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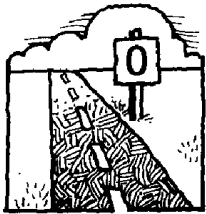
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FOUNDATIONS FOR ALGEBRA

SKILL BUILDERS

(Extra Practice)

Introduction to Students and Their Teachers

Learning is an individual endeavor. Some ideas come easily; others take time--sometimes lots of time--to grasp. In addition, individual students learn the same idea in different ways and at different rates. The authors of the *Foundations for Algebra: Years 1 and 2* textbooks designed the classroom lessons and homework to give students time--often weeks and months--to practice an idea and to use it in various settings. The skill builder resources offer students a brief review of 42 topics followed by examples and additional practice with answers. Not all students will need extra practice. Some will need to do a few topics, while others will need to do many of the sections to help develop their understanding of the ideas. The skill builders may also be useful to prepare for tests, especially final examinations.

How these problems are used will be up to your teacher, your parents, and yourself. In classes where a topic needs additional work by most students, your teacher may assign work from one of the skill builders that follow. In most cases, though, the authors expect that these resources will be used by individual students who need to do more than the textbook offers to learn an idea. This will mean that you are going to need to do some extra work outside of class. In the case where additional practice is necessary for you individually or for a few students in your class, you should not expect your teacher to spend time in class going over the solutions to the skill builder problems. After reading the examples and trying the problems, if you still are not successful, talk to your teacher about getting a tutor or extra help outside of class time.

Warning! Looking is not the same as doing. You will never become good at any sport just by watching it. In the same way, reading through the worked out examples and understanding the steps are not the same as being able to do the problems yourself. An athlete only gets good with practice. The same is true of developing your mathematics skills. How many of the extra practice problems do you need to try? That is really up to you. Remember that your goal is to be able to do problems of the type you are practicing on your own, confidently and accurately.

There are two additional sources for help with the topics in this course. One of them is the on-line homework help funded by CPM. Tutorial (that is, step by step) solutions to the homework problems are available at www.hotmath.com. Simply enter this website, select the course, then click on the icon of the CPM textbook. The other resource is the *Foundations for Algebra: Years 1 and 2 Parent Guide*. Information about ordering this resource can be found at the front of the student text at the end of the note to parents, students, and teachers. It is also available free at the CPM website: www.cpm.org. Homework help is provided by www.hotmath.org.

ADDITION AND SUBTRACTION OF FRACTIONS

#1

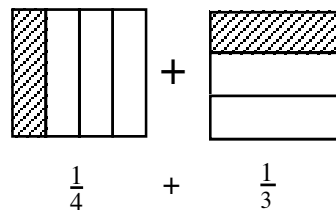
Before fractions can be added or subtracted, the fractions must have the same denominator, that is, a common denominator. There are three methods for adding or subtracting fractions.

AREA MODEL METHOD

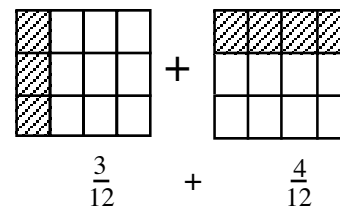
Step 1: Copy the problem.

$$\frac{1}{4} + \frac{1}{3}$$

Step 2: Draw and divide equal-sized rectangles for each fraction. One rectangle is cut horizontally. The other is cut vertically. Label each rectangle, with the fraction it represents.

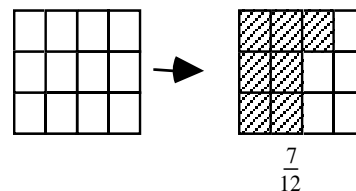


Step 3: Superimpose the lines from each rectangle onto the other rectangle, as if one rectangle is placed on top of the other one.



Step 4: Rename the fractions as twelfths, because the new rectangles are divided into twelve equal parts. Change the numerators to match the number of twelfths in each figure.

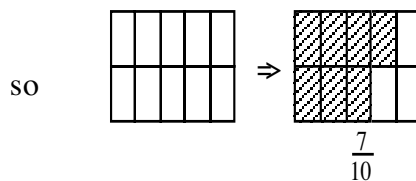
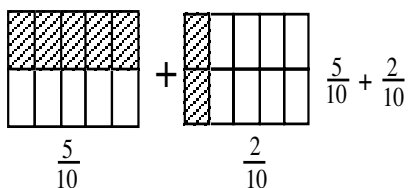
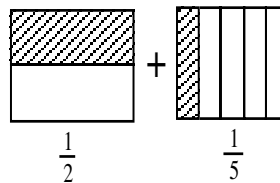
Step 5: Draw an empty rectangle with twelfths, then combine all twelfths by shading the same number of twelfths in the new rectangle as the total that were shaded in both rectangles from the previous step.



Step 6: Simplify if necessary.

Example 1

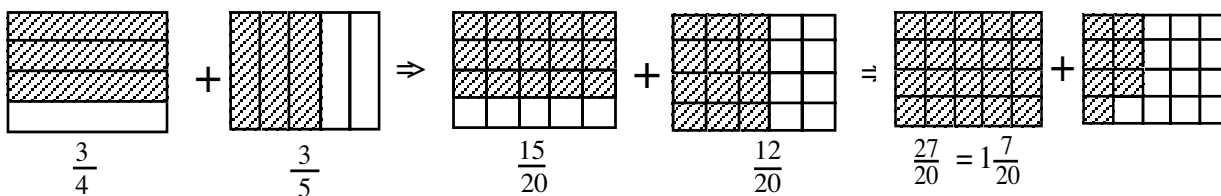
$\frac{1}{2} + \frac{1}{5}$ can be modeled as:



Thus, $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$.

Example 2

$\frac{3}{4} + \frac{3}{5}$ would be:



Problems

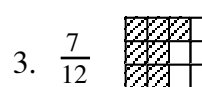
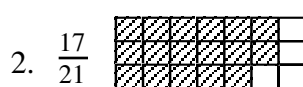
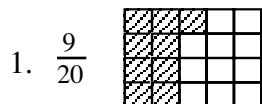
Use the area model method to add the following fractions.

1. $\frac{1}{4} + \frac{1}{5}$

2. $\frac{2}{3} + \frac{1}{7}$

3. $\frac{1}{3} + \frac{1}{4}$

Answers



IDENTITY PROPERTY OF MULTIPLICATION (Giant 1) METHOD

The Giant **1**, known in mathematics as the Identity Property of Multiplication, uses a fraction with the same numerator and denominator ($\frac{3}{3}$, for example) to write an equivalent fraction that helps to create common denominators.

Example

Add $\frac{2}{3} + \frac{1}{4}$ using the Giant **1**.

Step 1: Multiply both $\frac{2}{3}$ and $\frac{1}{4}$ by Giant **1**s to get a common denominator. $\frac{2}{3} \cdot \frac{4}{4} + \frac{1}{4} \cdot \frac{3}{3} = \frac{8}{12} + \frac{3}{12}$

Step 2: Add the numerators of both fractions to get the answer. $\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

RATIO TABLE METHOD

The **least common multiple**, that is, the smallest positive integer divisible by both (or all) of the denominators, is found by using ratio tables. The least common multiple is used as the common denominator of the fractions. The Giant **1** or another ratio table can be used to find the new numerators.

Example

Solve $\frac{3}{4} - \frac{1}{6}$ using a ratio table to find the least common denominator of the fractions.

Use a ratio table to find the least common denominator of the fractions. (This is the same as finding the least common multiple of the denominators, 4 and 6.)

4	8	(12)	16
6	(12)	18	24

You then use the Giant **1** to find the new numerator. $\frac{3}{4} - \frac{1}{6} \Rightarrow \frac{3}{4} \cdot \frac{3}{3} - \frac{1}{6} \cdot \frac{2}{2} \Rightarrow \frac{9}{12} - \frac{2}{12} \Rightarrow \frac{7}{12}$

Problems

Find each sum or difference. Use the method of your choice.

1. $\frac{1}{3} + \frac{3}{5}$

2. $\frac{5}{6} + \frac{1}{3}$

3. $\frac{5}{9} - \frac{1}{3}$

4. $\frac{1}{4} + \frac{5}{7}$

5. $\frac{3}{9} + \frac{3}{4}$

6. $\frac{5}{12} + \frac{2}{3}$

7. $\frac{4}{5} - \frac{2}{3}$

8. $\frac{3}{4} - \frac{2}{5}$

9. $\frac{5}{8} + \frac{3}{5}$

10. $\frac{1}{4} + \frac{2}{3}$

11. $\frac{1}{6} + \frac{2}{3}$

12. $\frac{7}{8} + \frac{3}{4}$

13. $\frac{5}{7} - \frac{1}{3}$

14. $\frac{3}{4} - \frac{2}{3}$

15. $\frac{4}{5} + \frac{1}{4}$

16. $\frac{6}{7} - \frac{3}{4}$

17. $\frac{2}{3} - \frac{3}{4}$

18. $\frac{3}{5} - \frac{9}{15}$

19. $\frac{4}{5} - \frac{2}{3}$

20. $\frac{4}{6} - \frac{11}{12}$

Answers

1. $\frac{14}{15}$

2. $\frac{7}{6} = 1\frac{1}{6}$

3. $\frac{2}{9}$

4. $\frac{27}{28}$

5. $\frac{39}{36} = 1\frac{3}{36} = 1\frac{1}{12}$

6. $\frac{13}{12} = 1\frac{1}{12}$

7. $\frac{2}{15}$

8. $\frac{7}{20}$

9. $\frac{49}{40} = 1\frac{9}{40}$

10. $\frac{11}{12}$

11. $\frac{5}{6}$

12. $\frac{13}{8} = 1\frac{5}{8}$

13. $\frac{8}{21}$

14. $\frac{1}{12}$

15. $\frac{21}{20} = 1\frac{1}{20}$

16. $\frac{3}{28}$

17. $-\frac{1}{12}$

18. 0

19. $\frac{2}{15}$

20. $-\frac{3}{12} = -\frac{1}{4}$

To summarize addition and subtraction of fractions:

1. Rename each fraction with equivalents that have a common denominator.
2. Add or subtract only the numerators, keeping the common denominator.
3. Simplify if possible.

SUBTRACTING MIXED NUMBERS

To subtract mixed numbers, change the mixed numbers into fractions greater than one, find a common denominator, then subtract.

Example

Find the difference: $2\frac{1}{5} - 1\frac{2}{3}$.

$$\begin{array}{r} 2\frac{1}{5} = \frac{11}{5} \cdot \frac{3}{3} = \frac{33}{15} \\ - 1\frac{2}{3} = \frac{5}{3} \cdot \frac{5}{5} = \frac{25}{15} \\ \hline \frac{8}{15} \end{array}$$

Problems

Find each difference.

1. $2\frac{1}{8} - 1\frac{3}{4}$

2. $4\frac{1}{3} - 2\frac{3}{6}$

3. $1\frac{1}{6} - \frac{3}{5}$

4. $4\frac{3}{4} - 2\frac{4}{5}$

5. $6 - 1\frac{2}{5}$

6. $4\frac{1}{8} - 1\frac{2}{3}$

Answers

1. $\frac{17}{8} - \frac{7}{4} \Rightarrow \frac{17}{8} - \frac{14}{8} \Rightarrow \frac{3}{8}$

2. $\frac{13}{3} - \frac{15}{6} \Rightarrow \frac{26}{6} - \frac{15}{6} \Rightarrow \frac{11}{6}$ or $1\frac{5}{6}$

3. $\frac{7}{6} - \frac{3}{5} \Rightarrow \frac{35}{30} - \frac{18}{30} \Rightarrow \frac{17}{30}$

4. $\frac{19}{4} - \frac{14}{5} \Rightarrow \frac{95}{20} - \frac{56}{20} \Rightarrow \frac{39}{20}$ or $1\frac{19}{20}$

5. $\frac{6}{1} - \frac{7}{5} \Rightarrow \frac{30}{5} - \frac{7}{5} \Rightarrow \frac{23}{5}$ or $4\frac{3}{5}$

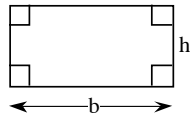
6. $\frac{33}{8} - \frac{5}{3} \Rightarrow \frac{99}{24} - \frac{40}{24} \Rightarrow \frac{59}{24}$ or $2\frac{11}{24}$

AREA

#2

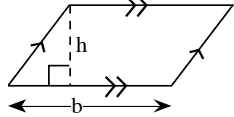
AREA is the number of square units in a flat region. The formulas to calculate the area of several kinds of polygons are:

RECTANGLE



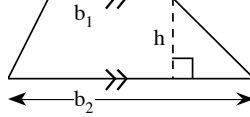
$$A = bh$$

PARALLELOGRAM



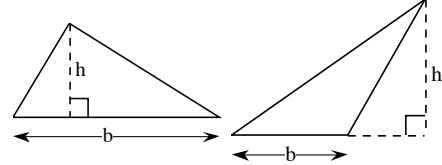
$$A = bh$$

TRAPEZOID



$$A = \frac{1}{2}(b_1 + b_2)h$$

TRIANGLE



$$A = \frac{1}{2}bh$$

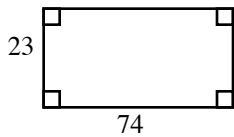
Note that the legs of any right triangle form a base and a height for the triangle (see Example 1, part (c)).

The area of a more complicated figure may be found by breaking it into smaller regions of the types shown above, calculating each area, and finding the sum or difference of the areas.

Example 1

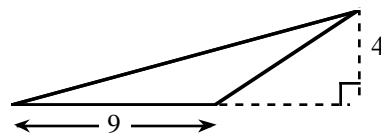
Find the area of each figure. All lengths are centimeters.

a)



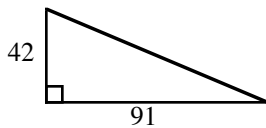
$$A = bh = (74)(23) = 1702 \text{ cm}^2$$

b)



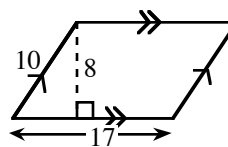
$$A = \frac{1}{2}bh = \frac{1}{2}(9)(4) = 18 \text{ cm}^2$$

c)



$$A = \frac{1}{2}bh = \frac{1}{2}(91)(42) = 1911 \text{ cm}^2$$

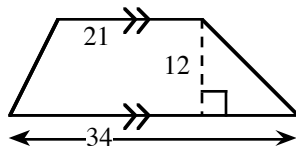
d)



$$A = bh = (17)8 = 136 \text{ cm}^2$$

Note that 10 is a side of the parallelogram, not the height.

e)



$$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(21 + 34)12 = \frac{1}{2}(55)(12) = 330 \text{ cm}^2$$

Example 2

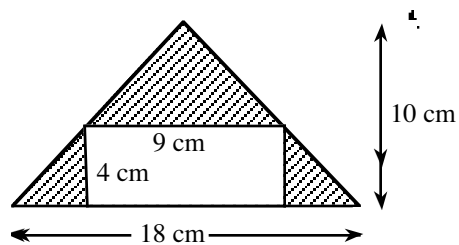
Find the area of the shaded region.

The area of the shaded region is the area of the triangle minus the area of the rectangle.

large triangle: $A = \frac{1}{2}(10)(18) = 90 \text{ cm}^2$

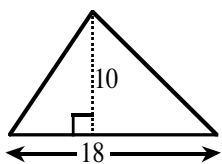
rectangle: $A = 9(4) = 36 \text{ cm}^2$

shaded region: $A = 90 - 36 = 54 \text{ cm}^2$

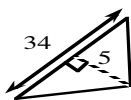


Find the area of the following triangles, parallelograms and trapezoids. Pictures are not drawn to scale. Round answers to the nearest tenth.

