

AR-5

<p><b>STUDY TEAM GUIDELINES</b></p> <p>1. Each member of the team is responsible for his or her own behavior.</p>	<hr/> <hr/> <hr/> <hr/>
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AR-12

<p><b>STUDY TEAM GUIDELINES</b></p> <p>2. Each member of the team must be willing to help any other team member who asks for help.</p>
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AR-49

<p><b>STUDY TEAM GUIDELINES</b></p> <p>3. When you have a question ask your partner or team first. If no one can answer the question, then ask the teacher for help.</p>	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/>
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AR-66

<p><b>STUDY TEAM GUIDELINES</b></p> <p>4. Use your team voice.</p>	<hr/> <hr/> <hr/>
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**MEAN**

The **MEAN** is the arithmetic average of a data set. One way to determine the mean is to add all values in a set of data and divide the sum by the number of values.

Example: Find the mean of this set of data:

57, 44, 52, 49, and 53.

- $57 + 44 + 52 + 49 + 53 = 255$
- $255 \div 5$  (the number of values) = 51,  
so the mean is 51.

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**MODE**

The **MODE** is the value in a data set that occurs more often than any other value. Data sets may have more than one mode, and some do not have any mode. The mode is useful when the data are not numeric, such as showing a “most popular” choice.

Example: Find the mode of this set of data: 11, 13, 14, 9, 7, 15, 14, 10.

11 13 (14) 9 7 15 (14) 10

The mode of this data set is 14.

**MEDIAN**

The **MEDIAN** is the middle value in a set of data arranged in numerical order. If there is an even number of values, the median is the mean of the two middle values.

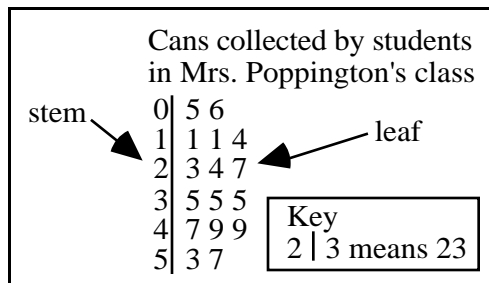
Example: Find the median of this set of data: 14, 103, 5, 38, 60, 6, 77, 13, 62, and 95.

- Arrange the data in order: 5, 6, 13, 14, 38, 60, 62, 77, 95, and 103.
- Find the middle value(s): 5, 6, 13, 14, (38), (60), 62, 77, 95, 103
- Where there are two middle values, find their mean:  
 $38 + 60 = 98$        $98 \div 2 = 49$  (the median)

### STEM-AND-LEAF PLOT

A **STEM-AND-LEAF PLOT** is a way to display data that shows the individual values from a set of data and how the values are distributed. The “stem” part on the graph represents the leading digit(s) of the number. The “leaf” part of the graph represents the other digit(s) of the number.

Example:



### A COMPLETE GRAPH

Graphs are ways of displaying and comparing information. A complete data graph has the following characteristics:

- All graphs are **neat** and easy to read and, when appropriate, constructed with a **straightedge**.
- The units (or numbers) along the axes are clearly **labeled**.
- The axes (the vertical and horizontal number lines) are labeled with **words** that explain the numbers on the axes.
- The units (numbers marked on the axes) follow **equal intervals** on each axis.
- All graphs have a **title**.
- A key or **legend** is included when it is necessary to explain any symbols that are used in the graph.

Graphs should be drawn on graph paper or resource pages.

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### MEASURES OF CENTRAL TENDENCY

Numbers that locate or approximate the “center” of a set of data are called **measures of central tendency**. Mean, median, and mode are measures of central tendency.

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AR-69

### SCALING THE AXES OF GRAPHS

- Find the difference between the low and high numbers you need to put on the axis.
- Determine how many spaces you have on your axis.
- Divide the difference by the number of spaces on your axis to find the width of each space.
- Label the axis.

Example: Suppose your x-values go from -22 to 68 and you have 20 spaces. You need for x to increase at least  $\frac{22+68}{20} = \frac{90}{20} = 4.5$  for each space. Most people would scale the axes by 5s.

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AR-75

### INTEGERS

**INTEGERS** are whole numbers and their opposites, including zero. These numbers can be positive, negative, or zero. On the number line, think of them as “whole steps.”

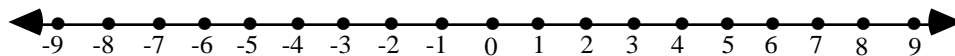
See number line below.

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**ADDITION OF INTEGERS**

If the signs are the same, combine by adding the value of each number and keep the same sign.

If the signs are different, ignore the signs. Then subtract the smaller number from the larger number and keep the sign of the original number that was farthest from zero on the number line.

Example 1:

add  $-5 + (-3)$

$$5 + 3 = 8$$

Both signs are  $-$ , so  $-5 + (-3) = -8$

Example 2:

add  $-4 + 2$

$$4 - 2 = 2$$

$-4$  is farthest from 0,  
so  $-4 + 2 = -2$

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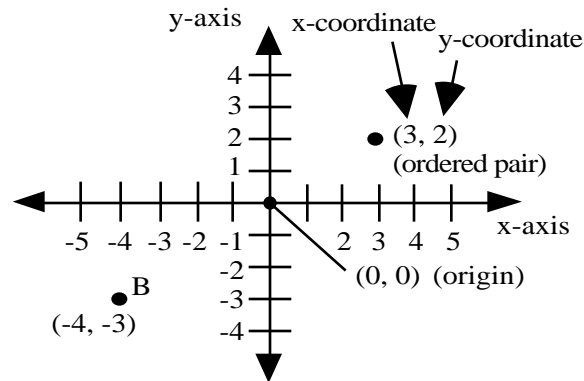
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## GRAPHING POINTS ON AN XY-COORDINATE GRID



Numerical data that you want to put on a two-dimensional graph is entered on the graph as **POINTS**.

