

College Prep Math Assessment in Algebra I and Geometry 1994 Results

Executive Summary

This report is a careful look at the results of a study of 9294 algebra students, 6568 geometry students and 1402 algebra 2 students who participated in an assessment during May of 1994. Each student worked on three of the exam questions shown in Attachment A and was required to show his or her work. A typical algebra 2 question was:

Susie started collecting CD's after she received several as gifts on her tenth birthday. Each month after that she bought the same number of CD's, but not as many as she had received as presents. After twelve months she had a total of 45, and after five years she had 189 CD's. Find the number of CD's she received on her tenth birthday.

Each student was given 30 minutes to work on three problems. Eighteen different versions of the examination were prepared with different mixes of questions, so that inter-question comparisons could be obtained. Algebra response papers were sent to three different sites: Davis, Riverside and San Diego. Geometry papers were graded at Davis and Riverside while algebra 2 papers were only graded at Davis.

Papers from CPM and non-CPM students were coded so that the students could not be identified and mixed together to guarantee uniform grading. Each paper was graded holistically on a scale of 0-4 with 3 representing satisfactory work and 4 representing excellent work.

For the third year in a row, the results favored CPM students in all three courses, most of the results significant at the .01 level or better. See the 1992 and 1993 reports for details about those results. A separate focus of the analysis this year was comparisons within schools at each grade level with the large results between schools demonstrating the importance of the quality of the schools and the teachers.

Methodology

In May, 1994, students and teachers from 32 schools participated in the 1994 CPM Spring Assessment. All of the schools participated in the algebra assessment and ten of the schools participated in the geometry assessment. A total of 7722 students returned examination forms for the algebra examination (5358 or 69% of them being CPM students) and 2951 students returned examination forms for the geometry examination (1786 or 60% of these being CPM students). The rest were from traditional classrooms, where we use the term “traditional” or “non-CPM” to denote a class using any one of the various standard textbook series--e.g. Houghton-Mifflin, Prentice-Hall, etc.

We asked the participating CPM teachers to find non-CPM teachers at their school or a similar one so that we could be assured of roughly comparable sets of students. All of the teachers involved were volunteers.

Each of the students was asked to fill out a brief questionnaire detailing their age, grade level, gender, ethnicity and their original language. Then students were given 30 minutes to work on the three questions from one of eighteen forms of the assessment. Two of the questions were developed by CPM teachers and the third question was either from IMP or from a national test. (A sample of the questions are shown in Attachment A.) Care was taken to ensure that each question was attempted by approximately equal numbers of students. Each teacher also filled out a questionnaire giving their gender, number of years of teaching experience, the text used, and the number of years they had taught from this particular text.

All of the geometry and about 70% of the algebra examinations were sent to the CRESS Center at UC Davis while the remaining algebra examinations were sent to a group at the county office in Riverside. At each site, every student's responses were coded by the same number on their information sheet and each of their three response sheets. Then the papers were separated into piles by question number and the papers within each pile were shuffled so that they would be graded in random order. Two questions were also common to the algebra and geometry examinations and the responses from these questions were shuffled.

During June of 1994, a group of 30 teachers in Davis and 20 teachers in Riverside participated in the grading which was done holistically on a 5-point scale. On this scale a 0 represented no useful work (either nothing done or nonsense); 1 represented an understanding of the problem, but little progress toward a solution; 2 a reasonable start toward a solution, but an approach with significant gaps or misconceptions; 3 an almost complete solution which might contain minor errors; and 4 a full and complete solution. For each problem, the group agreed on a rubric for the scoring by looking at several sample papers. After the group had come to an agreement the shuffled papers were each

graded by at least two people and, in case of disagreement, by a third person. The consensus score was recorded. The northern and southern California groups made their own separate decisions about how to assign points so the scores between the two groups are not comparable.

Analysis of the data

All of the data was analyzed by the SPSS statistical package with the assistance of Neil Willett of the UC Davis Statistical Consulting Laboratory. In the first pass of the data it was determined that there was no significant difference in the performance of males and females on the same problem, but there were significant in performance among different ethnic groups, grade levels, teachers, schools and programs. Because of differences between individual teachers and schools, the data was analyzed by a *random effects* analysis, where we assume that differences between schools and teachers exist as well as by a *fixed effects* analysis, where we assume that all teachers and schools are substantially identical. We are, of course, most interested in comparing the CPM program with non-CPM program students.

For each of the three studies--the two different groups who graded algebra examinations and the group who graded geometry examinations--CPM students outscored their non-CPM counterparts by a substantial and statistically significant margin. The levels of significance ranged from .0001 to .0304.

A different, although indirect, measure of the effectiveness of the CPM program is the fact that the proportion of ESL (English as a Second Language) students entering algebra was the essentially the same (12% for CPM and 13% for traditional classes), there was a marked difference in geometry. The data shows that ESL students still comprised 13% of the CPM geometry students, but only 6% of the students in traditional classes. Thus, the concerns that the intensive reading required for CPM discriminates against students who may have difficulty with English does not seem supported by the data. In fact, CPM appears to be considerably more beneficial to them.