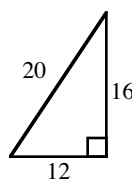
1.



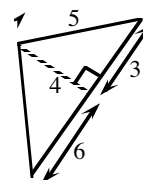
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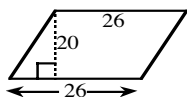
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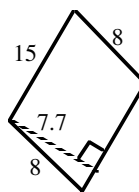
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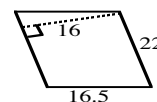
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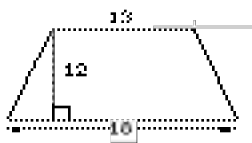
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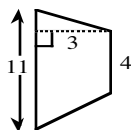
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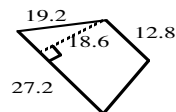
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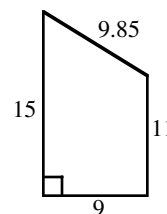
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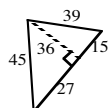
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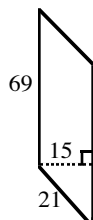
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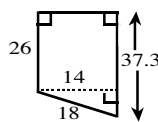
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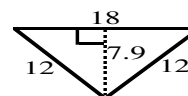
14.



15.

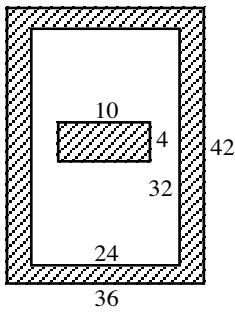


16.

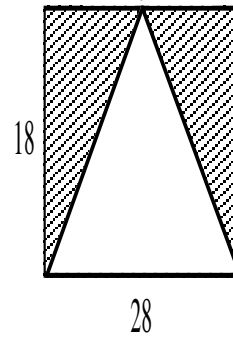


Find the area of the shaded region.

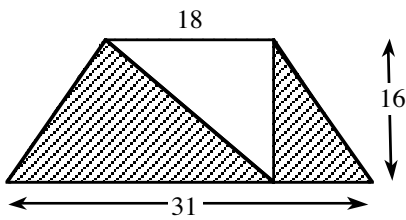
17. The figures are rectangles.



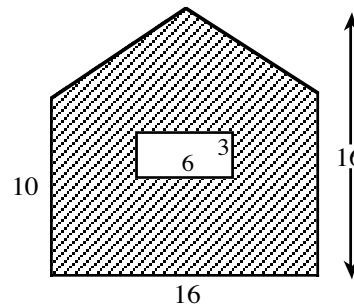
18. The triangle is inside a rectangle.



19. The outer border of the figure is a trapezoid. The triangle below right is a right triangle.



20. The lower corners are right angles and the two sloping sides are equal.



Answers (in square units)

- | | | | | | |
|----------|----------|-----------|----------|---------|---------|
| 1. 90 | 2. 85 | 3. 96 | 4. 18 | 5. 520 | 6. 133 |
| 7. 115.5 | 8. 352 | 9. 186 | 10. 22.5 | 11. 372 | 12. 117 |
| 13. 756 | 14. 1035 | 15. 443.1 | 16. 71.1 | 17. 784 | 18. 252 |
| 19. 248 | 20. 190 | | | | |

AREA OF A CIRCLE

#3

The **AREA** of a circle is the measure of the region inside it. To find the area of a circle when given the **radius**, use this formula:

$$A = \pi \cdot r \cdot r = \pi r^2$$

The radius of a circle needs to be identified in order to find the area of the circle. The radius is half the diameter. Next square the radius and multiply the result by π .

Example 1

Find the area of a circle with $r = 17$ feet.

$$\begin{aligned} A &= \pi r^2 \\ &\approx 3.14(17 \cdot 17) \\ &= 907.46 \text{ ft}^2 \end{aligned}$$

Example 2

Find the area of a circle with $d = 84$ cm.

$$\begin{aligned} \text{radius} &= \text{diameter} \div 2 \\ &= 84 \div 2 \\ &= 42 \text{ cm} \end{aligned}$$

$$\begin{aligned} A &= \pi r^2 \\ &= 3.14(42 \cdot 42) \\ &= 5538.96 \text{ cm}^2 \end{aligned}$$

Example 3

Find the radius of a circle with area 78.5 square meters.

$$\begin{aligned} 78.5 &= \pi r^2 \\ 78.5 &= 3.14r^2 \\ r^2 &\approx 78.5 \div 3.14 \approx 24.89 \end{aligned}$$

$$\begin{aligned} r &\approx \sqrt{24.89} \\ &\approx 5 \text{ meters} \end{aligned}$$

Example 4

Find the radius of a circle with area 50.24 square centimeters.

$$\begin{aligned} 50.24 &= \pi r^2 \\ 50.24 &= 3.14r^2 \\ r^2 &\approx 50.24 \div 3.14 = 16 \end{aligned}$$

$$\begin{aligned} r &\approx \sqrt{16} \\ &\approx 4 \text{ centimeters} \end{aligned}$$

Problems

Find the area of the circles with the following radius or diameter lengths. Use $\pi = 3.14$. Round your answers to the nearest hundredth.

1. $r = 9$ cm

2. $r = 5$ in.

3. $d = 20$ ft

4. $d = 8$ cm

5. $r = \frac{1}{4}$ m

6. $r = 7.2$ in.

7. $r = 4.6$ cm

8. $r = 6\frac{1}{4}$ in.

9. $d = 26.4$ ft

10. $r = 13.7$ m

Find the radius of each circle given the following areas. Use $\pi = 3.14$. Round your answers to the nearest tenth.

11. $A = 314$ m²

12. $A = 55.39$ cm²

13. $A \approx 140.95$ ft²

14. $A \approx 262.31$ in²

15. $A = 660.19$ km²

Answers

1. 254.34 cm²

2. 78.54 in.²

3. 314 ft²

4. 50.27 cm²

5. 0.20 m²

6. 162.78 in.²

7. 66.44 cm²

8. 122.66 in.²

9. 547.11 ft²

10. 589.35 m²

11. $r = 10$ m

12. $r = 4.2$ cm

13. $r \approx 6.7$ ft

14. $r = 9.1$ in.

15. $r = 14.5$ km

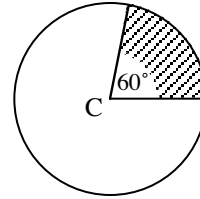
AREA OF SECTORS (SUBPROBLEMS WITH CIRCLES)

#4

A **SECTOR** of a circle is formed by the two radii of a central angle and the arc between their endpoints on the circle. When we find the area of a sector, we are finding a fractional part of the circle.

$$A_{\text{sector}} = (\text{fractional part})(\text{Area of Circle})$$

For example, a 60° sector looks like a slice of pizza. 60° is the part of the circle. The whole circle is 360° .

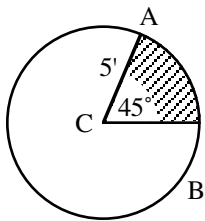


$$\frac{\text{part}}{\text{whole}} = \frac{60}{360} = \frac{1}{6}$$

The sector is $\frac{1}{6}$ of the circle, so $A_{\text{sector}} = \frac{1}{6}(\text{Area Circle})$.

Example 1

Find the area of the 45° sector.



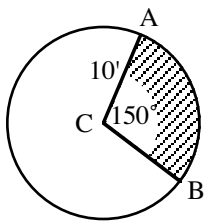
$$\text{Fractional part of circle: } \frac{\text{part}}{\text{whole}} = \frac{45^\circ}{360^\circ} = \frac{1}{8}$$

$$\text{Area of the circle: } A = \pi r^2 = 25\pi \approx 78.5 \text{ ft}^2$$

$$\text{Area of the sector: } \frac{1}{8} \cdot 78.5 \approx 9.81 \text{ ft}^2$$

Example 2

Find the area of the 150° sector.



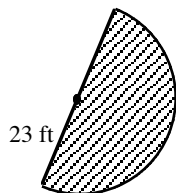
$$\text{Fractional part of circle: } \frac{\text{part}}{\text{whole}} = \frac{150^\circ}{360^\circ} = \frac{5}{12}$$

$$\text{Area of circle is } A = \pi r^2 = \pi 10^2 = 100\pi \approx 314 \text{ ft}^2$$

$$\text{Area of the sector: } \frac{5}{12} \cdot 314 \approx 130.83 \text{ ft}^2$$

Example 3

Find the area of the semicircle.



$$\text{Fractional part of circle: } \frac{\text{part}}{\text{whole}} = \frac{180^\circ}{360^\circ} = \frac{1}{2}$$

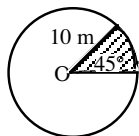
$$\text{Area of circle: } A = \pi r^2 = \pi 23^2 = 529\pi \approx 1661.06 \text{ ft}^2$$

$$\text{Area of the sector: } \frac{1}{2} \cdot 1661.06 \approx 830.53 \text{ ft}^2$$

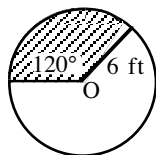
Problems

Calculate the area of the following shaded sectors. Point O is the center of each circle.
Use $\pi = 3.14$ in all problems.

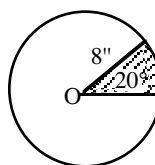
1.



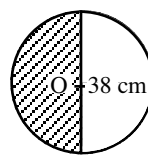
2.



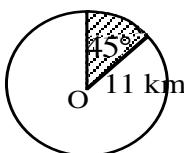
3.



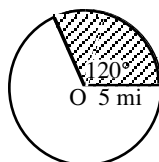
4.



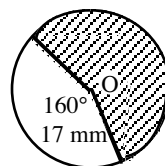
5.



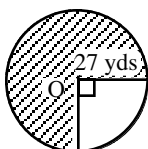
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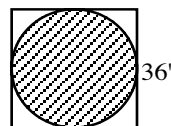
7.



8.

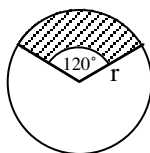


9. Find the area of a circular garden if the diameter of the garden is 60 feet.

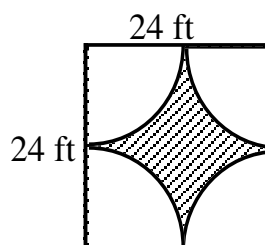


10. Find the area of a circle inscribed in a square whose edge is 36 feet long.

11. Find the radius. The shaded area is 157.84 cm^2 .



12. Find the area of the shaded region. The four points of the shaded region are midpoints of the sides.



Answers

1. 39.25 m^2

2. 37.68 ft^2

3. 11.16 in.^2

4. 566.77 cm^2

5. 47.49 km^2

6. 26.17 mi^2

7. 806.63 mm^2

8. 1716.80 yd^2

9. 2826 ft^2

10. 1017.36 ft^2

11. 12.28 cm^2

12. 123.84 ft^2

ARITHMETIC OPERATIONS WITH DECIMALS

#5

ADDING AND SUBTRACTING DECIMALS: Write the problem in column form with the decimal points in a vertical column. Write in zeros so that all decimal parts of the number have the same number of digits. Add or subtract as with whole numbers. Place the decimal point in the answer aligned with those above.

MULTIPLYING DECIMALS: Multiply as with whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the factors (numbers you multiplied). Sometimes zeros need to be added to place the decimal point.

DIVIDING DECIMALS: When dividing a decimal by a whole number, place the decimal point in the answer space directly above the decimal point in the number being divided. Divide as with whole numbers. Sometimes it is necessary to add zeroes to complete the division.

When dividing decimals or whole numbers by a decimal, the divisor must be multiplied by a power of ten to make it a whole number. The dividend must be multiplied by the same power of ten. Then divide following the same rules for division by a whole number.

Example 1

Add 47.37, 28.9, 14.56, and 7.8.

$$\begin{array}{r} 47.37 \\ 28.90 \\ 14.56 \\ + 7.80 \\ \hline 98.63 \end{array}$$

Example 2

Subtract 198.76 from 473.2.

$$\begin{array}{r} 473.20 \\ - 198.76 \\ \hline 274.44 \end{array}$$

Example 3

Multiply 27.32 by 14.53.

$$\begin{array}{r} 27.32 \text{ (2 decimal places)} \\ \times 14.53 \text{ (2 decimal places)} \\ \hline 8196 \\ 13660 \\ 10928 \\ 2732 \\ \hline 396.9596 \text{ (4 decimal places)} \end{array}$$

Example 4

Multiply 0.37 by 0.00004.

$$\begin{array}{r} 0.37 \text{ (2 decimal places)} \\ \times 0.0004 \text{ (4 decimal places)} \\ \hline 0.000148 \text{ (6 decimal places)} \end{array}$$

Example 5

Divide 32.4 by 8.

$$\begin{array}{r} 4.05 \\ 8 \overline{)32.40} \\ \underline{32} \\ 040 \\ \underline{40} \\ 0 \end{array}$$

Example 6

Divide 27.42 by 1.2. First multiply each number by 10^1 or 10.

$$\begin{array}{r} 1.2 \overline{)27.42} \Rightarrow 12 \overline{)274.2} \\ \underline{24} \\ 34 \\ \underline{24} \\ 102 \\ \underline{96} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

Problems

1. $4.7 + 7.9$
2. $3.93 + 2.82$
3. $38.72 + 6.7$
4. $58.3 + 72.84$
5. $4.73 + 692$
6. $428 + 7.392$
7. $42.1083 + 14.73$
8. $9.87 + 87.47936$
9. $9.999 + 0.001$
10. $0.0001 + 99.9999$
11. $0.0137 + 1.78$
12. $2.037 + 0.09387$
13. $15.3 + 72.894$
14. $47.9 + 68.073$
15. $289.307 + 15.938$
16. $476.384 + 27.847$
17. $15.38 + 27.4 + 9.076$
18. $48.32 + 284.3 + 4.638$
19. $278.63 + 47.0432 + 21.6$
20. $347.68 + 28.00476 + 84.3$
21. $8.73 - 4.6$
22. $9.38 - 7.5$
23. $8.312 - 6.98$
24. $7.045 - 3.76$
25. $6.304 - 3.68$
26. $8.021 - 4.37$
27. $14 - 7.431$
28. $23 - 15.37$
29. $10 - 4.652$
30. $18 - 9.043$
31. $0.832 - 0.47$
32. $0.647 - 0.39$
33. $1.34 - 0.0538$
34. $2.07 - 0.523$
35. $4.2 - 1.764$
36. $3.8 - 2.406$
37. $38.42 - 32.605$
38. $47.13 - 42.703$
39. $15.368 + 14.4 - 18.5376$
40. $87.43 - 15.687 - 28.0363$
41. $7.34 \cdot 6.4$
42. $3.71 \cdot 4.03$
43. $0.08 \cdot 4.7$
44. $0.04 \cdot 3.75$
45. $41.6 \cdot 0.302$
46. $9.4 \cdot 0.0053$
47. $3.07 \cdot 5.4$
48. $4.023 \cdot 3.02$
49. $0.004 \cdot 0.005$
50. $0.007 \cdot 0.0004$
51. $0.235 \cdot 0.43$
52. $4.32 \cdot 0.0072$
53. $0.0006 \cdot 0.00013$
54. $0.0005 \cdot 0.00026$
55. $8.38 \cdot 0.0001$
56. $47.63 \cdot 0.000001$
57. $0.078 \cdot 3.1$
58. $0.043 \cdot 4.2$
59. $350 \cdot 0.004$
60. $421 \cdot 0.00005$

>>Problems continue on the next page.>>

Divide. Round answers to the hundredth, if necessary.