The points on the graph are identified by two numbers, which together make an **ORDERED PAIR** written generally as  $(x, y)$ . One example of an ordered pair is  $(3, 2)$ . These two numbers are called **COORDINATES** because together they name the location of the point on the graph.

The first number of the ordered pair is the **X-COORDINATE** because it represents the horizontal distance from  $(0, 0)$ .

The second number of the ordered pair is the **Y-COORDINATE** because it represents the vertical distance from  $(0, 0)$ .

The ordered pair  $(3, 2)$  is located at a point that is right 3 units and up 2 units from the **ORIGIN**,  $(0, 0)$ . The scaled lines are called the **X-AXIS** (horizontal) and **Y-AXIS** (vertical).

Answer parts (a) and (b) in the space below.

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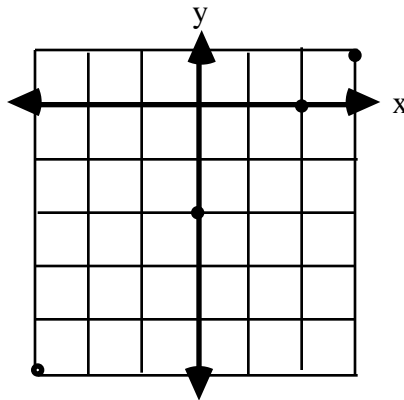
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### GRAPHING ALGEBRAIC RELATIONSHIPS

- Start by making a table to create ordered pairs for the relationship.
- Choose some x-values (inputs) and list them in the table from the smallest to the largest value.
- Substitute each x-value into the rule and calculate the corresponding y-value (output).
- Plot the points on a coordinate grid.



Example:  $y = x + (-2)$

input (x)	-3	-2	-1	0	1	2	3
output (y)	-5			-2		0	1

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### SOLVING EQUATIONS USING INVERSE OPERATIONS

Addition and subtraction are inverse operations.  
 Multiplication and division are inverse operations.

The order of operations in the equation  $5x + 7 = 37$  is to multiply by 5 and then add 7 to get 37. To solve the equation, work backward using the **INVERSE OPERATIONS**. First subtract 7 to get 30, then divide by 5 to get 6.

Check the solution by substituting 6 into the original equation for x:  $5(6) + 7 = 37$

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## RULES FOR MULTIPLYING INTEGERS

1. If you multiply two integers with the **same sign**, the product is **positive**.
2. If you multiply two integers with **different signs**, the product is **negative**.

Examples:  $(6)(-4) = -24$      $(-6)(-4) = 24$

## MP-1

These are the vocabulary words you practiced today:

### Angles

acute angle	An angle with measure greater than $0^\circ$ and less than $90^\circ$ .
right angle	An angle that measures $90^\circ$ .
obtuse angle	An angle with measure greater than $90^\circ$ and less than $180^\circ$ .
vertical angles	The opposite angles where two lines meet.

### Triangles

acute triangle	A triangle in which all three angle measures are less than $90^\circ$ .
right triangle	A triangle in which one angle is a right angle (measures $90^\circ$ ).
obtuse triangle	A triangle with an obtuse angle (greater than $90^\circ$ ).
scalene triangle	A triangle with no equal side lengths.
isosceles triangle	A triangle with two or more sides of equal length.
equilateral triangle	A triangle with all side lengths equal.

### Quadrilaterals

trapezoid	A quadrilateral with one pair of parallel sides.
parallelogram	A quadrilateral with both pairs of opposite sides parallel.
rectangle	A quadrilateral with four right ( $90^\circ$ ) angles.
rhombus	A quadrilateral with all four sides of equal length.
square	A special rectangle with all sides of equal length.

### Polygons

polygon	A two-dimensional closed figure of straight line segments (called edges or sides) connected end to end.
regular polygon	A polygon in which all side lengths are equal and all angle measures are equal.
quadrilateral	A polygon with four sides.
pentagon	A polygon with five sides.
hexagon	A polygon with six sides.
octagon	A polygon with eight sides.
vertex	The point where two consecutive sides of a polygon meet.
diagonal	A line segment that connects two vertices of a polygon and is not a side.

### Other terms

circle	A set of points at a fixed distance (radius) from a point (center).
parallel lines	Two straight lines on a two-dimensional plane that do not intersect no matter how far they are extended.
perpendicular lines	Two lines on a flat surface which make a right angle when they meet.
line segment	The part of a line between two points.
ray	Part of a line that starts at a point on the line and contains all the points on the line that are on one side of the starting point.
base	A special side of a polygon used to compute area. (See examples in problems MP-16, MP-43, and MP-55).
height	In a triangle, parallelogram, or trapezoid, the perpendicular distance from a vertex to the line containing the "opposite" side (possibly extended).