In addition to comparing overall means of the two groups, we compared the proportion of those who scored a 3 or a 4 on their responses. These were the responses that were considered essentially correct. Again, the proportion of CPM students who scored a 3 or a 4 was significantly higher than the proportion for students in traditional classes.

Appendix A

Questions from the Algebra 1 exam.

BUYING A CAR OR NOT

Kelly would like to buy a Chevy advertised for \$3,500. She currently has \$1,400 in her savings account. The value of the car decreases \$50 a month, and each month, Kelly puts \$150 into her savings account. In how many months will Kelly be able to afford the car (provided someone else hasn't already bought it)?

A complete response includes:

- a) your work CLEARLY shown so that other students could understand it;
- b) an appropriate equation (or equations);
- c) some OTHER method of solving the problem besides an equation, such as graphing, guess and check, arithmetic, or ...;
- d) your answer to the question.

WATER WORLD

A water slide is 90 feet long. The distance from the foot of the ladder to the bottom of the slide is three times the height of the ladder. Determine the height of the ladder.

A complete response includes:

- a) a clearly labeled diagram;
- b) your work CLEARLY shown so that other students could understand it;
- c) an appropriate equation (or equations);
- d) your answer to the question.

TWO GRAPHS

Find as accurately as you can where the graphs of $y = x^2 - 2$ and $y = 2x + 1$ intersect. Make sure you state both the x - and y -coordinates of the point(s) of intersection.

FILL IN THE BLANK

You are given the polynomial $x^2 + 8x + \underline{\hspace{2cm}}$

- Fill in the blank with some number so that one of the factors of this polynomial is $(x + 5)$ and show the polynomial in factored form.
- Is it possible to fill in the blank with a different number than you used in part (a) and still have $(x + 5)$ as a factor?
- If your answer is yes, give some examples. If it is no, explain clearly why not.

Questions from the Geometry exam.

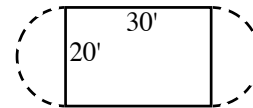
CARDBOARD TRIANGLE

A triangle is cut out of a piece of cardboard so that its sides measure 6 cm, 12 cm, and 15 cm. Its area is 34 square cm. Another triangle is cut out of the same piece of cardboard, but its sides measure only 2 cm, 4 cm and 5 cm.

Find the area of the smaller triangle.
Explain completely how you solved the problem.

THE GARDEN

A rectangular garden has two semicircular flower beds on opposite ends. Find the area of the entire figure and EXPLAIN CLEARLY how you did so.



CUTTING THE HEXAGON

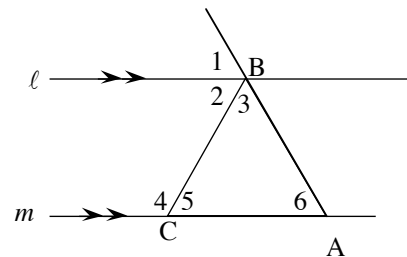
Suppose you have a regular hexagon ABCDEF.

- Find each angle of the hexagon.
- Divide the hexagon into a triangle and a pentagon by a line from A to C. Find each angle of the triangle.

ISOSCELES TRIANGLE

In the diagram at right, line l is parallel to line m , $\angle 1 \cong \angle 3$, and $m\angle 4 = 130^\circ$.

Prove that $\triangle ABC$ is isosceles.



Questions from the Algebra 2 exam.

THE CD COLLECTION

Susie started collecting CD's after she received several as gifts on her tenth birthday. Each month after that she bought the same number of CD's, but not as many as she had received as presents. After twelve months she had a total of 45, and after five years she had 189 CD's. Find the number of CD's she received on her tenth birthday.

A complete response includes

- your work CLEARLY shown so that other students could understand it,
- an appropriate equation or equations,
- your answer to the question.

TEMPERATURE OF MILK

If a carton of milk is left out on the counter for more than 20 minutes, its temperature can be approximated using the formula $T = 70 - \frac{500}{t}$, where t = time in minutes. Your little brother leaves the milk out quite often, and you would like a quick way to figure out how long it has been there. Rewrite the equation for t in terms of T .

SOLVE IT

Find x : $2^{(x^2 + 6x - 7)} = 8^3$

DEPRECIATION

Johanna bought a used Jeep Wrangler for \$9,500 and was told that it was a car that "keeps its value." If the depreciation is only 5% per year, how much will it be worth in 5 years? in 12 years? in x years?

A complete response includes

- an equation AND
- a description of what each part of the equation represents.

Attachment B

Algebra results.