61. $14.3 \div 8$

62. $18.32 \div 5$

63. $147.3 \div 6$

64. $46.36 \div 12$

65. $100.32 \div 24$

66. $132.7 \div 28$

67. $47.3 \div 0.002$

68. $53.6 \div 0.004$

69. $500 \div 0.004$

70. $420 \div 0.05$

71. $1.32 \div 0.032$

72. $3.486 \div 0.012$

73. $46.3 \div 0.011$

74. $53.7 \div 0.023$

75. $25.46 \div 5.05$

76. $26.35 \div 2.2$

77. $6.042 \div 0.006$

78. $7.035 \div 0.005$

79. $207.3 \div 4.4$

80. $306.4 \div 3.2$

Answers

1. 12.6

2. 6.75

3. 45.42

4. 131.14

5. 696.73

6. 435.392

7. 56.8383

8. 97.34936

9. 10.000

10. 100.0000

11. 1.7937

12. 2.13087

13. 88.194

14. 115.973

15. 305.245

16. 504.231

17. 51.856

18. 337.258

19. 347.2732

20. 459.98476

21. 4.13

22. 1.88

23. 1.332

24. 3.285

25. 2.624

26. 3.651

27. 6.569

28. 7.63

29. 5.348

30. 8.957

31. 0.362

32. 0.257

33. 1.2862

34. 1.547

35. 2.436

36. 1.394

37. 5.815

38. 4.427

39. 11.2304

40. 43.7067

41. 46.976

42. 14.9513

43. 0.376

44. 0.15

45. 12.5632

46. 0.04982

47. 16.578

48. 12.14946

49. 0.000020

50. 0.0000028

51. 0.10105

52. 0.031104

53. 0.000000078

54. 0.000000130

55. 0.000838

56. 0.0004763

57. 0.2418

58. 0.1806

59. 1.4

60. 0.02105

61. 1.7875 or 1.79

62. 3.664 or 3.66

63. 24.55

64. $3.86\bar{3}$ or 3.86

65. 4.18

66. 4.74

67. 23,650

68. 13,400

69. 125,000

70. 8400

71. 41.25

72. 29.05

73. 4209.09

74. 2334.78

75. 5.04

76. 11.98

77. 1007

78. 1407

79. 47.11

80. 95.75

ARITHMETIC OPERATIONS WITH INTEGERS

ADDITION OF INTEGERS

#6

Add numbers two at a time. If the signs are the same, add the numbers and keep the same sign. If the signs are different, ignore the signs (that is, use the absolute value of each number) and find the difference of the two numbers. The sign of the answer is determined by the number farthest from zero, that is, the number with the greater absolute value.

Follow the same rules for fractions and decimals.

Remember to apply the correct order of operations when you are working with more than one operation.

Example 1: same signs

a) $2 + 3 = 5$ or $3 + 2 = 5$

b) $-2 + (-3) = -5$ or $-3 + (-2) = -5$

Example 2: different signs

a) $-2 + 3 = 1$ or $3 + (-2) = 1$

b) $-3 + 2 = -1$ or $2 + (-3) = -1$

Problems: Addition

Simplify the following expression using the rules above without using a calculator.

1. $5 + (-2)$

2. $4 + (-1)$

3. $9 + (-7)$

4. $-10 + 5$

5. $-9 + 2$

6. $-12 + 8$

7. $-3 + (-7)$

8. $-12 + (-4)$

9. $-13 + (-16)$

10. $-7 + (-14)$

11. $-7 + 13$

12. $-24 + 11$

13. $-4 + 3 + 6$

14. $8 + (-10) + (-5)$

15. $5 + (-4) + (-2) + (-9)$

16. $-9 + (-3) + (-2) + 11$

17. $10 + (-7) + (-6) + 5 + (-8)$

18. $14 + (-13) + 18 + (-22)$

19. $55 + (-65) + 30$

20. $19 + (-16) + (-5) + 15$

Answers

1. 3

2. 3

3. 2

4. -5

5. -7

6. -4

7. -10

8. -16

9. -29

10. -21

11. 6

12. -13

13. 5

14. -7

15. -10

16. -3

17. -6

18. -3

19. 20

20. 13

SUBTRACTION OF INTEGERS

To find the difference of two values, change the subtraction sign to addition, change the sign of the number being subtracted, then follow the rules for addition.

Follow the same rules for fractions and decimals.

Remember to apply the correct order of operations when you are working with more than one operation.

Example 1

a) $2 - 3 \Rightarrow 2 + (-3) = -1$

b) $-2 - (-3) \Rightarrow -2 + (+3) = 1$

Example 2

a) $-2 - 3 \Rightarrow -2 + (-3) = -5$

b) $2 - (-3) \Rightarrow 2 + (+3) = 5$

Problems: Subtraction

Use the rule stated above to find each difference.

1. $8 - (-3)$

2. $8 - 3$

3. $-8 - 3$

4. $5 - (-8)$

5. $-38 - 62$

6. $-38 - (-62)$

7. $38 - 62$

8. $38 - (-62)$

9. $-5 - (-3) - 4 - 7$

10. $5 - (-8) - 3 - (-7)$

11. $-8 - (-3)$

12. $18 - 25$

13. $-7 - 5$

14. $-26 - 7$

15. $-3 - (-7)$

16. $10 - (-5)$

17. $-58 - 24$

18. $-62 - 73$

19. $-74 - (-47)$

20. $-37 - (-55)$

Answers

1. 11

2. 5

3. -11

4. 13

5. -100

6. 24

7. -24

8. 100

9. -13

10. 17

11. -5

12. -7

13. -12

14. -33

15. 4

16. 15

17. -82

18. -135

19. -27

20. 18

MULTIPLICATION AND DIVISION OF INTEGERS

Multiply and divide integers two at a time. If the signs are the same, their product will be positive. If the signs are different, their product will be negative.

Follow the same rules for fractions and decimals.

Remember to apply the correct order of operations when you are working with more than one operation.

Example

a) $2 \cdot 3 = 6$ or $3 \cdot 2 = 6$

b) $-2 \cdot (-3) = 6$ or $(+2) \cdot (+3) = 6$

c) $2 \div 3 = \frac{2}{3}$ or $3 \div 2 = \frac{3}{2}$

d) $(-2) \div (-3) = \frac{2}{3}$ or $(-3) \div (-2) = \frac{3}{2}$

e) $(-2) \cdot 3 = -6$ or $3 \cdot (-2) = -6$

f) $(-2) \div 3 = -\frac{2}{3}$ or $3 \div (-2) = -\frac{3}{2}$

g) $9 \cdot (-7) = -63$ or $-7 \cdot 9 = -63$

h) $-63 \div 9 = -7$ or $9 \div (-63) = -\frac{1}{7}$

Problems: Multiplication and Division

Use the rules above to find each product or quotient.

1. $(-4)(2)$

2. $(-3)(4)$

3. $(-12)(5)$

4. $(-21)(8)$

5. $(4)(-9)$

6. $(13)(-8)$

7. $(45)(-3)$

8. $(105)(-7)$

9. $(-7)(-6)$

10. $(-7)(-9)$

11. $(-22)(-8)$

12. $(-127)(-4)$

13. $(-8)(-4)(2)$

14. $(-3)(-3)(-3)$

15. $(-5)(-2)(8)(4)$

16. $(-5)(-4)(-6)(-3)$

17. $(-2)(-5)(4)(8)$

18. $(-2)(-5)(-4)(-8)$

19. $(-2)(-5)(4)(-8)$

20. $2(-5)(4)(-8)$

21. $10 \div (-5)$

22. $18 \div (-3)$

23. $96 \div (-3)$

24. $282 \div (-6)$

25. $-18 \div 6$

26. $-48 \div 4$

27. $-121 \div 11$

28. $-85 \div 85$

29. $-76 \div (-4)$

30. $-175 \div (-25)$

31. $-108 \div (-12)$

32. $-161 \div 23$

33. $-223 \div (-223)$

34. $354 \div (-6)$

35. $-1992 \div (-24)$

36. $-1819 \div (-17)$

37. $-1624 \div 29$

38. $1007 \div (-53)$

39. $994 \div (-14)$

40. $-2241 \div 27$

Answers

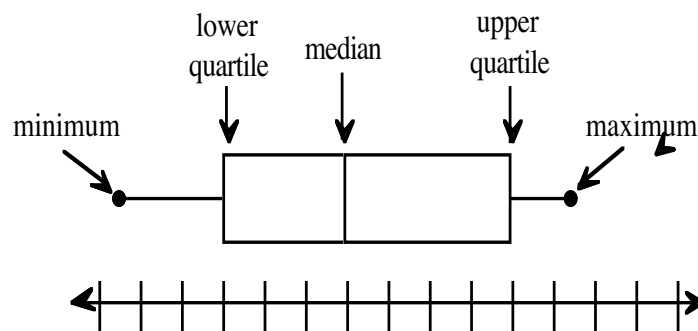
- | | | | | |
|---------|---------|---------|----------|---------|
| 1. -8 | 2. -12 | 3. -60 | 4. -168 | 5. -36 |
| 6. -104 | 7. -135 | 8. -735 | 9. 42 | 10. 63 |
| 11. 176 | 12. 508 | 13. 64 | 14. -27 | 15. 320 |
| 16. 360 | 17. 320 | 18. 320 | 19. -320 | 20. 320 |
| 21. -2 | 22. -6 | 23. -32 | 24. -47 | 25. -3 |
| 26. -12 | 27. -11 | 28. -1 | 29. 19 | 30. 7 |
| 31. 9 | 32. 7 | 33. 1 | 34. -59 | 35. 83 |
| 36. 107 | 37. -56 | 38. -19 | 39. -71 | 40. -83 |

BOX-AND-WHISKER PLOT

#7

A way to display data that shows how the data is grouped or clustered is a **BOX-AND-WHISKER PLOT**. The box-and-whisker plot displays the data using quartiles.

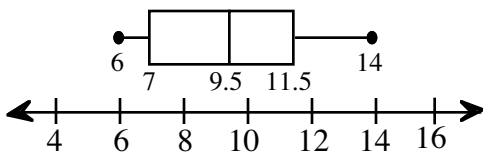
- Put the data set in order, lowest to highest.
- Create a number line which is slightly greater than the range of the data.
- Find the median of the data set. Place a vertical line segment (about two centimeters long) above the median. Write the number it represents below it.
- Find the median of the lower half of the data, that is, the numbers to the left of the median. Place a vertical line segment above this number and write the number the line segment represents below it. This number marks the **lower quartile** of the data.
- Find the median of the upper half of the data, that is, the numbers to the right of the median. Place a vertical line segment above this number and write the number the line segment represents below it. This number marks the **upper quartile** of the data.
- Draw a box between the upper quartile and the lower quartile, using the line segments you drew above each quartile as the vertical sides. The line segment for the median will be inside the box.
- Place two dots above the number line, one that labels the minimum (the smallest number) and another that labels the maximum (the largest number) values of the data set.
- Draw a horizontal line segment from the lower quartile to the dot representing the minimum value and a horizontal line segment from the upper quartile to the dot representing the maximum value. The line segments extending to the far right and left of the data display are called the **whiskers**.



Example 1

Display this data in a box-and-whisker plot:
6, 8, 10, 9, 7, 7, 11, 12, 6, 12, 14, and 10.

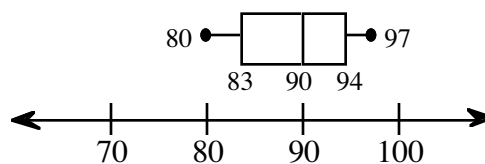
- Place the data in order from least to greatest: 6, 6, 7, 7, 8, 9, 10, 10, 11, 12, 12, and 14. The range is $14 - 6 = 8$. Thus you start with a number line with equal intervals from 4 to 16.
- The median of the set of data is 9.5. Draw a vertical line segment at this value above the number line.
- The median of the lower half of the data (the lower quartile) is 7. Draw a vertical line segment at this value above the number line.
- The median of the upper half of the data (the upper quartile) is 11.5. Draw a vertical line segment at this value above the number line.
- Draw a box between the upper and lower quartiles.
- Place a dot at the minimum value (6) and a dot at the maximum value (14). The horizontal line segments that connect these dots to the box are called the whiskers.



Example 2

Display this data in a box-and-whisker plot:
80, 90, 85, 83, 83, 92, 97, 91, and 95.

- Place the data in order from least to greatest: 80, 83, 83, 85, 90, 91, 92, 96, and 97. The range is $97 - 80 = 17$. Thus you want a number line with equal intervals from 70 to 100.
- Find the median of the set of data: 90. Draw the vertical line segment.
- Find the lower quartile: 83. Draw the vertical line segment.
- Find the upper quartile: $92 + 96 = 188$; $188 \div 2 = 94$. Draw the vertical line segment.
- Draw the box connecting the upper and lower quartiles. Place a dot at the minimum value (80) and a dot at the maximum value (97). Draw the whiskers.



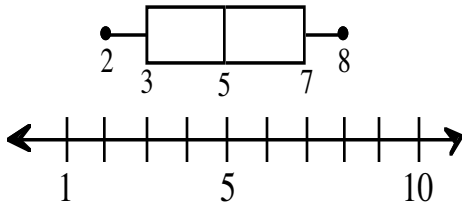
Problems

Make a box-and-whisker plot for each set of data.

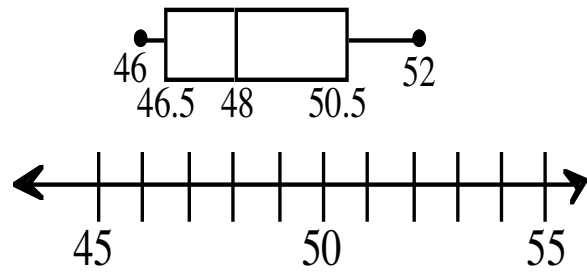
1. 5, 8, 3, 2, 7, 3, 7, 4, and 6.
2. 47, 52, 50, 47, 51, 46, 49, 46 and 48.
3. 20, 35, 16, 19, 25, 32, 17, 38, 16, and 36.
4. 70, 63, 62, 74, 67, 62, 70, 72, 60, and 61.
5. 72, 63, 70, 42, 50, 53, 65, 38, and 39.
6. 76, 90, 75, 72, 93, 82, 70, 85, and 80.

Answers

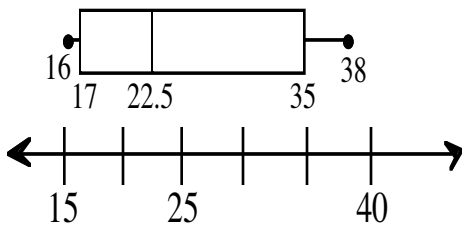
1.



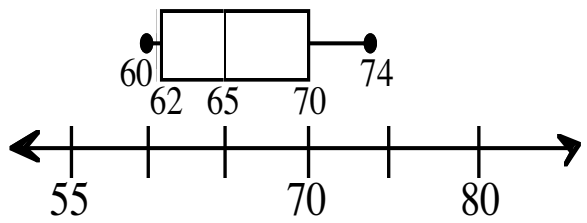
2.