MP-16

<p><b>AREA OF A RECTANGLE</b></p> <p>Area of a Rectangle = (base) · (height)</p> <p><math>A = b \cdot h</math></p> <div style="display: flex; align-items: center; justify-content: center;"><div style="text-align: right; margin-right: 10px;">3 height</div><div style="border: 1px solid black; width: 40px; height: 40px; display: grid; grid-template-columns: repeat(4, 1fr); grid-template-rows: repeat(3, 1fr);"></div><div style="text-align: left; margin-left: 10px;"><p>Area = base · height</p><p>= 4 · 3</p><p>= 12</p></div></div> <div style="text-align: center; margin-top: 10px;"><p>4 base</p></div>	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/>
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MP-20

<p><b>CONGRUENCE</b></p> <p>Figures that have exactly the same size and shape are called <b>CONGRUENT</b>. When one congruent figure is placed on top of the other, all sides and angles match. (One figure may have to be rotated or flipped.) Congruent figures have the same area and perimeter.</p> <div style="display: flex; justify-content: space-around; align-items: center;"><div style="text-align: center;"><p>5</p><div style="border: 1px solid black; width: 60px; height: 40px; margin: 0 auto;"></div><p>3</p><p>Perimeter = 16 units Area = 15 square units</p></div><div style="text-align: center;"><p>3</p><div style="border: 1px solid black; width: 40px; height: 60px; margin: 0 auto;"></div><p>5</p><p>Perimeter = 16 units Area = 15 square units</p></div></div>	
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MP-26

<p><b>BASE AND HEIGHT OF A PARALLELOGRAM</b></p> <p>Any side of a parallelogram can be its <b>BASE</b>. The <b>HEIGHT</b> of the parallelogram is the length of a perpendicular line segment from a vertex (or any other point on the side opposite the base) to the base or an extension of the base.</p>	
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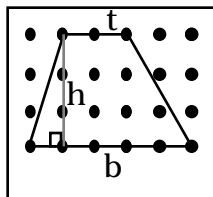


### AREA OF A TRAPEZOID

Since the area of the trapezoid is half the area of a parallelogram made with two congruent copies of the trapezoid, we have:

$$\text{area of a trapezoid} = \frac{1}{2} \cdot (b + t) \cdot h$$

or  $\frac{b+t}{2} \cdot h$




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### RATIOS

A **RATIO** is a comparison of two quantities by division. It can be written in several ways,

such as:  $\frac{2}{5}$      $2 : 5$      $2$  to  $5$

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### MIXED NUMBERS AND FRACTIONS GREATER THAN ONE

The number  $4\frac{1}{4}$  is called a **MIXED NUMBER**, because it is made from a whole number, 4, and a fraction,  $\frac{1}{4}$ . It is a mix of a whole number and a fraction.

The number  $\frac{17}{4}$  is a fraction that is greater than one, because the numerator is greater than the denominator. Sometimes fractions that are greater than one are called "improper fractions." "Improper" does not mean "wrong." In mathematics, keeping a fraction in its "improper form" is often more useful than changing it to a mixed number.

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GO-14

### EQUIVALENT FRACTIONS

Two fractions that name the same number are called **EQUIVALENT FRACTIONS**.

For example, because  $\frac{1}{2} = \frac{3}{6}$ , the fractions  $\frac{1}{2}$  and  $\frac{3}{6}$  are equivalent.

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GO-41

### INEQUALITY SYMBOLS

The symbol  $>$  means “**is greater than**” and the symbol  $<$  means “**is less than.**” These symbols are called **INEQUALITY SYMBOLS** and are used to compare and order numbers. For example,  $5 > 3$  and  $1 < 7$ . The symbol  $\geq$  means “is greater than or equal to” and  $\leq$  means “is less than or equal to.”

Here is an easy way to remember what each inequality symbol means: the pointed part of the symbol always points to the smaller number, and the open part of the symbol always points to the larger number.

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GO-52

### IDENTITY PROPERTY OF MULTIPLICATION

Any number multiplied by 1 equals itself; that is, it remains unchanged. This is called the **IDENTITY PROPERTY OF MULTIPLICATION**.

$$7 \cdot 1 = 7 \quad -3 \cdot 1 = -3 \quad x \cdot 1 = x$$

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<p style="text-align: center;"><b>PROPORTION</b></p> <p>A <b>PROPORTION</b> is an equation that states that two fractions are equal.</p> $\frac{3}{8} = \frac{6}{16} \quad \frac{4}{12} = \frac{6}{18} \quad \frac{2}{3} = \frac{x}{30}$	<hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/>
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<p style="text-align: center;"><b>SOLVING PROPORTIONS USING CROSS MULTIPLICATION</b></p> <p>When a proportion has one variable, multiplying the diagonals, known as <b>CROSS MULTIPLICATION</b>, results in an equation without fractions.</p> <p>Example: If Cindy can read 3 books in 2 weeks, how many books can she read in 18 weeks?</p>	<div style="text-align: center; margin-bottom: 20px;"> <math>\frac{3 \text{ books}}{2 \text{ weeks}} = \frac{x \text{ books}}{18 \text{ weeks}}</math>  <math>\frac{3}{2} = \frac{x}{18}</math>  <math>3(18) = 2x</math>  <math>54 = 2x</math>  <math>27 = x</math> </div> <div style="text-align: center; border-top: 1px solid black; border-bottom: 1px solid black; height: 40px;"> <math>\frac{\quad}{\quad} = \frac{\quad}{\quad}</math> </div>
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<p style="text-align: center;"><b>LEAST COMMON MULTIPLES</b></p> <p>The <b>LEAST COMMON MULTIPLE</b> of two or more integers is the smallest positive integer that is divisible by <u>both</u> (or all) of the integers.</p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">3</td> <td style="border-right: 1px solid black; padding: 0 5px;">6</td> <td style="border-right: 1px solid black; padding: 0 5px;">9</td> <td style="border-right: 1px solid black; padding: 0 5px;">12</td> <td style="padding: 0 5px;">(15)</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">5</td> <td style="border-right: 1px solid black; padding: 0 5px;">10</td> <td style="border-right: 1px solid black; padding: 0 5px;">(15)</td> <td style="border-right: 1px solid black; padding: 0 5px;">20</td> <td style="padding: 0 5px;">25</td> </tr> </table> <p>Example: Use a ratio table to find the least common multiple of 3 and 5.</p> <p>15 is the least common multiple because it is the smallest positive integer divisible by both 3 and 5.</p>	3	6	9	12	(15)	5	10	(15)	20	25	<hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/> <hr style="border: 0; border-top: 1px solid black; margin-bottom: 5px;"/>
3	6	9	12	(15)							
5	10	(15)	20	25							

