AP Calculus Content Strands

Note 1: Chapter 1 includes review of precalculus concepts essential for the understanding of Calculus. Chapter 9 includes precalculus background on vectors, polar graphs, and parametric equations necessary for the BC additional material on those topics.
Note 2: This guide shows where the topics are introduced and studied. Once a topic is introduced and presented, problems occur throughout the rest of the text as review in the homework section. Those problems spaced throughout the text are not listed in this guide.
Note 3: Students are encouraged throughout the text to explore concepts using their graphing calculator as well as understanding concepts analytically. They are encouraged to connect their geometric and analytic understanding of concepts.

I. Functions, Graphs, and Limits	Chapter and Lessons
Analysis of graphs With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.	The analysis of graphs is covered throughout the text showing the connection between the geometric and analytic representations.
Limits of functions (including one-sided limits)	
• An intuitive understanding of the limiting process	**2.2.1** Students will explore ideas leading to an intuitive definition of a limit.
• Calculating limits using algebra	**2.2.4** Students learn to evaluate limits with algebraic manipulations.
• Estimating limits from graphs or tables of data	**2.2.1-2.2.4** Students will estimate limits from graphs or tables.
Asymptotic and unbounded behavior	
• Understanding asymptotes in terms of graphical behavior	**1.2.2 & 1.2.3** Students will understand asymptotes in terms of graphical behavior. They will recognize holes, vertical asymptotes. They will also examine rational functions writing approach statements for them.

• Describing asymptotic behavior in terms of limits involving infinity	**1.2.2 & 1.2.3** Students will describe asymptotic behavior in terms of limits involving infinity including end behavior and horizontal asymptotes.
• Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth)	**1.3.3** Students write descriptions of the rate of change of various functions and relations.

Continuity as a property of functions

• An intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)	**1.2.1** Students will gain an intuitive notion of continuity.
• Understanding continuity in terms of limits	**2.2.2** Students will continue to explore ideas of continuity including determining the formal definition of continuity involving an understanding of limits.
• Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)	**2.2.3** Students learn the Intermediate Value Theorem. **5.1.3** Students learn the Extreme Value Theorem.
***BC* - Parametric, polar, and vector functions The analysis of planar curves includes those given in parametric form, polar form, and vector form.**	**11.1.1** Students will approximate the area within a polar curve and conjecture a general formula. **11.1.2** Students will find areas bounded by polar graphs. **11.1.3** Students will find polar area bound by two polar curves.

II. Derivatives

Concept of the derivative

• Derivative presented graphically, numerically, and analytically	**2.3.1- 2.3.2** Students approximate average velocity and instantaneous velocity through the Ramp Lab and Sudden Impact investigation. 3.2.2 Students find derivatives using multiple strategies i.e. the definition of the derivative, power rule, or a graphing calculator.

• Derivative interpreted as an instantaneous rate of change	**2.3.1 - 2.3.2** Students calculate average rate of change and estimate instantaneous rate of change in the Ramp Lab. **3.2.1** Students formulate the Definition of a Derivative interpreted as instantaneous rate of change. **5.1.1** Students calculate instantaneous rate of change in applied situations.
• Derivative defined as the limit of the difference quotient	**3.1.2** Students explore the difference between Average Rate of Change (AROC) and Instantaneous Rate of Change (IROC) by drawing secant lines between two points. Students write the rate as a difference quotient and let the run approach 0 to find IROC.
• Relationship between differentiability and continuity	**3.4.2** Students will identify points of non-differentiability and discuss reasons including a cusp and non-continuity.

Derivative at a point

• Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.	**3.4.1** Students learn/explore conditions for function to be differentiable at a point. **3.4.2** Students continue to explore what it means to be differentiable at a point giving examples of situations that are not differentiable. They are also introduced to the formal definition of a limit: $\lim_{h \to 0} \left(f(x+h) - f(x) \right) / h$ **3.4.3** Students explore functions that appear to be differentiable on a calculator but an analytic analysis may show there are points which are not differentialbe.
• Tangent line to a curve at a point and local linear approximation	**2.3.3** Students study local linearity first on a graphing calculator zooming in close to a point and describing what they see. They discuss whether the tangent line is an over/under approximation of the height as you get close to the point.
• Instantaneous rate of change as the limit of average rate of change	**3.1.2** Students explore the difference between Average Rate of Change (AROC) and Instantaneous Rate of Change (IROC) by drawing secant lines between two points. Students write the rate as a difference quotient and let the run approach 0 to find IROC.

• Approximate rate of change from graphs and tables of values	**3.2.1** Students explore a variety of situations with a table of values, function, or graph writing a quotient function to find AROC and letting the horizontally distance between to points approach 0 to find IROC.