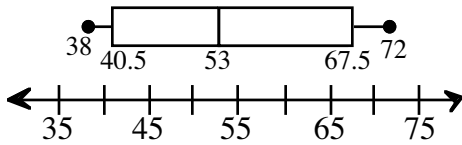
3.



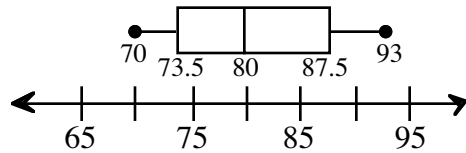
4.



5.



6.



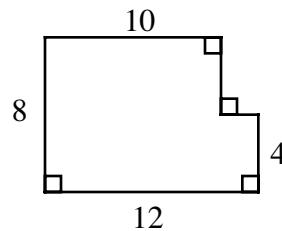
CALCULATING COMPOUND AREAS USING SUBPROBLEMS

#8

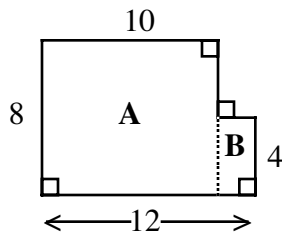
Every polygon can be dissected (or broken up) into rectangles and triangles which have no interior points in common. This is an example of the problem solving strategy of **SUBPROBLEMS**. Finding simpler problems that you know how to solve will help you solve the larger problem.

Example 1

Find the area of the figure at right.



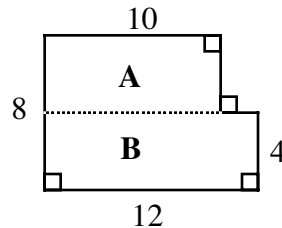
Method #1



Subproblems:

1. Find the area of rectangle A:
 $8 \cdot 10 = 80$ square units.
2. Find the area of rectangle B:
 $4 \cdot (12 - 10) = 4 \cdot 2 = 8$ square units.
3. Add the area of rectangle A to the area of rectangle B:
 $80 + 8 = 88$ square units.

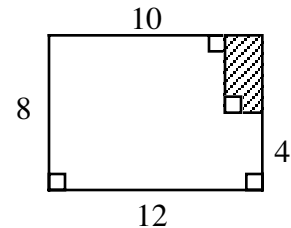
Method #2



Subproblems:

1. Find the area of rectangle A:
 $10 \cdot (8 - 4) = 10 \cdot 4 = 40$ square units.
2. Find the area of rectangle B:
 $12 \cdot 4 = 48$ square units.
3. Add the area of rectangle A to the area of rectangle B:
 $40 + 48 = 88$ square units.

Method #3

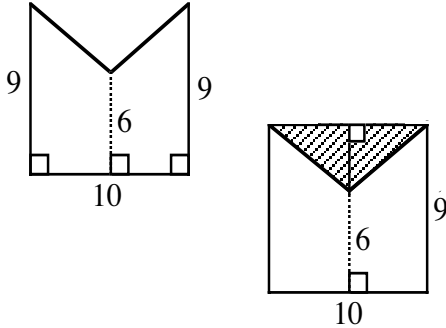


Subproblems:

1. Make a large rectangle by enclosing the upper right corner.
2. Find the area of the new, larger rectangle:
 $8 \cdot 12 = 96$ square units.
3. Find the area of the shaded rectangle:
 $(8 - 4) \cdot (12 - 10) = 4 \cdot 2 = 8$ square units.
4. Subtract the shaded rectangle from the larger rectangle:
 $96 - 8 = 88$ square units.

Example 2

Find the area of the figure below. The vertical 6 unit segment cuts the 10 unit segment in half.

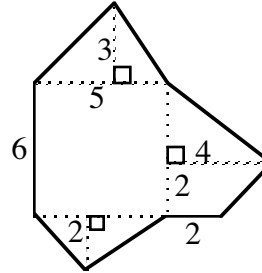


Subproblems:

1. Make a rectangle out of the figure by enclosing the top.
2. Find the area of the entire rectangle:
 $10 \cdot 9 = 90$ square units
3. Find the area of the shaded triangle.
Use the formula $A = \frac{1}{2} bh$.
 $b = 10$ and $h = 9 - 6 = 3$,
so $A = \frac{1}{2} (10 \cdot 3) = \frac{30}{2}$
 $= 15$ square units
4. Subtract the area of the triangle from the area of the rectangle:
 $90 - 15 = 75$ square units

Example 3

Find the area of the figure below. The quadrilateral in the middle is a rectangle.



This figure consists of five familiar figures: a central rectangle, 5 units by 6 units; three triangles, one on top with $b = 5$ and $h = 3$, one on the right with $b = 4$ and $h = 4$, and one on the bottom with $b = 5$ and $h = 2$; and a trapezoid with an upper base of 4, a lower base of 2 and a height of 2.

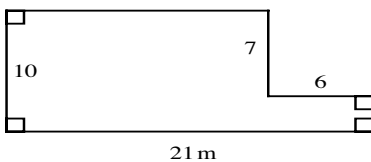
The area is:

rectangle	$5 \cdot 6$	$= 30$
top triangle	$\frac{1}{2} \cdot 5 \cdot 3$	$= 7.5$
bottom triangle	$\frac{1}{2} \cdot 5 \cdot 2$	$= 5$
right side triangle	$\frac{1}{2} \cdot 4 \cdot 4$	$= 8$
trapezoid	$\frac{(4+2) \cdot 2}{2}$	$= 6$
total area		$= 56.5 \text{ u}^2$

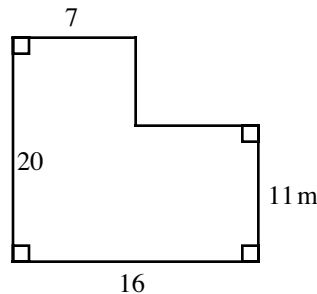
Problems

Find the area of each of the following figures. Assume that anything that looks like a right angle is a right angle.

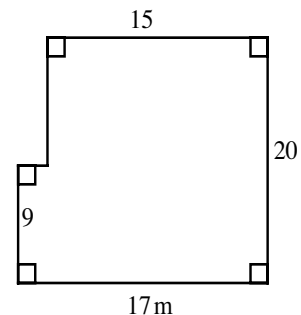
1.



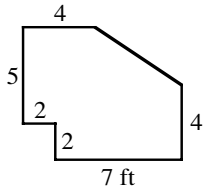
2.



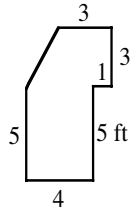
3.



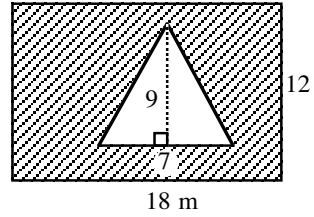
19.



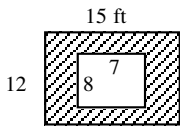
20.



21. Find the area of the shaded region inside the rectangle.

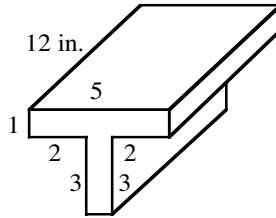


22. Find the area of the shaded region between the two rectangles.

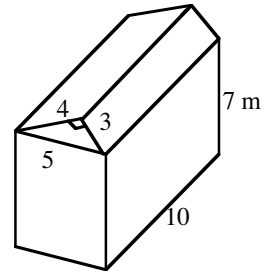


For figures 23 and 24, find the surface area.

23.



24.



Answers

- | | | | |
|--------------------------|-------------------------|--------------------------|-------------------------|
| 1. 168 m ² | 2. 239 m ² | 3. 318 m ² | 4. 45 ft ² |
| 5. 48 yds ² | 6. 60 in. ² | 7. 234 in. ² | 8. 286 m ² |
| 9. 90 ft ² | 10. 192 ft ² | 11. 28 ft ² | 12. 408 cm ² |
| 13. 95 cm ² | 14. 40 mm ² | 15. 40 in. ² | 16. 41 ft ² |
| 17. 38 cm ² | 18. 43 cm ² | 19. 51.5 ft ² | 20. 32 ft ² |
| 21. 184.5 m ² | 22. 124 ft ² | 23. 232 in. ² | 24. 342 m ² |

CIRCUMFERENCE

#9

Circumference is the perimeter of a circle, that is, the distance around the circle.

$$C = \pi d \quad \text{or} \quad C = 2\pi r$$

d = diameter
 r = radius
 $\pi \approx 3.14$

Example 1

Find the circumference of a circle with a diameter of 15 inches.

$$d = 15 \text{ inches}$$

$$\begin{aligned} C &= \pi d \\ &= \pi(15) \text{ or } 3.14(15) \\ &= 47.1 \text{ inches} \end{aligned}$$

Example 2

Find the circumference of a circle with a radius of 12 units.

$$r = 12, \text{ so } d = 2(12) = 24$$

$$\begin{aligned} C &= 3.14(24) \\ &= 75.36 \text{ units} \end{aligned}$$

Example 3

Find the diameter of a circle with a circumference of 254.34 inches.

$$\begin{aligned} C &= \pi d \\ 254.34 &= \pi d \\ 254.34 &= 3.14d \\ d &= \frac{254.34}{3.14} \\ &= 81 \text{ inches} \end{aligned}$$

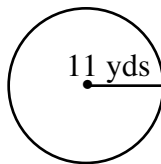
Problems

Find the circumference of each circle given the following diameter or radius lengths. Round your answer to the nearest hundredth. Use $\pi = 3.14$.

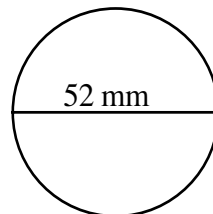
1. $d = 53 \text{ ft}$ 2. $d = 8.5 \text{ ft}$ 3. $r = 7.3 \text{ m}$ 4. $d = 63 \text{ m}$ 5. $r = 2.12 \text{ cm}$

Find the circumference of each circle shown below. Round your answer to the nearest hundredth. Use $\pi = 3.14$.

6.



7.



Find the diameter of each circle given the circumference. Round your answer to the nearest tenth. Use $\pi = 3.14$.

8. $C = 54.636 \text{ mm}$ 9. $C = 135.02 \text{ km}$ 10. $C = 389.36 \text{ km}$

Answers

1. 166.42 ft 2. 26.69 ft 3. 45.84 m 4. 197.82 m
5. 13.31 cm 6. 69.08 yds 7. 163.28 mm 8. 17.4 mm
9. 43 km 10. 124 km

DISTRIBUTIVE PROPERTY

#10

The **DISTRIBUTIVE PROPERTY** shows how to express sums and products in two ways:
 $a(b + c) = ab + ac$. This can also be written $(b + c)a = ab + ac$.

Factored form
 $a(b + c)$

Distributed form
 $a(b) + a(c)$

Simplified form
 $ab + ac$

To simplify: Multiply each term on the inside of the parentheses by the term on the outside.
Combine terms if possible.

Example 1

$$\begin{aligned}2(47) &= 2(40 + 7) \\ &= (2 \cdot 40) + (2 \cdot 7) \\ &= 80 + 14 \\ &= 94\end{aligned}$$

Example 2

$$\begin{aligned}3(x + 4) &= (3 \cdot x) + (3 \cdot 4) \\ &= 3x + 12\end{aligned}$$

Example 3

$$\begin{aligned}4(x + 3y + 1) &= (4 \cdot x) + (4 \cdot 3y) + 4(1) \\ &= 4x + 12y + 4\end{aligned}$$

Problems

Simplify each expression below by applying the Distributive Property.

- $6(9 + 4)$
- $4(9 + 8)$
- $7(8 + 6)$
- $5(7 + 4)$
- $3(27) = 3(20 + 7)$
- $6(46) = 6(40 + 6)$
- $8(43)$
- $6(78)$
- $3(x + 6)$
- $5(x + 7)$
- $8(x - 4)$
- $6(x - 10)$
- $(8 + x)4$
- $(2 + x)5$
- $-7(x + 1)$
- $-4(y + 3)$
- $-3(y - 5)$
- $-5(b - 4)$
- $-(x + 6)$
- $-(x + 7)$
- $-(x - 4)$
- $-(-x - 3)$
- $x(x + 3)$
- $4x(x + 2)$
- $-x(5x - 7)$
- $-x(2x - 6)$

Answers

- $(6 \cdot 9) + (6 \cdot 4) = 54 + 24 = 78$
- $(4 \cdot 9) + (4 \cdot 8) = 36 + 32 = 68$
- $56 + 42 = 98$
- $35 + 20 = 55$
- $60 + 21 = 81$
- $240 + 36 = 276$
- $320 + 24 = 344$
- $420 + 48 = 468$
- $3x + 18$
- $5x + 35$
- $8x - 32$
- $6x - 60$
- $4x + 32$
- $5x + 10$
- $-7x - 7$
- $-4y - 12$
- $-3y + 15$
- $-5b + 20$
- $-x - 6$
- $-x - 7$
- $-x + 4$
- $x + 3$
- $x^2 + 3x$
- $4x^2 + 8x$
- $-5x^2 + 7x$
- $-2x^2 + 6x$

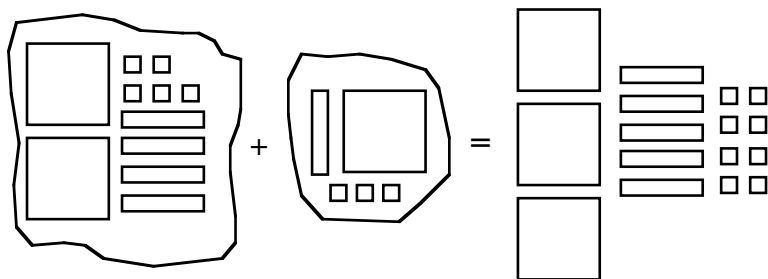
COMBINING LIKE TERMS

#11

LIKE TERMS are terms that are exactly the same except for their coefficients. Like terms can be combined into one quantity by adding and/or subtracting the coefficients of the terms. Terms are usually listed in the order of decreasing powers of the variable. Combining like terms using algebra tiles is shown in the first two examples.

Example 1

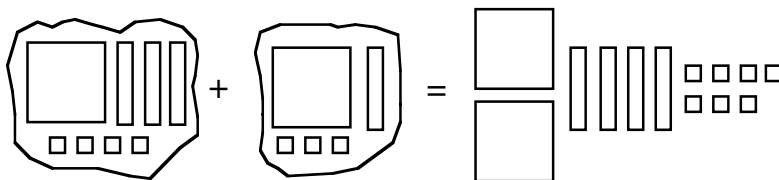
Simplify $(2x^2 + 4x + 5) + (x^2 + x + 3)$ means combine $2x^2 + 4x + 5$ with $x^2 + x + 3$.



$$(2x^2 + 4x + 5) + (x^2 + x + 3) = 3x^2 + 5x + 8.$$

Example 2

Simplify $(x^2 + 3x + 4) + (x^2 + x + 3)$.



$$(x^2 + 3x + 4) + (x^2 + x + 3) = 2x^2 + 4x + 7$$

Example 3

$$(4x^2 + 3x - 7) + (-2x^2 - 2x - 3) = 4x^2 + (-2x^2) + 3x + (-2x) - 7 + (-3) = 2x^2 + x - 10$$

Example 4

$$\begin{aligned} (-3x^2 - 2x + 5) - (-4x^2 + 7x - 6) &= -3x^2 - (-4x^2) - 2x - (7x) + 5 - (-6) \\ &= -3x^2 + 4x^2 - 2x - 7x + 5 + 6 = x^2 - 9x + 11 \end{aligned}$$

Problems

Combine like terms for each expression below.