**CONVERTING FRACTIONS TO DECIMALS**

Think of a fraction as a part of something.  $\frac{3}{4}$  would be three parts of 4. To write  $\frac{3}{4}$  as a decimal, consider the fraction as a division problem:  $3 \div 4$  and perform long division.

$$\begin{array}{r}
 .75 \\
 4 \overline{)3.00} \\
 \underline{-28} \phantom{0} \\
 20 \\
 \underline{-20} \\
 0
 \end{array}
 \qquad
 \frac{3}{4} = 0.75$$

Sometimes the decimal number ends, and sometimes it repeats. If it repeats, you will need to use bar notation.

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**PERCENTS USING PROPORTIONS**

- Proportions can be used to find percents.
- Set up your proportion with the percent numbers on one side and the quantities you wish to compare on the other side.

Example: What number is 15% of 140?

$$\begin{array}{l}
 \text{part} \rightarrow \frac{15}{100} = \frac{x}{140} \\
 \text{whole} \rightarrow
 \end{array}$$

$$15 \cdot 140 = 100x$$

- Cross multiply to write an equation without fractions.
- Solve for the unknown.

$$x = 21$$

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## ADDING AND SUBTRACTING FRACTIONS

To add or subtract two fractions, the fractions must have the same denominator. One way to convert two fractions with different denominators into fractions with the same denominator is to use the Giant **1**. Below are examples of adding and subtracting two fractions with different denominators. In general,

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \boxed{\frac{d}{d}} + \frac{c}{d} \cdot \boxed{\frac{b}{b}} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$$

Addition Example

$$\frac{1}{5} + \frac{1}{3} = \frac{1}{5} \cdot \boxed{\frac{3}{3}} + \frac{1}{3} \cdot \boxed{\frac{5}{5}} = \frac{3}{15} + \frac{5}{15} = \frac{8}{15}$$

Subtraction Example

$$\frac{5}{6} - \frac{1}{4} = \frac{5}{6} \cdot \boxed{\frac{2}{2}} - \frac{1}{4} \cdot \boxed{\frac{3}{3}} = \frac{10}{12} - \frac{3}{12} = \frac{7}{12}$$

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### ADDING AND SUBTRACTING MIXED NUMBERS

You can add or subtract mixed numbers by adding or subtracting their parts. Add the whole-number parts and the fraction parts separately. Adjust if the fraction is greater than one or less than zero.

OR

You can add or subtract mixed numbers by changing the mixed numbers into fractions greater than one. Add or subtract as you would with fractions between zero and one.

Example:  $3\frac{1}{5} + 1\frac{2}{3}$

$$\begin{array}{r} 3\frac{1}{5} = 3 + \frac{1}{5} \quad \boxed{\frac{3}{3}} \\ 1\frac{2}{3} = 1 + \frac{2}{3} \quad \boxed{\frac{5}{5}} \\ \hline 4\frac{13}{15} \end{array}$$

OR

$$\begin{array}{r} 3\frac{1}{5} + 1\frac{2}{3} \\ \frac{16}{5} + \frac{5}{3} \\ \frac{16}{5} \boxed{\frac{3}{3}} + \frac{5}{3} \boxed{\frac{5}{5}} \\ \frac{48}{15} + \frac{25}{15} = \frac{73}{15} = 4\frac{13}{15} \end{array}$$

### MULTIPLYING TWO FRACTIONS

Multiplying two fractions gives you a new fraction (which you might be able to simplify to a whole number). The numerator of the new fraction is the product of the two original numerators. The denominator is the product of the two original denominators.

For example:

$$\frac{2}{3} \cdot \frac{2}{7} = \frac{2 \cdot 2}{3 \cdot 7} = \frac{4}{21}$$

In general:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Sometimes you will need to simplify the product.

$$\frac{2}{3} \cdot \frac{3}{5} = \frac{2 \cdot 3}{3 \cdot 5} = \frac{6}{15} = \left(\frac{2}{5} \cdot \frac{3}{3}\right) = \frac{2}{5}$$

Sometimes it is easier to simplify the fractions before multiplying.

$$\left(\frac{8}{12}\right)\left(\frac{6}{15}\right) = \left(\frac{2}{3} \cdot \frac{4}{4}\right)\left(\frac{2}{5} \cdot \frac{3}{3}\right) = \left(\frac{2}{3}\right)\left(\frac{2}{5}\right) = \frac{4}{15}$$

### MULTIPLYING MIXED NUMBERS (based on the Distributive Property)

To multiply mixed numbers, you may use a generic rectangle (based on the Distributive Property).