Derivative as a function

• Corresponding characteristics of graphs of f and f'	**1.4.1 – 1.5.1** Students will explore and compare position graphs with their respective velocity graphs. They will draw a position graph given a velocity graph. Or they will draw a velocity graph given a position graph.
• Relationship between the increasing and decreasing behavior of f and the sign of f'	**3.3.1 – 3.3.4** Students learn the corresponding characteristics of f and f′. By comparing graphs, students explore the relationship between the increasing and decreasing behavior of f and the sign if f′.
• The Mean Value Theorem and its geometric interpretation	**6.4.1 - 6.4.2** Students will explore concepts involving the mean value and its geometric interpretaion. They will learn the Mean Value Theorem.
• Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.	**5.1.3** Students will describe the shape of a curve given information about the first and second derivative for a particular interval and a particular point.

Second derivatives

• Corresponding characteristics of the graphs of f, f' , and f''	**5.1.3-5.1.4** Students will describe the shape of a curve given information about the first and second derivative for a particular interval and a particular point.
• Relationship between the concavity of f and the sign of f''	**3.3.4** Students chart the signs of various curve segments for f, f' and f''.
• Points of inflection as places where concavity changes	**3.3.4** Students discover that the point of inflection is where concavity changes.

Applications of derivatives

	3.4.4 Students will apply their knowledge of tangent lines and derivatives to solve a problem. **5.1.3** Students will describe the shape of a curve given information about the first and second derivative for a particular interval and a particular point.
• Analysis of curves, including the notions of monotonicity and concavity	**5.1.4** Students will apply the first and second derivative tests.
***BC* - Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration**	**11.2.1** Students will find velocity vectors and their slopes. **11.2.2** Students will write acceleration vectors and determine their slopes. **11.2.3** Students will determine the slope of vectors without eliminating the parameter. **11.2.4** Students will find arclength of parametric curves. **11.3.1-11.3.2** Students will develop two methods for determining slopes of polar curves. **11.4.1** Students will apply their knowledge of parametric equations to simulate a robot race and determine the winner.
• Optimization, both absolute (global) and relative (local) extrema	**5.1.2** Students will find the dimensions of each box design that will maximize the volume. **5.3.1 - 5.3.3** Students solve a variety of optimization problems.
• Modeling rates of change, including related rates problems	**7.1.1** Students will describe the rates of change of objects that are directly related. **7.1.1 - 7.1.5** Students will solve a variety of related Rates problems.
• Use of implicit differentiation to find the derivative of an inverse function	**6.3.4** Students will find derivative of inverse functions and derive a formula to find the derivative of an inverse function.
• Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration	**5.1.1** Students will find velocity and acceleration from a position function. **6.4.2 - 6.4.3** Students solve average value problems.
• Geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations	**7.3.3 - 7.3.5** Students will sketch slope fields and use slope fields to sketch particular solutions for differential equations.

***BC* - Numerical solution of differential equations using Euler's method**

7.4.1 Students will use Euler's Method to approximate a curve.

5.5.1-5.5.2 Students will use their GC to find the limit of indeterminate limits. They will explore the slopes of the numerator and denominator of indeterminate limits. They will learn and apply L'hopital's Rule to find limits of indeterminate forms.

***BC* - L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series**

10.1.7 Students will develop and apply the Limit Comarison Test for series convergence.

Computation of derivatives

3.1.1-3.1.2 Students learn the power rule to find a derivative.

3.2.3 Students figure out what the derivative of the sin and cos are through graphical interpretation.

5.2.5 Students derive the derivatives of the main trigonometric functions from the derivatives of sin x and cos x.

6.1.1 Students will explore the value of "e" and gain an intuitive understanding of the derivative of e^x.

6.1.2-6.1.4 Students will find derivatives of a variety of functions including exponential and trigonmetric using a variety of derivative rules.

• Knowledge of derivatives of basic functions, including power, exponential,logarithmic, trigonometric, and inverse trigonometric functions

6.3.1-6.3.2 Students will find derivatives of inverse trigonometric functions.

6.3.3 Students will find derivative of logarithmic functions.

5.2.1 Students will analyze a proof of the Product Rule and apply the product rule on a variety of problems.

• Derivative rules for sums, products, and quotients of functions

5.2.4 Students discover and apply the Quotient Rule to find derivatives.

5.2.2 Students will find a process for taking the derivative of composite functions and apply that knowledge.

5.2.4 Students will apply the Chain Rule with power and product rule to a set of functions to find the derivative.

• Chain rule and implicit differentiation

6.2.1-6.2.2 Students will explore relations describing implicit functions and differentiate them implicitly.

***BC* - Derivatives of parametric, polar, and vector functions**

11.3.1 Students calculuate derivatives of Polar Curves

III. Integrals

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval: $\int_a^b f'(x)\,dx = f(b) - f(a)$
- Basic properties of definite integrals (examples include additivity and linearity)

4.1.1 -4.1.2 Riemann sums are formally defined. Students explore the accuracy of Riemann sums.