- $(x^2 + 3x + 4) + (x^2 + 3x + 2)$
- $(x^2 + 4x + 3) + (x^2 + 2x + 5)$
- $(2x^2 + 2x + 1) + (x^2 + 4x + 5)$
- $(3x^2 + x + 7) + (3x^2 + 2x + 4)$
- $(2x^2 + 4x + 3) + (x^2 + 3x + 5)$
- $(4x^2 + 2x + 8) + (2x^2 + 5x + 1)$
- $(4x^2 + 2x + 8) + (3x^2 + 5x + 3)$
- $(3x^2 + 4x + 1) + (2x^2 + 4x + 5)$
- $(5x^2 + 4x - 7) + (3x^2 + 2x + 3)$
- $(3x^2 - 4x + 2) + (2x^2 + 2x + 4)$
- $(3x^2 - x + 2) + (4x^2 + 3x - 1)$
- $(2x^2 - 2x + 7) + (5x^2 + 4x - 3)$
- $(2x^2 - 3x - 3) + (5x^2 - 4x + 4)$
- $(3x^2 - 3x + 6) + (2x^2 - x - 4)$
- $(-4x^2 + x + 2) + (6x^2 - 3x + 2)$
- $(-3x^2 + 4x + 2) + (5x^2 - 6x - 1)$
- $(x^2 - 4) + (-x^2 + x - 3)$
- $(3x^2 + x) + (-2x^2 + 4)$
- $(3x^2 + 4) + (x^2 - 2x + 3)$
- $(-2x^2 - x) + (4x^2 - 3)$
- $(7x^2 - 2x + 3) - (3x^2 - 4x + 7)$
- $(x^2 - 3x - 2) - (4x^2 + 3x - 3)$
- $(8x^2 + 4x - 7) - (-4x^2 + 3x - 4)$
- $(-2x^2 + 14) - (3x^2 + 4x - 7)$

Answers

- $2x^2 + 6x + 6$
- $2x^2 + 6x + 8$
- $3x^2 + 6x + 6$
- $6x^2 + 3x + 11$
- $3x^2 + 7x + 8$
- $6x^2 + 7x + 9$
- $7x^2 + 7x + 11$
- $5x^2 + 8x + 6$
- $8x^2 + 6x - 4$
- $5x^2 - 2x + 6$
- $7x^2 + 2x + 1$
- $7x^2 + 2x + 4$
- $7x^2 - 7x + 1$
- $5x^2 - 4x + 2$
- $2x^2 - 2x + 4$
- $2x^2 - 2x + 1$
- $x - 7$
- $x^2 + x + 4$
- $4x^2 - 2x + 7$
- $2x^2 - x - 3$
- $4x^2 + 2x - 4$

DIVISION OF FRACTIONS USING AN AREA MODEL

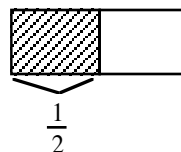
#12

Fractions can be divided using a rectangular area model. The division problem $8 \div 2$ means, "In 8, how many groups of 2 are there?" Similarly, $\frac{1}{2} \div \frac{1}{4}$ means, "In $\frac{1}{2}$, how many fourths are there?"

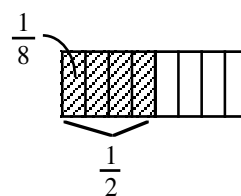
Example 1

Use the rectangular model to divide: $\frac{1}{2} \div \frac{1}{8}$.

Step 1: Using the rectangle, we first divide it into 2 equal pieces. Each piece represents $\frac{1}{2}$.
Shade $\frac{1}{2}$ of it.



Step 2: Then divide the original rectangle into eight equal pieces. Each section represents $\frac{1}{8}$.
In the shaded section, $\frac{1}{2}$, there are 4 eighths.



Step 3: Write the equation.

$$\frac{1}{2} \div \frac{1}{8} = 4$$

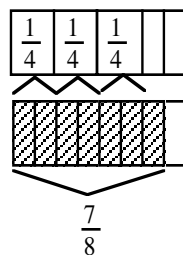
Example 2

In $\frac{7}{8}$, how many $\frac{1}{4}$ s are there?

That is, $\frac{7}{8} \div \frac{1}{4} = ?$



Start with $\frac{7}{8}$.



In $\frac{7}{8}$ there are three full $\frac{1}{4}$ s shaded and half of another one (that is, half of one-fourth).

So: $\frac{7}{8} \div \frac{1}{4} = 3\frac{1}{2}$
(three and one-half fourths)

Problems

Use the rectangular model to divide.

1. $1\frac{2}{3} \div \frac{1}{9}$ 2. $\frac{3}{2} \div \frac{1}{4}$ 3. $1 \div \frac{1}{5}$ 4. $1\frac{1}{8} \div \frac{1}{2}$ 5. $2\frac{1}{3} \div \frac{5}{6}$

Answers

1. 15 2. 6 3. 5 4. $\frac{9}{7}$ or $2\frac{1}{7}$ 5. $\frac{14}{7}$ or $2\frac{4}{7}$

DIVISION OF FRACTIONS USING RECIPROCAL

Two numbers that have a product of 1 are reciprocals. For example, $\frac{1}{3} \cdot \frac{3}{1} = 1$, $\frac{1}{8} \cdot \frac{8}{1} = 1$, and $\frac{1}{5} \cdot \frac{5}{1} = 1$, so $\frac{1}{3}$ and $\frac{3}{1}$, $\frac{1}{8}$ and $\frac{8}{1}$, and $\frac{1}{5}$ and $\frac{5}{1}$ are all pairs of reciprocals.

There is another way to divide fractions: invert the divisor, that is, write its reciprocal, then proceed as you do with multiplication. (The divisor is the number after the division sign.) After inverting the divisor, change the division sign to a multiplication sign and multiply. Simplify if possible.

Example 1

$$\frac{3}{8} \div \frac{1}{2} \Rightarrow \frac{3}{8} \cdot \frac{2}{1} \Rightarrow \frac{6}{8} = \frac{3}{4}$$

Example 2

$$1\frac{1}{5} \div \frac{1}{6} \Rightarrow \frac{6}{5} \cdot \frac{6}{1} \Rightarrow \frac{36}{5} \Rightarrow 7\frac{1}{5}$$

The examples above were written horizontally, but a division of fractions problem can also be written in the vertical form such as $\frac{\frac{1}{2}}{\frac{1}{3}}$, $\frac{\frac{1}{4}}{\frac{1}{2}}$, and $\frac{1\frac{1}{2}}{\frac{1}{6}}$. They still mean the same thing:

$\frac{\frac{1}{2}}{\frac{1}{3}}$ means, "In $\frac{1}{2}$, how many $\frac{1}{3}$ s are there?" $\frac{\frac{1}{4}}{\frac{1}{2}}$ means, "In $\frac{1}{4}$, how many $\frac{1}{2}$ s are there?"

$\frac{1\frac{1}{2}}{\frac{1}{6}}$ means, "In $1\frac{1}{2}$, how many $\frac{1}{6}$ s are there?"

You can use a Super Giant **1** to solve these vertical division problems. This Super Giant **1** uses the reciprocal of the divisor.

Example 3

$$\frac{\frac{1}{2}}{\frac{1}{3}} \cdot \boxed{\begin{array}{c} 3 \\ 1 \\ \hline 3 \\ 1 \end{array}} = \frac{\frac{3}{2}}{1} = \frac{3}{2} = 1\frac{1}{2}$$

Example 4

$$\frac{\frac{1}{4}}{\frac{1}{2}} \cdot \boxed{\begin{array}{c} 2 \\ 1 \\ \hline 2 \\ 1 \end{array}} = \frac{\frac{2}{4}}{1} = \frac{2}{4} = \frac{1}{2}$$

Example 5

$$1\frac{1}{2} \div \frac{1}{6} = \frac{3}{2} \div \frac{1}{6} = \frac{3}{2} \cdot \frac{6}{1} = \frac{18}{2} = \frac{18}{2} = 9$$

Example 6

$$\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \cdot \frac{3}{1} = \frac{6}{5} = 1\frac{1}{5}$$

Compared to:

$$\frac{2}{5} \cdot \frac{3}{1} = \frac{6}{5} = 1\frac{1}{5}$$

Problems

Solve these division problems. Use any method.

1. $\frac{3}{5} \div \frac{3}{8}$

2. $2\frac{1}{2} \div \frac{7}{8}$

3. $\frac{4}{5} \div \frac{2}{3}$

4. $1\frac{1}{5} \div \frac{3}{5}$

5. $\frac{6}{7} \div \frac{3}{8}$

6. $\frac{3}{10} \div \frac{5}{6}$

7. $1\frac{2}{7} \div \frac{1}{3}$

8. $7 \div \frac{1}{4}$

9. $1\frac{5}{9} \div \frac{2}{5}$

10. $3\frac{1}{3} \div \frac{5}{9}$

11. $2\frac{1}{3} \div \frac{1}{6}$

12. $2\frac{1}{2} \div \frac{3}{4}$

13. $\frac{7}{8} \div 1\frac{1}{4}$

14. $5\frac{1}{3} \div \frac{2}{9}$

15. $\frac{3}{5} \div 9$

Answers

1. $\frac{8}{5}$ or $1\frac{3}{5}$

2. $\frac{20}{7}$ or $2\frac{6}{7}$

3. $\frac{6}{5}$ or $1\frac{1}{5}$

4. 2

5. $\frac{16}{7}$ or $2\frac{2}{7}$

6. $\frac{9}{25}$

7. $\frac{27}{7}$ or $3\frac{6}{7}$

8. 28

9. $\frac{35}{9}$ or $3\frac{8}{9}$

10. 6

11. 14

12. $\frac{10}{3}$ or $3\frac{1}{3}$

13. $\frac{7}{10}$

14. 24

15. $\frac{1}{15}$

DRAWING A GRAPH FROM A TABLE

#13

One way to organize the points needed to graph an equation is to place them in an xy -table. In this course, linear equations will usually be written in y -form, such as $y = mx + b$. Make a table with rows for the x - and y -values. Choose some values for x . Substitute each x -value in the rule (the $mx + b$ part or the expression that is equal to y), evaluate, and record the result as the corresponding y -value. Select an appropriate scale for your axes and plot the graph.

Example

Complete a table to graph $y = 5x - 8$, then graph the equation.

x	-3	-2	-1	0	1	2	3	4	5
y									

- Make a table with x -values
- Each y -value is found by:
 - substituting the value for x .
 - multiplying it by 5.
 - then subtracting 8.

For Example:

$$\begin{aligned}y &= 5(-3) - 8 \\ &= -15 - 8 \\ &= -23\end{aligned}$$

The point $(-3, -23)$ is on the graph.

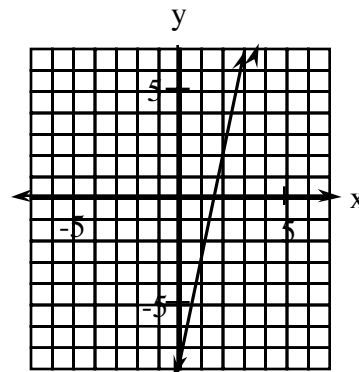
The completed table is shown below. Not all points are necessary to create a meaningful graph.

$$y = 5x - 8$$

x	-3	-2	-1	0	1	2	3	4	5
y	-23	-18	-13	-8	-3	2	7	12	17

- x -values may be referred to as inputs.
The set of all input values is the **domain**.
- y -values may be referred to as outputs.
The set of all output values is the **range**.

Use the pairs of xy -values in the table to graph the equation. A portion of the graph is shown at right.



Problems

Copy and complete each table. Graph each rule.

1. $y = 4x - 3$

x	-3	-2	-1	0	1	2	3
y							

2. $y = -x + 5$

x	-3	-2	-1	0	1	2	3
y							

3. $y = -2x - 3$

x	-3	-2	-1	0	1	2	3
y							

5. $y = \frac{1}{2}x - 3$

x	-5	-1	0	1	2	3	6
y							

7. $y = \frac{1}{5}x + 3$

x	-6	-5	-2	0	2	4	5
y							

9. $y = -2x + 4$

x	-2	-1	0	1	2	3	4
y							

11. $y = x^2 - 5$

x	-3	-2	-1	0	1	2	3
y							

13. $y = x^2 - 3x - 2$

x	-2	-1	0	1	2	3	4	5
y								

4. $y = -5x - 0.5$

x	-3	-2	-1	0	1	2	3
y							

6. $y = \frac{3}{2}x - 4$

x	-4	-2	0	1	2	4	6
y							

8. $y = -\frac{3}{4}x - 2$

x	-4	-2	0	1	2	3	4
y							

10. $y = -\frac{2}{3}x + 5$

x	-6	-3	0	1	3	6	9
y							

12. $y = -x^2 + 1$ (Careful! Square first, then change the sign.)