Example:  $2\frac{1}{2} \cdot 1\frac{1}{4} = (2 + \frac{1}{2}) \cdot (1 + \frac{1}{4})$

	$1 + \frac{1}{4}$	
$2$	$2$	$\frac{2}{4}$
$+$		
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{8}$

$$\begin{aligned}
 &= 2(1) + 2(\frac{1}{4}) + \frac{1}{2}(1) + \frac{1}{2}(\frac{1}{4}) \\
 &= 2 + \frac{2}{4} + \frac{1}{2} + \frac{1}{8} \\
 &= 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8} \\
 &= 2 + 1 + \frac{1}{8} \\
 &= 3\frac{1}{8}
 \end{aligned}$$

### RECIPROCAL

Two numbers that have a product of 1 are called **reciprocals**.

Examples:

$$\frac{5}{8} \cdot \frac{8}{5} = \frac{40}{40} = 1 \qquad 3\frac{1}{4} \cdot \frac{4}{13} = \frac{13}{4} \cdot \frac{4}{13} = \frac{52}{52} = 1$$

$$0.25 \cdot 4 = 1.00$$

In general:

$$a \cdot \frac{1}{a} = 1 \text{ and } \frac{a}{b} \cdot \frac{b}{a} = 1 \text{ where } a \text{ and } b \text{ are not } 0.$$

## DIVISION OF FRACTIONS

### Division of Fractions with a Super Giant 1 Method:

$$F \div \frac{N}{D} = \frac{F}{\frac{N}{D}} = \frac{F \cdot \frac{D}{N}}{1} = F \cdot \frac{D}{N}$$

$$\frac{3}{4} \div \frac{2}{5} = \frac{\frac{3}{4}}{\frac{2}{5}} = \frac{\frac{3}{4} \cdot \frac{5}{2}}{1} = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$$

OR

### Division of Fractions with the Invert and Multiply Method:

$$F \div \frac{N}{D} = F \cdot \frac{D}{N}$$

Invert the divisor (switch the numerator and the denominator) and then multiply the fractions as usual.

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$$

Answer parts (a) through (c) in the space below.

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### MULTIPLYING DECIMAL NUMBERS

The answer to a multiplication problem is called the product of the factors.

To place the decimal point correctly in the product, count the decimal places in each of the factors and count that many places from the right in the product.

Examples:

one place   two places   three places $2.\underline{3} \cdot 5.\underline{06} = 11.\underline{638}$	$1 + 2 \text{ places} = 3 \text{ places}$ from the right
four places   two places   six places $0.\underline{0004} \cdot 3.\underline{42} = 0.\underline{001368}$	$4 + 2 \text{ places} = 6 \text{ places}$ from the right
one place   one place   two places $0.\underline{5} \cdot 0.\underline{2} = 0.\underline{10}$	$1 + 1 \text{ places} = 2 \text{ places}$ from the right

### DIVIDING BY A DECIMAL NUMBER

When you are dividing by a decimal number, count how many digits the decimal point must move to the right to become an integer (whole number).

Then move the decimal point in the dividend the same direction and the same number of digits.

Example:

$$\text{divisor} \longrightarrow 4.\underline{07} \overline{)8.\underline{30}\overset{\cdot}{\uparrow}} \longleftarrow \text{dividend}$$

Moving the decimal point two places to the right is multiplying by both numbers by 100.

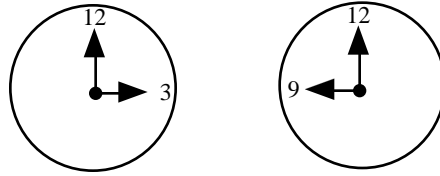
The Giant **1** (Identity Property) proves this.

$$8.3 \div 4.07 = \frac{8.3}{4.07} \cdot \frac{\boxed{100}}{\boxed{100}} = \frac{830}{407}$$

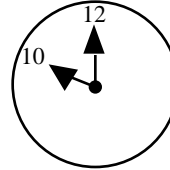
Choose either way to divide by a decimal number.

### TYPES OF ANGLES

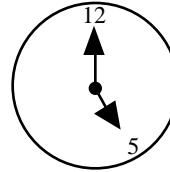
Clocks can help you understand angle classification. When a clock shows 3 o'clock or 9 o'clock, the hands make an angle of  $90^\circ$  or a **RIGHT ANGLE**.



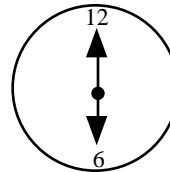
Angles that measure more than  $0^\circ$  but less than  $90^\circ$  are called **ACUTE ANGLES**.



Angles that measure more than  $90^\circ$  but less than  $180^\circ$  are called **OBTUSE ANGLES**.



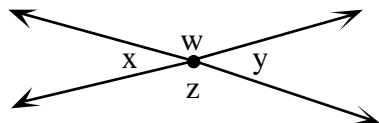
Angles that measure  $180^\circ$  are called **STRAIGHT ANGLES**.



Draw your examples in the space below.