The next three tables, give the least square mean (lsm) scores for CPM and non-CPM students for the three sites where the examinations were graded. The least-square means are determined from the raw scores by computing a number which reflects the makeup of the class. For example, a class with many eighth graders generally scores higher than a class consisting of ninth and tenth graders. So the raw score of a class of eighth graders is decreased by a certain fraction while the raw score of a class consisting primarily of tenth graders would be raised. The program does the computation automatically to correlate with a fictitious class whose makeup consists a proportional fraction of the total population of students taking the examination.

Notice the wide differences in grading standards, as reflected in different mean scores, which were adopted by the different groups of graders. In general, the graders at Riverside tended to be more generous than those at Davis or San Diego, and a statistical analysis of the data confirms this observation. So we can only compare scores within each of the grading groups. The measures of the effectiveness of CPM also vary widely among the three schools with a p-values at Davis of .0002, which is highly significant, to a p-value at San Diego of .73 which is indistinguishable from random chance. In further analyzing the data (see pages B7-B9) the major reason for the difference seems to be two extremely good non-CPM classes whose exams were graded in San Diego.

Algebra 1 - San Diego

School Number	p(fixed)	p(random)	lsm (CPM)	lsm (nCPM)
601	.6524	.5730	1.281	1.177
602	.0001	.0214	1.266	1.907
603	.0077	.2985	2.117	1.715
604	.	.	1.955	.
605	.1712	.2092	1.322	2.335
606	.0006	.0460	2.007	1.512
607	.	.	1.988	.
608	.0001	.2501	2.599	1.826
609	.0001	.	2.947	1.633
610	.	.	.	2.306
611	.	.	1.641	.

Algebra 1 - Davis

School Number	p(fixed)	p(random)	lsm (CPM)	lsm (nCPM)
101	.	.	2.061	.
103	.	.	1.696	.
104	.0013	.0095	1.994	1.457
106	.0001	.0233	3.376	2.661
107	.0249	.2221	1.807	1.484
109	.	.	1.336	.
110	.	.	.	1.785
111	.	.	2.209	.
112	.	.	2.963	.
113	.	.	2.212	.
114	.	.	1.496	.
115	.3677	.	2.483	2.274
116	.	.	2.403	.
117	.2273	.3909	1.336	1.176
118	.	.	.	1.442
119	.	.	3.264	.
123	.	.	1.994	.
125	.0001	.0483	2.737	1.807

Algebra 1 - Riverside

School Number	p(fixed)	p(random)	lsm (CPM)	lsm (nCPM)
401	.0168	.2056	2.113	1.771
404	.0212	.3284	1.721	2.081
407	.0052	.1745	1.980	2.396
410	.0001	.0201	2.873	2.258
411	.0537	.1350	2.382	2.109
413	.9378	.8653	1.776	1.790
414	.0001	.0025	3.498	2.405

Geometry.

The geometry results are much the same as last year and still favor CPM, but to a lesser degree. Because the differences between males and females were virtually non-existent, separate means were not computed for the two genders. Separate means were computed for the major ethnic groups and the tables showing the comparative mean scores are shown below. The least-square mean data for Davis and Riverside are shown on the next two tables.

Geometry 1 - Davis

School Number	p(fixed)	p(random)	lsm (CPM)	lsm (nCPM)
201	.0039	.2802	2.525	2.214
203	.	.	2.002	.
205	.0139	.0000	2.623	2.954
208	.0648	.1403	2.369	2.100
209	.0842	.0114	3.095	2.801
213	.	.	2.357	.
214	.	.	2.125	.
218	.	.	.	2.398
223	.	.	2.857	.
225	.0079	.4360	2.275	1.819

Geometry - Riverside

School Number	p(fixed)	p(random)	lsm (CPM)	lsm (nCPM)
501	.0001	.0795	1.956	2.610
502	.6574	.6799	1.675	2.096
503	.1361	.2560	2.562	2.353
505	.0600	.2321	2.862	3.223
508	.0043	.1834	3.166	2.671
509	.3636	.5807	1.763	1.915
510	.0138	.1589	2.519	2.279
512	.0875	.0999	2.329	2.070
514	.0003	.0843	2.850	2.320
515	.1152	.5098	2.619	2.453
516	.2410	.5132	2.476	2.239
517	.1918	.2509	1.718	1.978
518	.3686	.6070	2.231	2.390

Algebra 2.

This is the first year that we have had sufficient students to do a study of the CPM Algebra 2 students. The results here strongly favor CPM students at most schools as indicated in the table below.

Algebra 2 - Davis

School Number	p(fixed)	p(random)	lsm (CPM)	lsm (nCPM)
301	.	.	3.077	.
305	.3746	.	2.371	2.560
314	.0006	.1552	2.266	1.637
320	.	.	1.828	.
321	.0001	.0518	2.382	1.570
322	.0008	.0000	1.682	0.593
323	.	.	2.670	.
324	.2003	.	2.671	3.293
326	.0001	.1297	1.999	1.040
327	.2252	.6355	2.004	1.832
328	.	.	1.728	.