x	-3	-2	-1	0	1	2	3
y							

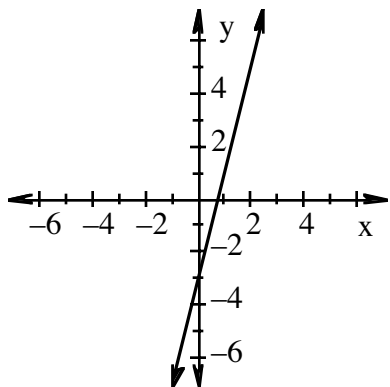
14. $y = x^2 - 4x - 2$

x	-1	0	1	2	3	4	5
y							

Answers

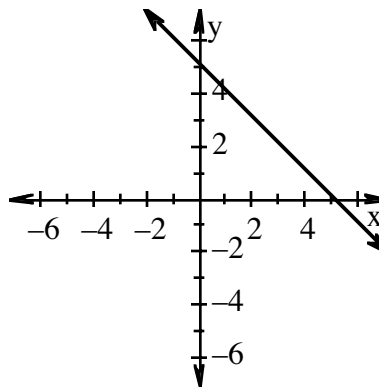
1. $y = 4x - 3$

x	-3	-2	-1	0	1	2	3
y	-15	-11	-7	-3	1	5	9



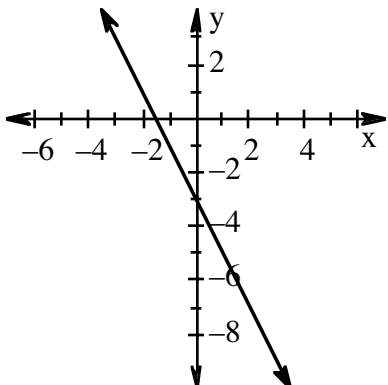
2. $y = -x + 5$

x	-3	-2	-1	0	1	2	3
y	8	7	6	5	4	3	2



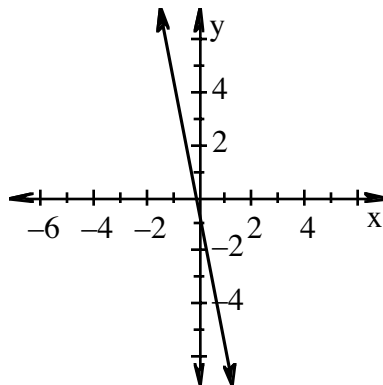
3. $y = -2x - 3$

x	-3	-2	-1	0	1	2	3
y	3	1	-1	-3	-5	-7	-9



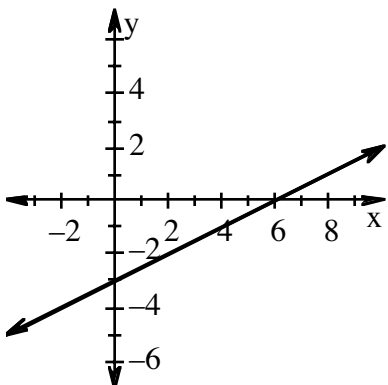
4. $y = -5x - 0.5$

x	-3	-2	-1	0	1	2	3
y	14.5	9.5	4.5	-0.5	-5.5	-10.5	-15.5



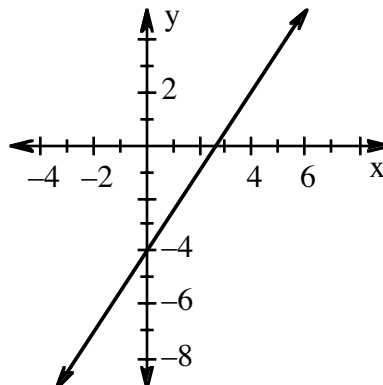
5. $y = \frac{1}{2}x - 3$

x	-5	-1	0	1	2	3	-6
y	-5.5	-3.5	-3	-2.5	-2	-1.5	0



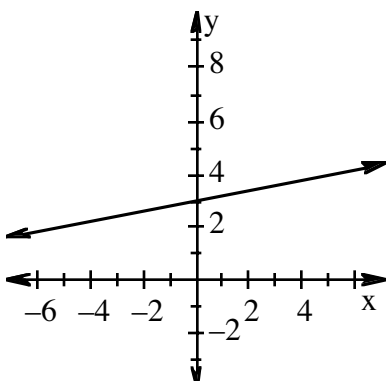
6. $y = \frac{3}{2}x - 4$

x	-4	-2	0	1	2	4	6
y	-10	-7	-4	-2.5	-1	2	5



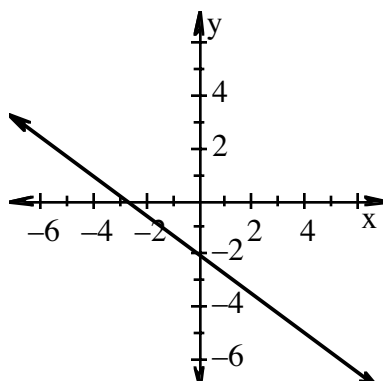
7. $y = \frac{1}{5}x + 3$

x	-6	-5	-2	0	2	4	5
y	1.8	2	2.6	3	3.4	3.8	4



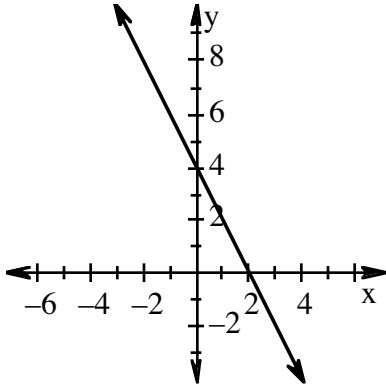
8. $y = -\frac{3}{4}x - 2$

x	-4	-2	0	1	2	3	4
y	1	-0.5	-2	-2.75	-3.5	-4.25	-5



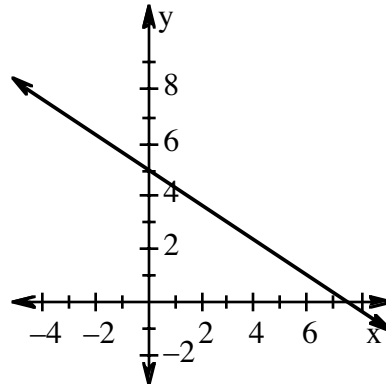
9. $y = -2x + 4$

x	-2	-1	0	1	2	3	4
y	8	6	4	2	0	-2	-4



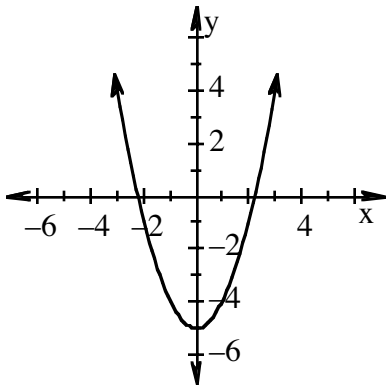
10. $y = -\frac{2}{3}x + 5$

x	-6	-3	0	1	3	6	9
y	9	7	5	4.33	3	1	-1



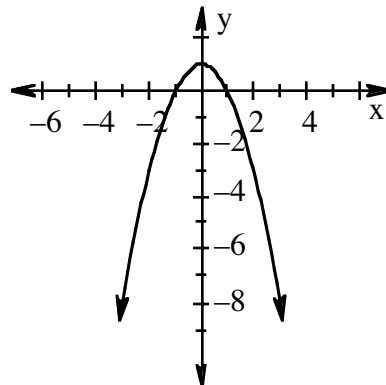
11. $y = x^2 - 5$

x	-3	-2	-1	0	1	2	3
y	4	-1	-4	-5	-4	-1	4



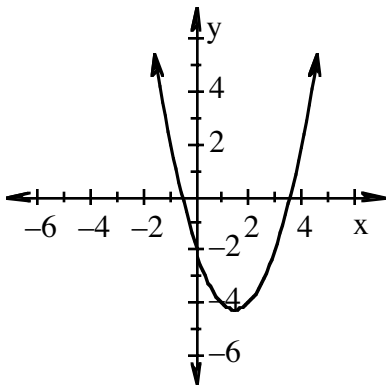
12. $y = -x^2 + 1$

x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8



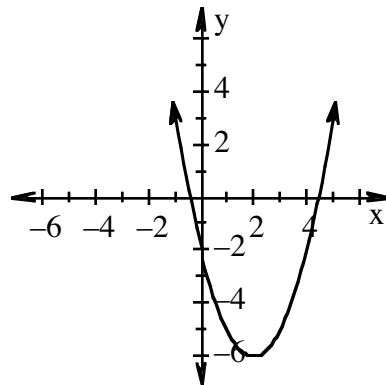
13. $y = x^2 - 3x - 2$

x	-2	-1	0	1	2	3	4	5
y	8	2	-2	-4	-4	-2	2	8



14. $y = x^2 - 4x - 2$

x	-1	0	1	2	3	4	5
y	3	-2	-5	-6	-5	-2	3



EQUIVALENT FRACTIONS

#14

Fractions that name the same value are called equivalent fractions, such as $\frac{2}{3} = \frac{6}{9}$.

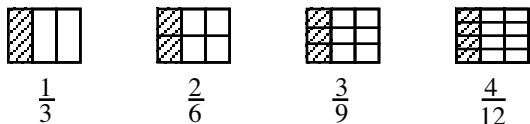
Three methods for finding equivalent fractions are using a ratio table, a rectangular area model, and the Identity Property of Multiplication (the Giant **1**). The ratio table method is discussed in the "Ratio Applications" skill builder on page 73.

RECTANGULAR AREA MODEL

This method for finding equivalent fractions is based on the fact that the area of a rectangle is the same no matter how it is dissected (cut up). Draw, divide (using vertical lines), and shade a rectangle to represent the original fraction. Next, add horizontal lines to the rectangle to divide the area equally so that the rectangle has the same number of equal pieces as the number in the denominator of the second fraction. Note that each rectangle has the same amount of shaded area. Renaming the shaded area in terms of the new, smaller pieces gives the equivalent fraction.

Example 1

Use the rectangular area model to find three equivalent fractions for $\frac{1}{3}$.

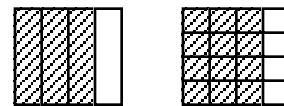


The horizontal line in the second figure creates two rows, which is the same as using a Giant **1**, $\frac{2}{2}$. The same idea is used in the third and fourth figures, $\frac{3}{3}$ and $\frac{4}{4}$.

Example 2

Use the rectangular area model to find the specified equivalent fraction.

$$\frac{3}{4} \cdot 1 = \frac{\quad}{16}$$



$$\frac{3}{4} \cdot \frac{4}{4} = \frac{12}{16}$$

After drawing the fraction $\frac{3}{4}$, the diagram is divided into four horizontal rows because $16 \div 4$ equals 4. The diagram now shows 12 shaded parts out of 16 total parts. This area model shows the equivalent fractions: $\frac{3}{4} = \frac{12}{16}$.

Problems

Draw rectangular models to find the specified equivalent fraction.

1. $\frac{4}{7} = \frac{\quad}{21}$


2. $\frac{7}{8} = \frac{\quad}{24}$

3. $\frac{4}{9} = \frac{\quad}{36}$

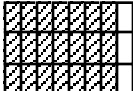
4. $\frac{5}{3} = \frac{\quad}{9}$

Answers


1. $\frac{12}{21}$



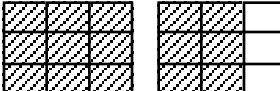
2. $\frac{21}{24}$



3. $\frac{16}{36}$



4. $\frac{15}{9}$



THE IDENTITY PROPERTY OF MULTIPLICATION or THE GIANT 1

Multiplying by 1 does not change the value of a number. The Giant **1** uses a fraction that has the same numerator and denominator, such as $\frac{2}{2}$, to find an equivalent fraction.

Example 1

Find three equivalent fractions for $\frac{1}{3}$.

$$\frac{1}{3} \cdot \boxed{\frac{2}{2}} = \frac{2}{6}$$

$$\frac{1}{3} \cdot \boxed{\frac{3}{3}} = \frac{3}{9}$$

$$\frac{1}{3} \cdot \boxed{\frac{4}{4}} = \frac{4}{12}$$

Example 2

Use the Giant **1** to find an equivalent fraction to $\frac{5}{12}$ using 48ths: $\frac{5}{12} \cdot \boxed{1} = \frac{\square}{48}$

Since $48 \div 12 = 4$, the Giant **1** is $\frac{4}{4}$: $\frac{5}{12} \cdot \boxed{\frac{4}{4}} = \frac{20}{48}$

Problems

Use the Giant **1** to find the specified equivalent fraction. Your answer should include the Giant **1** you use and the equivalent numerator.

$$1. \quad \frac{5}{3} \cdot \underline{\quad} = \frac{?}{21}$$

$$2. \quad \frac{7}{9} \cdot \underline{\quad} = \frac{?}{45}$$

$$3. \quad \frac{9}{5} \cdot \underline{\quad} = \frac{?}{45}$$

$$4. \quad \frac{3}{4} \cdot \underline{\quad} = \frac{?}{24}$$

$$5. \quad \frac{7}{6} \cdot \underline{\quad} = \frac{?}{30}$$

$$6. \quad \frac{8}{5} \cdot \underline{\quad} = \frac{?}{30}$$

Answers

$$1. \quad \frac{7}{7}, 35$$

$$2. \quad \frac{5}{5}, 35$$

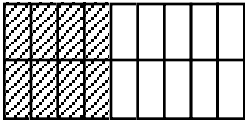
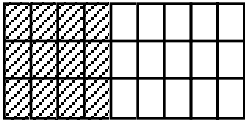
$$3. \quad \frac{9}{9}, 81$$

$$4. \quad \frac{6}{6}, 18$$

$$5. \quad \frac{5}{5}, 35$$

$$6. \quad \frac{6}{6}, 48$$

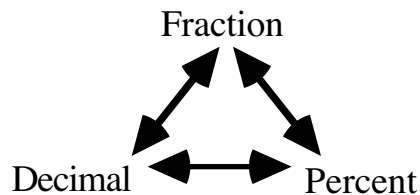
The following table summarizes the three methods for finding equivalent fractions.

Fraction	Ratio Table	Giant 1	Rectangular Model			
$\frac{4}{9}$	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td style="padding: 5px;">$\frac{4}{9}$</td> <td style="padding: 5px;">$\frac{8}{18}$</td> <td style="padding: 5px;">$\frac{12}{27}$</td> </tr> </table>	$\frac{4}{9}$	$\frac{8}{18}$	$\frac{12}{27}$	$\frac{4}{9} \cdot \underline{\frac{2}{2}} = \frac{8}{18}$	 $\frac{4}{9} = \frac{8}{18}$
		$\frac{4}{9}$	$\frac{8}{18}$	$\frac{12}{27}$		
$\frac{4}{9} \cdot \underline{\frac{3}{3}} = \frac{12}{27}$	 $\frac{4}{9} = \frac{12}{27}$					

FRACTION, DECIMAL, AND PERCENT EQUIVALENTS

#15

Fractions, decimals, and percents are different ways to represent the same number.



Examples

Decimal to percent:

Multiply the decimal by 100.

$$(0.27)(100) = 27\%$$

Percent to decimal:

Divide the percent by 100.

$$47.3\% \div 100 = 0.473$$

Fraction to percent:

Write a proportion to find an equivalent fraction using 100 as the denominator. The numerator is the percent.

$$\frac{3}{5} = \frac{x}{100} \quad \text{so} \quad \frac{3}{5} = \frac{60}{100} = 60\%$$

Percent to fraction:

Use 100 as the denominator. Use the percent as the numerator. Simplify as needed.

$$24\% = \frac{24}{100} = \frac{6}{25}$$

Decimal to fraction:

Use the decimal as the numerator. Use the decimal place value name as the denominator. Simplify as needed.

$$\text{a) } 0.4 = \frac{4}{10} = \frac{2}{5} \quad \text{b) } 0.37 = \frac{37}{100}$$

Fraction to decimal:

Divide the numerator by the denominator.

$$\frac{5}{8} = 5 \div 8 = 0.625$$

Problems

Convert the fraction, decimal, or percent as indicated.

1. Change $\frac{1}{5}$ to a decimal.
2. Change 40% to a fraction in lowest terms.
3. Change 0.54 to a fraction in lowest terms.
4. Change 35% to a decimal.
5. Change 0.43 to a percent.
6. Change $\frac{1}{4}$ to a percent.
7. Change 0.7 to a fraction.
8. Change $\frac{3}{8}$ to a decimal.
9. Change $\frac{2}{3}$ to a decimal.
10. Change 0.07 to a percent.
11. Change 67% to a decimal.
12. Change $\frac{4}{5}$ to a percent.
13. Change 0.6 to a fraction in lowest terms.
14. Change 85% to a fraction in lowest terms.
15. Change $\frac{2}{9}$ to a decimal.
16. Change 135% to a fraction in lowest terms.
17. Change $\frac{9}{5}$ to a decimal.
18. Change 5.25 to a percent.
19. Change $\frac{1}{18}$ to a decimal, then change the decimal to a percent.
20. Change 47% to a fraction, then change the fraction to a decimal.
21. Change $\frac{3}{7}$ to a decimal.
22. Change 0.625 to a percent.
23. Change $\frac{5}{8}$ to a decimal, then change the decimal to a percent.
24. Change 45% to a decimal, then change the decimal to a fraction.