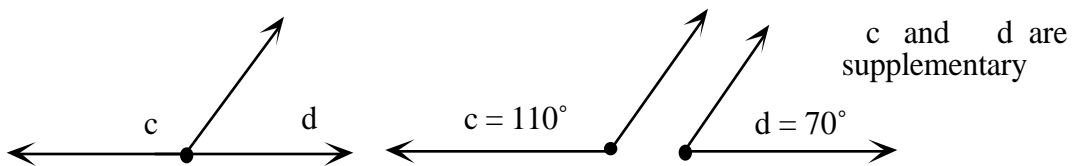
### PROPERTIES OF ANGLE PAIRS

Intersecting lines form four angles. The pairs of angles across from each other are called **VERTICAL ANGLES**. The measures of vertical angles are equal.

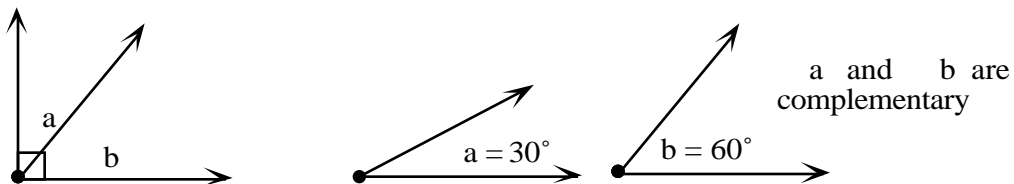


$x$  and  $y$  are vertical angles  
 $w$  and  $z$  are vertical angles

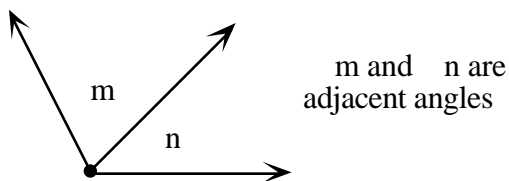
If the sum of the measures of two angles is exactly  $180^\circ$ , then they are called **SUPPLEMENTARY ANGLES**.



If the sum of the measures of two angles is exactly  $90^\circ$ , then they are called **COMPLEMENTARY ANGLES**.



Angles that share a vertex and one side but have no common interior points (that is, do not overlap each other) are called **ADJACENT ANGLES**.



$m$  and  $n$  are adjacent angles

Write your notes in the space below.

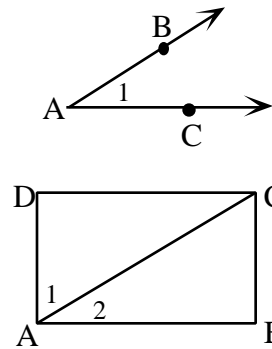
MC-25.

### NAMING ANGLES

There are several ways to label and refer to the angles in a geometric figure. When an angle is alone we may name it  $\angle 1$ ,  $\angle A$ ,  $\angle CAB$ , or  $\angle BAC$ .

When an angle is part of a diagram, using just one letter may be confusing. There are three angles with vertex A. You can always use three points to name each angle:  $\angle DAB$ ,  $\angle CAB$ , and  $\angle DAC$ . Notice the vertex is always the middle letter. Two of the angles can also be named by number:

$$\angle DAC = \angle 1; \quad \angle CAB = \angle 2.$$

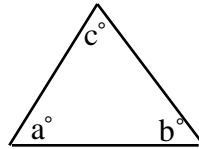


MC-53

### ANGLES IN A TRIANGLE

The sum of the measures of the **angles in any triangle** is exactly  $180^\circ$ .

For any triangle,  $a^\circ + b^\circ + c^\circ = 180^\circ$ .

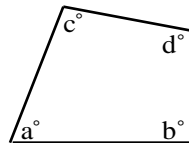


MC-57

### ANGLES IN A QUADRILATERAL

The sum of the measures of the angles in a quadrilateral is exactly  $360^\circ$ .

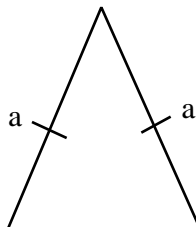
In general, for any quadrilateral,  $a^\circ + b^\circ + c^\circ + d^\circ = 360^\circ$ .



### CLASSIFYING TRIANGLES BY SIDE LENGTH

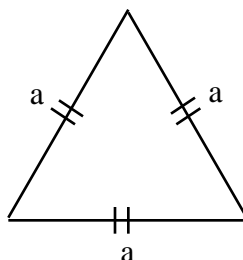
#### Isosceles Triangle

A triangle with at least two congruent sides is called an **ISOSCELES TRIANGLE**. We know that the triangle at right is isosceles because the two sides have the same label, a. The single slash on each side is another way to indicate that the two sides are congruent.



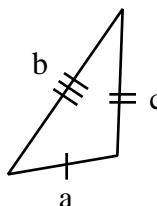
#### Equilateral Triangle

A triangle with all three of its sides congruent is called an **EQUILATERAL TRIANGLE**. We know that the triangle at right is equilateral because the three sides have the same label, a, and the same number of slash marks.



#### Scalene Triangle

A triangle with no congruent sides is called a **SCALED TRIANGLE**. The different number of marks on each side indicates that no pair of sides has the same length. Usually, a scalene triangle will have no marks at all.




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### GUIDELINES FOR COMBINING LIKE TERMS

**TERMS** are separated by addition or subtraction symbols.

**LIKE TERMS** are terms that have the same variable or variables, such as  $2G$  and  $3G$ ,  $4x$  and  $5x$ ,  $6y^2$  and  $2y^2$ , and  $3xy$  and  $-2xy$ .