4.2.3-4.2.4 Students develop both parts of the Fundamental Theorem of Calculus

4.1.3 Students explore properties of definite integrals

*** Applications of integrals Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen , the emphasis is on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from a rate of change.**

4.3.1-4.3.3 The Case of the Heavy-Footed Teacher: Students will use their knowledge of velocity, position, acceleratons and the FTC to solve a speeding violation case.
4.4.1-4.4.3 Students will set up integrals for finding the area between two or more curves.
8.1.1-8.3.3 Students use integration to find volumes of known cross sections. These include solids of revolution using disk and shell method as well as solids with non-circular cross sections.
8.4.1 Students will develop a method to find the length of a curve by approximating the length of a curve using a sum of line segment lengths.

Fundamental Theorem of Calculus

	4.2.2 Students distinquish between a definite and indefinite integral. They use area functions to numerically integrate. **4.2.4** Students practice using the twoparts of the FTC.
• Use of the Fundamental Theorem to evaluate definite integrals	**5.4.1** Students will explore and solve integrals where the limits of integration are functions
• Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined	**4.2.3** Students evaluate definite integrals using the FTC. They explore the relationship between an integral and its derivative. They develop both parts of the FTC.

Techniques of antidifferentiation

• Antiderivatives following directly from derivatives of basic functions	**3.4.1** Students find the general antiderivative for a polynomial function. **6.1.4** Students integrate exponential functions.
BC - Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only)	**7.2.1 -7.2.4** Students learn antiderivatives by substitution of variables including change of limits of definite integrals. **7.4.2-7.4.3** Students will learn integration by parts. **7.4.4** Students will use partial fractions to integrate.
BC - Improper integrals (as limits of definite integrals)	**6.5.1** Students will evaluate improper integrals.

Applications of antidifferentiation

• Finding specific antiderivatives using initial conditions, including applications to motion along a line	**5.1.1** Students find velocity functions from an acceleration function and position functions from a velocity function. They explore graphs and functions to find initial conditions in order to get the exact functions.
• Solving separable differential equations and using them in modeling (including the study of the equation y ky and exponential growth)	**7.3.1-7.3.2** Students learn to solve separable differential equations and apply this knowledge in a variety of applications.
BC - Solving logistic differential equations and using them in modeling	**10.2.1** Students will investigate and graph a scenario involving logistic differential equations. **10.2.2** Students will solve logistic differential equations.

Numerical approximations to definite integrals

Use of Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values	**2.1.1 – 2.1.3; 2.4.1** Students will use Riemann sums using left and right endpoints to approximate definite integrals of functions represented algebraically, graphically, and by tables of values. They will also use the Trapezoidal Rule to approximate the area under a curve.

*IV. Polynomial Approximations and Series

*** Concept of series A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.**

9.1.2 Students work with sequence of partial sums and convergence of the limit of partial sums.

*** Series of constants**

BC - Motivating examples, including decimal expansion

6.1.1 Students explore the decimal expansion of "e".

BC - Geometric series with applications

9.1.1-9.1.3 Students study Infinite Geometric Series in various applications and their convergence or divergence.

BC - The harmonic series

10.1.4 Students test a harmonic series for convergence.

BC - Alternating series with error bound

10.1.3 Students will learn and apply the Alternating Series Test.

12.2.1 Students will investigate error of alternating Taylor polynomials graphically.

10.1.1 -10.1.2 Students will investigate convergence and divergence of series.

BC - Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series

10.1.4 Students will learn and apply the Integra Test and its use in testing the convergence of p-series.

10.1.5 Students wil learn and apply the p-Series Test for convergence.

BC - The ratio test for convergence and divergence

10.1.8 Students will develop and apply the Ratio Test for convergence and divergence.

BC - Comparing series to test for convergence or divergence

10.1.6 Students will learn and apply the Comparison Test for series convergence or divergence.

***BC* - Taylor series**

BC - Maclaurin series and the general Taylor series centered at x = a

12.1.1 Students find a polynomial given minimal information about the function at a point.

BC - Maclaurin series for the functions ex, sinx, cosx, and 1/(1-x)

12.1.2 Maclaurin polynomials are formalized.

BC - Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series *BC* - Functions defined by power series *BC* - Radius and interval of convergence of power series *BC* - Lagrange error bound for Taylor polynomials	**12.1.3** Students will develop Taylor polynomials as a translation of a MacLaurin polynominal. **12.1.4** Students will define Taylor Series **12.1.5** Students will use substitution to approximate complicated composite functions with Taylor and MacLaurin polynomials **10.3.1** Students will define a power series and its interval of convergence. **12.2.3** Students will determine the interval of convergence for Taylor series. **12.2.2** Students will derive a formula for Lagrange error of Taylor polynomials.