Answers

- | | | | |
|---------------------|-----------------------------------|-------------------------------------|----------------------------|
| 1. 0.20 | 2. $\frac{40}{100} = \frac{2}{5}$ | 3. $\frac{54}{100} = \frac{27}{50}$ | 4. 0.35 |
| 5. 43% | 6. 25% | 7. $\frac{7}{10}$ | 8. 0.325 |
| 9. $0.\overline{6}$ | 10. 7% | 11. 0.67 | 12. 80% |
| 13. $\frac{3}{5}$ | 14. $\frac{17}{50}$ | 15. $0.\overline{2}$ | 16. $\frac{27}{20}$ |
| 17. 1.8 | 18. 525% | 19. $0.0\overline{5}$; 5.6% | 20. $\frac{47}{100}$; 47% |
| 21. 0.429 | 22. 62.5% | 23. 0.625; 62.5% | 24. 0.45; $\frac{9}{20}$ |

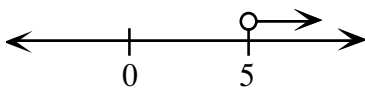
GRAPHING INEQUALITIES

#16

The solution(s) to an equation can be represented as a point (or points) on the number line. The solutions to inequalities are represented by rays or segments with solid or open endpoints. Solid endpoints indicate that the endpoint is included in the solution (\leq or \geq), while the open dot indicates that it is not part of the solution ($<$ or $>$).

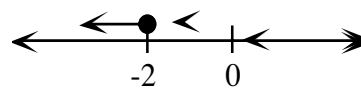
Example 1

$$x > 5$$



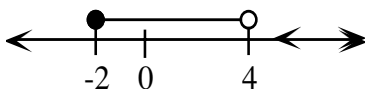
Example 2

$$x \leq -2$$



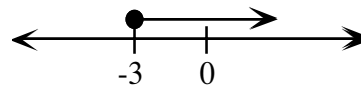
Example 3

$$-2 \leq m < 4$$



Example 4

$$q \geq -3$$



Problems

Graph each inequality on a number line.

- $m < 4$
- $x \leq -3$
- $y \geq 2$
- $x \leq 5$
- $-8 < x < -4$
- $-2 < x \leq 3$
- $m > -7$
- $x \neq 4$
- $-2 \leq x \leq 2$
- $x \geq -2$

Answers

-
-
-
-
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-
-
-

LAWS OF EXPONENTS

#17

In the expression 5^3 , 5 is the **base** and 3 is the **exponent**. For x^a , x is the base and a is the exponent. 5^3 means $5 \cdot 5 \cdot 5$. 5^4 means $5 \cdot 5 \cdot 5 \cdot 5$, so you can write $\frac{5^7}{5^4}$ (which means $5^7 \div 5^4$) or you can write it as: $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5}$.

You can use the Giant **1** to find the matching pairs of numbers in the numerator and denominator. There are four Giant **1**s, namely, $\frac{5}{5}$ four times so $\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5} = 5^3$ or 125. Writing 5^3 is usually sufficient.

When there is a variable, it is treated the same way. $\frac{x^6}{x^2}$ means $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$. The Giant **1** here is $\frac{x}{x}$ (two of them). The answer is x^4 .

$5^2 \cdot 5^3$ means $(5 \cdot 5)(5 \cdot 5 \cdot 5)$ which is 5^5 .

$(5^2)^3$ means $(5^2)(5^2)(5^2)$ or $(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$ which is 5^6 .

When the problems have variables such as $x^3 \cdot x^5$, you only need to add the exponents. The answer is x^8 . If the problem is $(x^3)^5$ (x^3 to the fifth power) it means $x^3 \cdot x^3 \cdot x^3 \cdot x^3 \cdot x^3$. The answer is x^{15} . You multiply exponents in this case.

If the problem is $\frac{x^8}{x^3}$, you subtract the bottom exponent from the top exponent ($8 - 3$).

The answer is x^5 . You can also have problems like $\frac{x^8}{x^{-3}}$. You still subtract, $8 - (-3)$ is 11, and the answer is x^{11} .

You need to be sure the bases are the same to use these laws. For example, $x^4 \cdot y^5$ cannot be further simplified, since x and y are not the same base.

In general, the **LAWS OF EXPONENTS**, where $x \neq 0$, are:

$$x^a \cdot x^b = x^{(a+b)}$$

$$(x^a)^b = x^{ab}$$

$$\frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1$$

$$(x^a y^b)^c = x^{ac} y^{bc}$$

Examples

a) $x^6 \cdot x^5 = x^{6+5} = x^{11}$

b) $\frac{x^{18}}{x^{12}} = x^{18-12} = x^6$

c) $(z^5)^3 = z^{5 \cdot 3} = z^{15}$

d) $(x^2 y^3)^5 = x^{2 \cdot 5} y^{3 \cdot 5} = x^{10} y^{15}$

e) $\frac{x^5}{x^{-2}} = x^{5-(-2)} = x^7$

f) $(3x^2 y^2)^2 = 3^2 x^{2 \cdot 2} y^{2 \cdot 2} = 9x^4 y^4$

g) $(3x^3 y^{-2})^3 = 3^3 x^{3 \cdot 3} y^{-2 \cdot 3} = 27x^9 y^{-6}$ or $\frac{27x^9}{y^6}$

h) $\frac{x^7 y^5 z^3}{x^3 y^6 z^{-2}} = x^{7-3} y^{5-6} z^{3-(-2)} = x^4 y^{-1} z^5$ or $\frac{x^4 z^5}{y}$

Problems

Simplify each expression.

1. $5^3 \cdot 5^4$

2. $x^3 \cdot x^5$

3. $\frac{5^{18}}{5^{14}}$

4. $\frac{x^{11}}{x^4}$

5. $(5^4)^3$

6. $(x^5)^3$

7. $(2x^3 y^4)^4$

8. $\frac{5^4}{5^{-4}}$

9. $5^6 \cdot 5^{-3}$

10. $(y^2)^{-4}$

11. $(3a^4 b^{-3})^3$

12. $\frac{x^7 y^5 z^2}{x^4 y^3 z^2}$

13. $\frac{x^8 y^4 z^4}{x^{-2} y^3 z^{-1}}$

14. $5x^3 \cdot 3x^4$

Answers

1. 5^7

2. x^8

3. 5^4

4. x^7

5. 5^{12}

6. x^{15}

7. $16x^{12} y^{16}$

8. 5^8

9. 5^3

10. y^{-8} or $\frac{1}{y^8}$

11. $27a^{12} b^{-9}$ or $\frac{27a^{12}}{b^9}$

12. $x^3 y^2$

13. $x^{10} yz^5$

14. $15x^7$

MEASURES OF CENTRAL TENDENCY

#18

The **MEASURES OF CENTRAL TENDENCY** are numbers that locate or approximate the “center” of a set of data. Mean, median, and mode are the most common measures of central tendency.

The **MEAN** is the arithmetic average of a data set. Add all the values in a set and divide this sum by the number of values in the set.

The **MODE** is the value in a data set that occurs most often. Data sets may have one mode, more than one mode, or no mode.

The **MEDIAN** is the middle number in a set of data arranged in numerical order. If there are an even number of values, the median is the mean of the two middle numbers.

The **RANGE** of a set of data is the difference between the highest value and the lowest value.

Example 1

Find the mean of this set of data: 23, 31, 46, 23, 38, 47, 23.

- Add the numbers: $23 + 31 + 46 + 23 + 38 + 47 + 23 = 231$
- Divide by the number of values: $231 \div 7 = 33$

The mean of this set of data is 33.

Example 2

Find the mean of this set of data: 82, 72, 70, 82, 68, 65, 85, 64.

- Add the numbers: $82 + 72 + 70 + 82 + 68 + 65 + 85 + 64 = 588$
- Divide by the number of values: $588 \div 8 = 73.5$

The mean of this set of data is 73.5.

Example 3

Find the mode of this set of data: 34, 31, 36, 34, 37, 38, 42, 34.

34 is the mode of this set of data since there are three 34s and only one of each of the other numbers.

Example 4

Find the mode of this set of data: 23, 46, 23, 47, 48, 46, 23, 46.

There are two modes for this data, 23 and 46. Since there are two modes, this data set is said to be **bimodal**.

Example 5

Find the median of this set of data: 23, 34, 25, 37, 35, 22, 30.

- Arrange the data in order: 22, 23, 25, 30, 34, 35, 37
- Find the middle value: 30, since there are the same number of values on either side.

Therefore, the median of this data set is 30.

Example 6

Find the median of this set of data: 52, 54, 58, 42, 53, 51, 25, 28.

- Arrange the data in numerical order: 25, 28, 42, 51, 52, 53, 54, 58.
- Find the middle value(s): 51 and 52, since there are three values on either side.
- Since there are two middle values, find their mean: $51 + 52 = 103$, so $103 \div 2 = 51.5$

The median of this data set is 51.5

Example 7

Find the range of this set of data: 15, 46, 13, 23, 32, 40, 38, 18, 27, 16.

- The highest value is 46.
- The lowest value is 13.
- Subtract the lowest value from the highest: $46 - 13 = 33$

The range of this set of data is 33.

Problems

Identify the mean, median, mode and range for each set of data

1. 10, 8, 7, 9, 12, 14, 9, 9, 10
2. 21, 44, 32, 27, 38, 36, 32, 45, 47, 40, 23
3. 68, 55, 53, 55, 64, 60, 35, 42, 47, 46
4. 120, 88, 74, 82, 78, 80, 100, 110, 74, 78
5. 82, 83, 84, 82, 77, 82, 77, 70, 70, 77, 70, 85
6. 18, 32, 37, 42, 56, 78, 82, 95, 100, 7

Answers

- | | | | | | |
|----|------------------------|-------------------------|----|---------------------------------|---------------------------|
| 1. | Mean: 9.78
Mode: 9 | Median: 9
Range: 7 | 4. | Mean: 88.4
Mode: 74, 78 | Median: 81
Range: 46 |
| 2. | Mean: 35
Mode: 32 | Median: 36
Range: 26 | 5. | Mean: 78.25
Mode: 70, 77, 82 | Median: 79.5
Range: 15 |
| 3. | Mean: 52.5
Mode: 55 | Median: 54
Range: 33 | 6. | Mean: 54.7
Mode: None | Median: 49
Range: 93 |

MULTIPLICATION OF FRACTIONS WITH AN AREA MODEL #19

When multiplying fractions using a rectangular area model, lines that represent one fraction are drawn vertically and the correct number of parts are shaded. Then lines that represent the second fraction are drawn horizontally and part of the shaded region is darkened that represents the product of the two fractions.

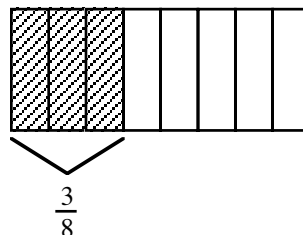
The rule for multiplying fractions derived from the area model is to multiply the numerators, then multiply the denominators. Simplify the product when possible. In general,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

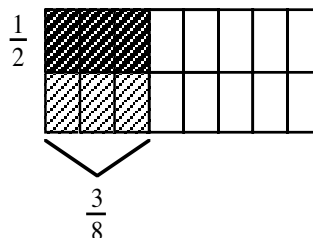
Example 1

$$\frac{1}{2} \cdot \frac{3}{8} \text{ (that is, } \frac{1}{2} \text{ of } \frac{3}{8} \text{)}$$

Step 1: Draw a unit rectangle and divide it into 8 pieces vertically. Lightly shade 3 of those pieces. Label it $\frac{3}{8}$.



Step 2: Use a horizontal line and divide the unit rectangle in half. Darkly shade $\frac{1}{2}$ of $\frac{3}{8}$ and label it.



Step 3: Write an equation.

$$\frac{1}{2} \cdot \frac{3}{8} = \frac{3}{16}$$

Example 2

$$\text{a) } \frac{2}{3} \cdot \frac{1}{5} \Rightarrow \frac{2 \cdot 1}{3 \cdot 5} \Rightarrow \frac{2}{15}$$

$$\text{b) } \frac{3}{5} \cdot \frac{5}{9} \Rightarrow \frac{3 \cdot 5}{5 \cdot 9} \Rightarrow \frac{15}{45} \Rightarrow \frac{1}{3}$$

Problems

Draw an area model for each of the following multiplication problems and write the answer.

1. $\frac{1}{3} \cdot \frac{5}{6}$

2. $\frac{3}{4} \cdot \frac{2}{5}$

3. $\frac{1}{3} \cdot \frac{4}{9}$

Use the rule for multiplying fractions to find each product. Simplify when possible.

4. $\frac{2}{3} \cdot \frac{2}{7}$

5. $\frac{2}{3} \cdot \frac{2}{5}$

6. $\frac{3}{7} \cdot \frac{2}{5}$

7. $\frac{4}{9} \cdot \frac{1}{3}$

8. $\frac{2}{3} \cdot \frac{5}{8}$

9. $\frac{5}{7} \cdot \frac{2}{5}$

10. $\frac{4}{7} \cdot \frac{3}{4}$

11. $\frac{5}{12} \cdot \frac{2}{3}$

12. $\frac{4}{7} \cdot \frac{1}{2}$

13. $\frac{5}{8} \cdot \frac{4}{5}$

14. $\frac{5}{9} \cdot \frac{3}{5}$

15. $\frac{7}{10} \cdot \frac{2}{7}$

16. $\frac{5}{12} \cdot \frac{6}{9}$

17. $\frac{5}{6} \cdot \frac{3}{20}$

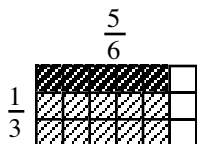
18. $\frac{10}{13} \cdot \frac{3}{5}$

19. $\frac{7}{12} \cdot \frac{3}{7}$

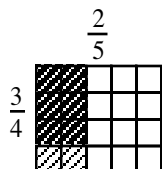
20. $\frac{7}{10} \cdot \frac{5}{14}$

Answers

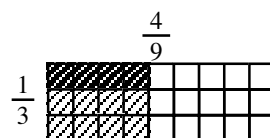
1. $\frac{5}{18}$



2. $\frac{6}{20} = \frac{3}{10}$



3. $\frac{4}{27}$



4. $\frac{4}{21}$

5. $\frac{4}{15}$

6. $\frac{6}{35}$

7. $\frac{4}{27}$

8. $\frac{10}{24} = \frac{5}{12}$

9. $\frac{10}{35} = \frac{2}{7}$

10. $\frac{12}{28} = \frac{3}{7}$

11. $\frac{10}{36} = \frac{5}{18}$

12. $\frac{4}{14} = \frac{2}{7}$

13. $\frac{20}{40} = \frac{1}{2}$

14. $\frac{15}{45} = \frac{1}{3}$

15. $\frac{14}{70} = \frac{1}{5}$

16. $\frac{30}{108} = \frac{5}{18}$

17. $\frac{15}{120} = \frac{1}{8}$

18. $\frac{30}{65} = \frac{6}{13}$

19. $\frac{21}{84} = \frac{1}{4}$

20. $\frac{35}{140} = \frac{1}{4}$

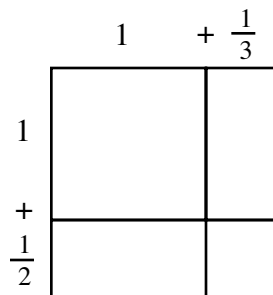
MULTIPLICATION OF MIXED NUMBERS

There are two ways to multiply mixed numbers. One is with generic rectangles. You can also multiply mixed numbers by changing them to fractions greater than 1, then multiplying the numerators and multiplying the denominators. Simplify if possible.

Example 1

Find the product: $1\frac{1}{2} \cdot 1\frac{1}{3}$.

Step 1: Draw the generic rectangle. Label the top $1 + \frac{1}{3}$. Label the side $1 + \frac{1}{2}$.