Like terms such as  $2G$  and  $3G$  can be combined into one term by addition or subtraction so  $2G + 3G = 5G$ . Similarly,  $4x + 5x = 9x$ ,  $6y^2 - 2y^2 = 4y^2$ , and  $3xy - 2xy = xy$ .

The terms  $2G$  and  $3N$  must remain separate terms as  $2G + 3N$  because  $G$  and  $N$  are different variables and can represent different numbers. Similarly,  $5x$  and  $6y^2$  cannot be combined, nor can  $5x$  and  $3xy$ .

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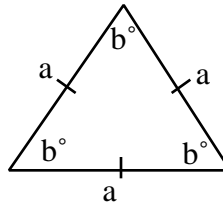
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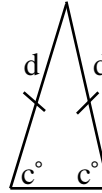
**ANGLES IN EQUILATERAL TRIANGLES**

In an equilateral triangle, all three angles are congruent and each measures  $60^\circ$ . Thus,  $m \angle b = 60^\circ$ .



**ANGLES IN ISOSCELES TRIANGLES**

In an isosceles triangle, the angles opposite the congruent sides are congruent. Thus,  $m \angle c = m \angle c$ .



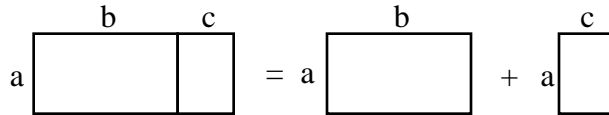
**DISTRIBUTIVE PROPERTY**

The **DISTRIBUTIVE PROPERTY** allows us to separate or group quantities in multiplication problems. We can represent the Distributive Property symbolically, numerically, and pictorially.

Symbolically:

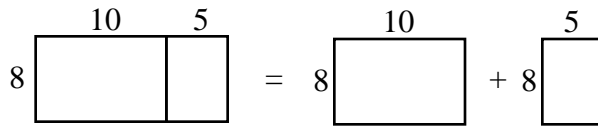
$$a(b + c) = ab + ac$$

Pictorially



Numerically:

$$\begin{aligned} 8(10 + 5) &= 8(10) + 8(5) \\ &= 80 + 40 \\ &= 120 \end{aligned}$$



Show the example of the Distributive Property in the space below.

### COMMUTATIVE PROPERTY

The **COMMUTATIVE PROPERTY** allows us to change the order in which we add or multiply two numbers. We represent the Commutative Property symbolically and numerically as:

#### Symbolically

$$a + b = b + a \quad \text{or} \quad a \cdot b = b \cdot a$$

(addition)  (multiplication)

#### Numerically

$$-7 + 13 = 13 + (-7) \quad \text{or} \quad (-4) \cdot 17 = 17 \cdot (-4)$$

$6 = 6$    $-68 = -68$

### ASSOCIATIVE PROPERTY

The **ASSOCIATIVE PROPERTY** allows us to regroup in an addition problem with three terms or a multiplication problem with three factors. We represent the Associative Property symbolically and numerically as:

#### Symbolically

$$a + (b + c) = (a + b) + c \quad \text{or} \quad a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

(addition)  (multiplication)

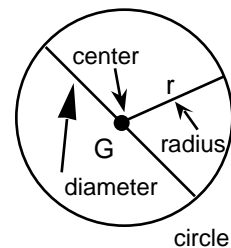
#### Numerically

$$7 + (-7 + 3) = (7 + (-7)) + 3 \quad \text{or} \quad 4(-25 \cdot 9) = (4 \cdot (-25)) \cdot 9$$

$7 + -4 = 0 + 3$    $4(-225) = (-100)(9)$   
 $3 = 3$    $-900 = -900$

### CIRCLE

A **CIRCLE** is the set of all points that are the same distance from a fixed point,  $G$ . We use  $\odot G$  as the **symbol to represent a circle**, in this case circle  $G$ , where  $G$  is the center. The fixed point is called the **CENTER** of the circle and the distance from the center to the points on the circle is called the **RADIUS** (usually denoted  $r$ ). A line segment drawn through the center of the circle with both endpoints on the circle is called a **DIAMETER** (denoted  $d$ ). Note:  $d = 2r$ .

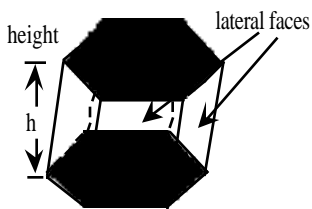


You can think of a circle as the rim of a bicycle wheel. The center of the circle is the hub where the wheel is bolted to the bicycle's frame. The radius is a spoke of the wheel.



### PRISMS

A **PRISM** is a three-dimensional figure composed of polygonal faces (called sides or lateral sides) and two congruent, parallel faces called bases. No holes are permitted in the solid. The remaining faces are parallelograms (or rectangles). A prism is named for the shape of its base.



### VOLUME OF A PRISM

The **VOLUME** of a prism is the area of either base ( $A$ ) times the height ( $h$ ) of the prism.

