the following Mat plans.

49.

	1	4
	2	3

50.

	3	3
4	2	1
5	2	1

51.

2	4	
1	3	1
2	1	2

52.

		3
	2	2

53.

3	2	4
3	2	1

54.

3	2	2
1	3	2
2	2	1

55.

		2
2	2	1

56.

		3
		2
2	2	1

57.

		2
	3	1
2	3	2

58.

		3
	3	1
4	3	1

59.

	3	3
	3	2
5	4	3

60.

	3	3
4	3	1
4	2	1

61.

	3	2
2	2	1

62.

3	3	3
3		2
3	2	2

63.

4		
3	2	
3	1	

64.

	3	2
		1
	4	2

65.

	3	2
	3	1
4	3	

66.

3	4	3
3	2	1
4	1	

Find the volume of each solid represented by each Mat plan below.

67.

	1	4
	2	3

68.

	3	3
4	2	1
5	2	1

69.

2	4	
1	3	1
2	1	2

70.

		3
	2	2

71.

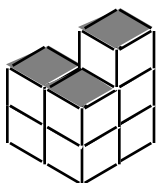
3	2	4
3	2	1

72.

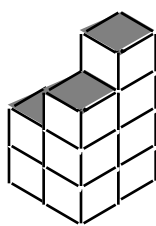
3	2	2
1	3	2
2	2	1

Create a Mat plan for each of the following isometric drawings. Be sure to indicate possible hidden cubes.

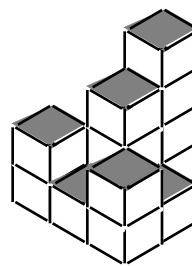
73.



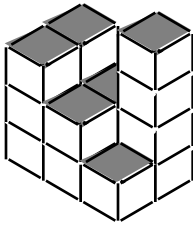
74.



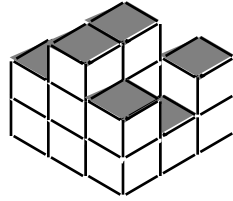
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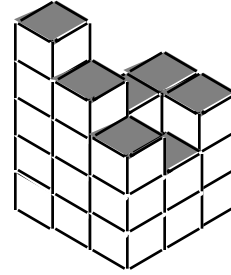
76.



77.

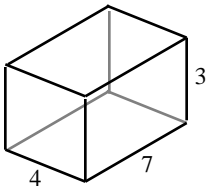


78.

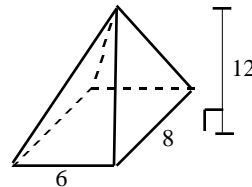


Find the volume of the following.

79.

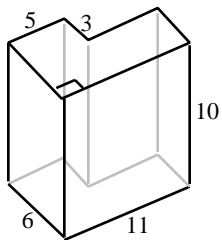


80.

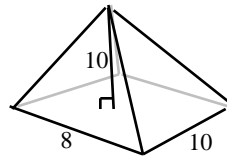


Calculate the surface area of each of the following. Show the subproblems.

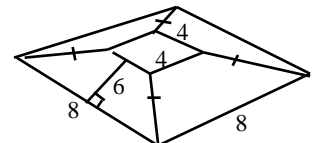
81.



82.

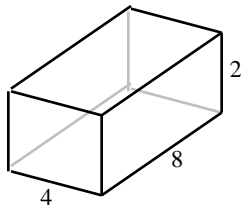


83.

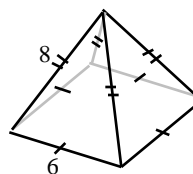


Find the surface area of each polyhedron.

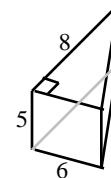
84.



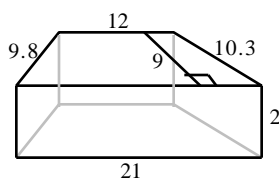
85.



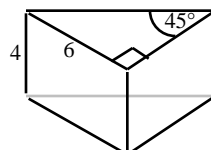
86.



87.



88.



89.