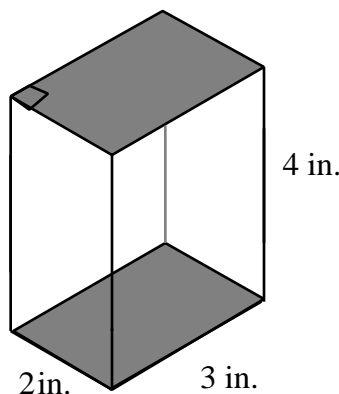
$$V = (\text{Area of base}) \cdot (\text{height}) \text{ or } V = Ah$$

Example:

$$\begin{aligned} \text{Area of base} &= (2 \text{ in.})(3 \text{ in.}) \\ &= 6 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} (\text{Area of base})(\text{height}) &= (6 \text{ in.}^2)(4 \text{ in.}) \\ &= 24 \text{ in.}^3 \end{aligned}$$

$$\text{Volume} = 24 \text{ in.}^3$$




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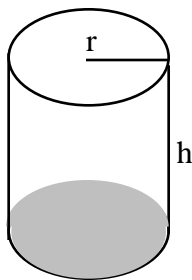
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### VOLUME OF A CYLINDER

The volume of a cylinder is the area of its base multiplied by its height:

$$V = A \cdot h.$$

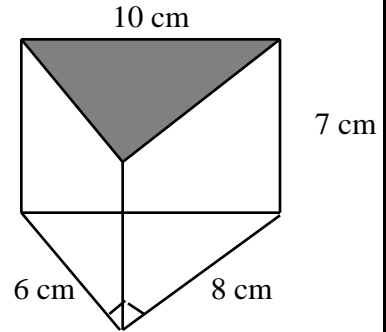
Since the base of a cylinder is a circle of area  $A = r^2$ , we can write:  $V = r^2h$ .



### SURFACE AREA OF PRISMS AND CYLINDERS

The **SURFACE AREA OF A PRISM** is the sum of the areas of all of its faces, including the bases. Surface area is expressed in **square units**.

Example: Find the surface area of the triangular prism at right.



Subproblem 1: Area of the two bases

$$2\left[\frac{1}{2}(6\text{cm})(8\text{ cm})\right] = 48\text{ cm}^2$$

Subproblem 2: Area of the 3 sides (lateral faces)

Area of side 1:  $(6\text{ cm})(7\text{ cm}) = 42\text{ cm}^2$

Area of side 2:  $(8\text{ cm})(7\text{ cm}) = 56\text{ cm}^2$

Area of side 3:  $(10\text{ cm})(7\text{ cm}) = 70\text{ cm}^2$

Subproblem 3: Surface Area of Prism = sum of bases and lateral faces

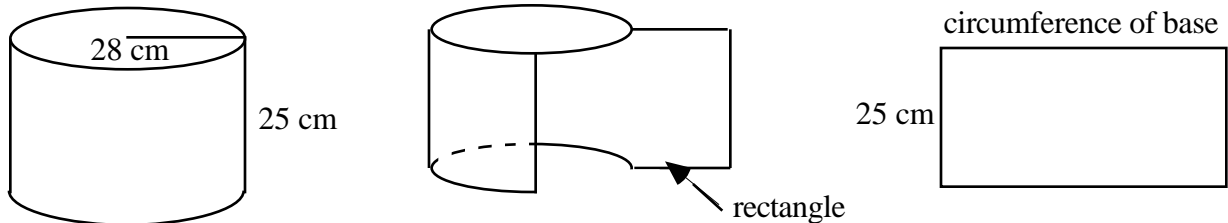
$$48\text{ cm}^2 + 42\text{ cm}^2 + 56\text{ cm}^2 + 70\text{ cm}^2 = 216\text{ cm}^2$$

The **SURFACE AREA OF A CYLINDER** is the sum of the two base areas and the lateral surface area. The formula for the surface area is:

$$\text{S.A.} = 2\text{ r}^2 + \text{dh} = 2\text{ r}^2 + 2\text{ rh}$$

where  $r$  = radius,  $d$  = diameter, and  $h$  = height of the cylinder.

Example: Find the surface area of the cylinder below.



Subproblem 1: Area of the two circular bases

$$2[(28\text{ cm})^2] = 1568\text{ cm}^2$$

Subproblem 2: Area of the lateral face

$$(56)25 = 1400\text{ cm}^2$$

Subproblem 3: Surface area of the cylinder

$$1568\text{ cm}^2 + 1400\text{ cm}^2 = 2968\text{ cm}^2$$

$$9324.25\text{ cm}^2$$

**VOCABULARY TERMS FOR PROBABILITY**

**SAMPLE SPACE:** All possible outcomes of a situation. For example, there are six possible outcomes when a six-sided die is rolled and two possible outcomes when flipping a coin.

**OUTCOME:** Any possible or actual result or consequence of the action(s) considered, such as rolling a five on a die or getting tails when flipping a coin.

**EVENT:** An outcome or group of outcomes from an experiment, such as rolling an even number on a die.

**PROBABILITY:** A number between zero and one that states the likelihood of an event. It is the ratio of desired outcomes to all possible outcomes (the sample space).

**IMPOSSIBILITY:** When an event has a probability of zero; that is, an event that cannot occur, such as rolling a seven on a six-sided die.

**CERTAINTY:** When an event has a probability of one; such as rolling a number between one and six on a standard die.

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### POPULATION AND SAMPLES

A **POPULATION** is a collection of objects or a group of people about whom information is gathered.

A **SAMPLE** is a subgroup of the population. For example, if you want to conduct a survey at your school about what foods to serve in the cafeteria, the population would be the entire student body.

If you choose to sample 10% of the students, you need to include the correct fraction of students from each grade and an equal number of male and female students. A **REPRESENTATIVE SAMPLE** is a subgroup of the population that matches the general characteristics of the entire population.