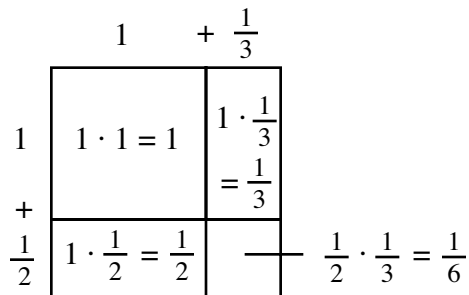


Step 2: Write the area of each smaller rectangle in each of the four parts of the drawing.

Find the total area:

$$1 + \frac{1}{3} + \frac{1}{2} + \frac{1}{6} \Rightarrow 1 + \frac{2}{6} + \frac{3}{6} + \frac{1}{6}$$

$$\Rightarrow 1\frac{6}{6} \Rightarrow 2$$

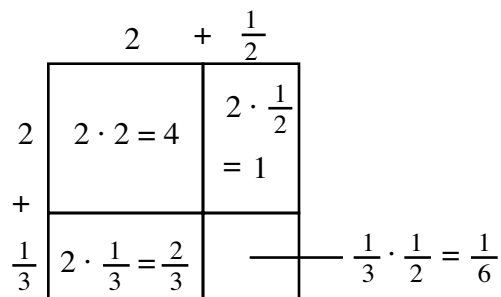


Step 3: Write an equation: $1\frac{1}{2} \cdot 1\frac{1}{3} = 2$

Example 2

Find the product: $2\frac{1}{3} \cdot 2\frac{1}{2}$.

$$4 + 1 + \frac{2}{3} + \frac{1}{6} \Rightarrow 5 + \frac{4}{6} + \frac{1}{6} \Rightarrow 5\frac{5}{6}$$



Problems

Use a generic rectangle to find each product.

1. $2\frac{1}{4} \cdot 1\frac{1}{2}$ 2. $2\frac{1}{6} \cdot 2\frac{1}{3}$ 3. $2\frac{3}{4} \cdot 2\frac{1}{2}$ 4. $1\frac{1}{3} \cdot 2\frac{1}{6}$ 5. $2\frac{1}{2} \cdot 1\frac{1}{6}$

Answers

1. $\frac{27}{8}$ or $2\frac{3}{8}$ 2. $\frac{91}{18}$ or $5\frac{1}{18}$ 3. $\frac{55}{8}$ or $6\frac{7}{8}$ 4. $\frac{52}{18}$ or $2\frac{16}{18}$
or $2\frac{8}{9}$ 5. $\frac{35}{12}$ or $2\frac{11}{12}$

	$2 + \frac{1}{4}$		$2 + \frac{1}{3}$		$2 + \frac{1}{2}$		$2 + \frac{1}{6}$		$1 + \frac{1}{6}$																							
1	2	+	$\frac{1}{4}$	2	4	+	$\frac{2}{3}$	2	4	+	1	1	2	+	$\frac{1}{6}$	2	2	+	$\frac{2}{6} = \frac{1}{3}$													
+		$\frac{1}{2}$		+		$\frac{1}{6}$		+		$\frac{3}{4}$		+		$\frac{1}{3}$		+		$\frac{1}{2}$		+		$\frac{1}{2}$		$\frac{1}{6}$		$\frac{1}{2}$		$\frac{1}{18}$		$\frac{1}{2}$		$\frac{1}{12}$
	1		$\frac{1}{8}$		$\frac{2}{6}$		$\frac{1}{18}$		$\frac{6}{4}$		$\frac{3}{8}$		$\frac{2}{3}$		$\frac{1}{18}$		$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{12}$											

Example 3

$$1\frac{1}{2} \cdot 1\frac{1}{3} \Rightarrow \frac{3}{2} \cdot \frac{4}{3} \Rightarrow \frac{3 \cdot 4}{2 \cdot 3} \Rightarrow \frac{12}{6} \Rightarrow 2$$

Problems

Find each product, using the method of your choice. Simplify when possible.

1. $2\frac{1}{4} \cdot 2\frac{5}{8}$ 2. $2\frac{1}{5} \cdot 2\frac{1}{5}$ 3. $1\frac{3}{8} \cdot 1\frac{5}{6}$ 4. $2\frac{5}{9} \cdot 2\frac{5}{6}$
5. $1\frac{2}{7} \cdot 1\frac{1}{7}$ 6. $2\frac{4}{7} \cdot 3\frac{8}{9}$ 7. $2\frac{3}{7} \cdot 1\frac{1}{12}$ 8. $2\frac{7}{9} \cdot 2\frac{4}{5}$
9. $2\frac{1}{3} \cdot 1\frac{2}{7}$ 10. $2\frac{2}{5} \cdot 2\frac{3}{10}$

Answers

1. $\frac{189}{32}$ or $5\frac{29}{32}$ 2. $\frac{121}{25}$ or $4\frac{21}{25}$ 3. $\frac{121}{48}$ or $2\frac{25}{48}$ 4. $\frac{391}{54}$ or $7\frac{13}{54}$
5. $\frac{72}{49}$ or $1\frac{23}{49}$ 6. 10 7. $\frac{221}{84}$ or $2\frac{53}{84}$ 8. $\frac{70}{9}$ or $7\frac{7}{9}$
9. 3 10. $\frac{138}{25}$ or $5\frac{13}{25}$

ORDER OF OPERATIONS

#20

The **ORDER OF OPERATIONS** establishes the necessary rules so that expressions are evaluated in a consistent way by everyone.

1. Circle the terms in the expression. A term is each part (a number, a variable, a product or a quotient of numbers and variables) of the expression that is separated by addition (+) or subtraction (-) symbols unless the sum or difference is inside parentheses.
2. Simplify each term until it is one number by:
 - evaluating each exponential number.
 - performing the operations inside the parentheses.
 - multiplying and dividing from left to right.
3. Finally, perform all addition and subtraction from left to right.

Example 1

- Circle the terms.
- Simplify each term until it is one number.
- Add the terms going from left to right.

$$7 + 3 \cdot 8$$

$$(7) + (3 \cdot 8)$$

$$7 + 24$$

$$31$$

Example 2

- Circle the terms.
- Simplify each term until it is one number.
 - Evaluate 2^2 .
 - Subtract 2 from 5.
 - Multiply within each term, left to right.
 - Add the numbers.

$$2^2 \cdot 4 + 4(5 - 2) + 7$$

$$(2^2 \cdot 4) + (4(5 - 2)) + (7)$$

$$(4 \cdot 4) + (4(3)) + (7)$$

$$16 + 12 + 7$$

$$35$$

Example 3

- Circle the terms.
- Simplify each term until it is one number.
 - Evaluate 3^2 first.
 - Add $4 + 3$ in the parentheses.
 - Multiply and divide left to right in each term.
 - Add and subtract the numbers from left to right.

$$7 - 9 \div 3^2 + 4(4 + 3) - 7$$

$$(7) - (9 \div 3^2) + (4(4 + 3)) - (7)$$

$$(7) - (9 \div 9) + (4(7)) - (7)$$

$$(7) - (1) + (28) - (7)$$

$$27$$

Example 4

- Circle the terms.
- Simplify each term until it is one number.
 - Subtract the numerator.
 - Evaluate 3^2 .
 - Divide.
 - Add or subtract the numbers from left to right.

$$18 + \frac{12-2}{5} - 3^2 + 18 \div 6$$
$$\textcircled{18} + \textcircled{\frac{12-2}{5}} - \textcircled{3^2} + \textcircled{18 \div 6}$$
$$\textcircled{18} + \textcircled{\frac{10}{5}} - \textcircled{9} + \textcircled{3}$$
$$18 + 2 - 9 + 3$$
$$14$$

Problems

Circle the terms, then simplify each expression.

- $7 \cdot 3 + 5$
- $8 \div 4 + 3$
- $2(12 - 4) + 4$
- $4(9 + 3) + 10 \div 2$
- $24 \div 3 + 7(9 + 1) - 4$
- $\frac{12}{3} + 5 \cdot 4^2 - 2(12 - 5)$
- $\frac{20}{3+2} + 9 \cdot 2 \div 3$
- $\frac{4+24}{7} + 5^2 - 27 \div 9$
- $3^2 + 8 - 16 \div 4^2 \cdot 2$
- $16 - 4^2 + 4 - 2^2$
- $5(19 - 3^2) + 5 \cdot 3 - 7$
- $(6 - 2)^2 + (8 + 1)^2$
- $4^2 + 8(2) \div 4 + (6 - 2)^2$
- $\frac{16}{2^2} + \frac{7 \cdot 3}{7}$
- $3(8 - 2)^2 + 10 \div 5 - 6 \cdot 5$
- $18 \div 2 + 7 \cdot 8 \div 2 - (9 - 4)^2$
- $\frac{24}{3} + 16 - 12 \div 3 - (3 + 5)^2$
- $22 \cdot 2 \div 4 - (7 + 3)^2 + 3(7 - 2)^2$
- $\left(\frac{22+3}{5}\right)^2 + 4^2 - (2 \cdot 3)^2$
- $5^2 - \left(\frac{40+4}{4}\right)^2 + (3 \cdot 4)^2$

Answers

- 26
- 5
- 20
- 53
- 74
- 70
- 10
- 26
- 9
- 0
- 58
- 97
- 36
- 7
- 80
- 12
- 44
- 14
- 5
- 48

PERCENTAGE OF INCREASE OR DECREASE

#21

Finding the percentage of increase or decrease can be done using proportions.

The proportion used to find percent is:

$$\frac{\%}{100} = \frac{\text{part}}{\text{whole}}$$

For percentage of increase or decrease, the same concept is used. The proportion becomes:

$$\frac{\%}{100} = \frac{\text{change (increase or decrease)}}{\text{original amount}}$$

Example 1

A town's population grew from 1,437 to 6,254 over five years. What was the percentage of increase?

- Subtract to find the change:

$$6,254 - 1437 = 4817$$

- Put the known numbers in the proportion:

$$\frac{\%}{100} = \frac{4817 \text{ increase}}{1437 \text{ original}}$$

- The percentage becomes x , the unknown:

$$\frac{x}{100} = \frac{4817}{1437}$$

- Cross multiply:

$$1437x = 481,700$$

- Divide each side by 1437:

$$x = 335.2\% \text{ increase}$$

Example 2

A Sumo wrestler retired from Sumo wrestling and went on a diet. When he retired he weighed 428 pounds. After two years he weighed 253 pounds. What was the percentage of decrease in his weight?

- Subtract to find the change:

$$428 - 253 = 175$$

- Put the known numbers in the proportion:

$$\frac{\%}{100} = \frac{175 \text{ decrease}}{428 \text{ original}}$$

- The percentage becomes x , the unknown:

$$\frac{x}{100} = \frac{175}{428}$$

- Cross multiply:

$$428x = 17,500$$

- Divide each side by 428:

$$x = 40.89\%$$

His weight decreased by 40.89%.

Problems

Solve the following problems.

1. Thirty years ago gasoline cost \$0.30 per gallon. Today gasoline averages about \$1.65 per gallon. What is the percentage of increase in the cost of gasoline?
2. When Spencer was 3, he was 26 inches tall. Today he is 5 feet 8 inches tall. What is the percentage of increase in Spencer's height?
3. The cars of the early 1900s cost \$500. Today a new car costs an average of \$28,500. What is the percentage of increase of the cost of an automobile?
4. The population of the U.S. at the first census in 1790 was 3,929,000 people. By 2000 the population had increased to 284,000,000! What is the percentage of increase in the population?
5. In 2002 the rate for a first class U.S. postage stamp increased to \$0.37. This represents a \$0.34 increase since 1917. What is the percentage of increase of postage since 1917?
6. In 1880, Americans consumed an average of 28.75 gallons of whole milk. By 1998 the average consumption was 8.32 gallons. What is the percentage of decrease in consumption of whole milk?
7. In 1980 there were 150 students for each computer in U.S. public schools. By 1998 there were 6.1 students for each computer. What is the percentage of decrease in the ratio of students to computers?
8. Sara bought a dress on sale for \$28. She saved 40%. What was the original cost of the dress?
9. Pat was shopping and found a jacket with the original price of \$120 on sale for \$19.99. What was the percentage of decrease in the price of the jacket?
10. The price of a pair of pants decreased from \$69.99 to \$19.95. What was the percentage of decrease in the price of the pants?

Answers

1. $1.65 - 0.30 = 1.35$; $\frac{x}{100} = \frac{1.35}{0.30}$; $x = 450\%$
2. $68 - 26 = 42$; $\frac{x}{100} = \frac{42}{26}$; $x = 161.5\%$
3. $28,500 - 500 = 28,000$; $\frac{x}{100} = \frac{28000}{500}$; $x = 5600\%$
4. $284,000,000 - 3,929,000 = 280,071,000$; $\frac{x}{100} = \frac{280,071,000}{3929000}$; $x = 7,128.3\%$
5. $0.37 - 0.34 = 0.03$; $\frac{x}{100} = \frac{0.34}{0.03}$; $x = 1133.3\%$
6. $28.75 - 8.32 = 20.43$; $\frac{x}{100} = \frac{20.43}{28.75}$; $x = 71.06\%$
7. $150 - 6.1 = 143.9$; $\frac{x}{100} = \frac{143.9}{150}$; $x = 95.93\%$
8. $100 - 40 = 60\%$; $\frac{60}{100} = \frac{28}{x}$; $x = \$46.67$
9. $120 - 19.99 = 100.01$; $\frac{x}{100} = \frac{100.01}{120}$; $x = 83.34\%$
10. $69.99 - 19.95 = 50.04$; $\frac{x}{100} = \frac{50.04}{69.99}$; $x = 71.5\%$

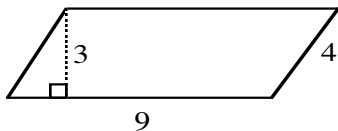
PERIMETER

#22

The **PERIMETER** of a polygon is the distance around the outside of the figure. The perimeter is found by adding the lengths of all of the sides.

Example 1

Find the perimeter of the parallelogram below.

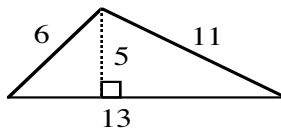


$$P = 9 + 4 + 9 + 4 = 26 \text{ units}$$

(Parallelograms have opposite sides equal.)

Example 2

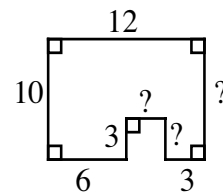
Find the perimeter of the triangle below.



$$P = 6 + 11 + 13 = 30 \text{ units}$$

Example 3

Find the perimeter of the figure below.



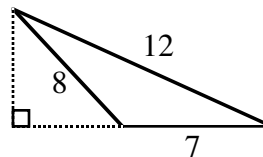
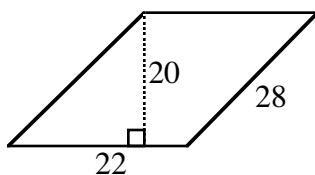
$$P = 10 + 12 + 10 + 3 + 3 + 3 + 3 + 6 = 50 \text{ units}$$

(You need to look carefully to find the missing lengths of the sides.)

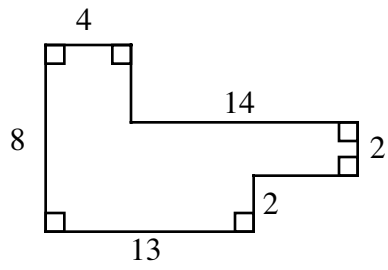
Problems

Find the perimeter of each shape.

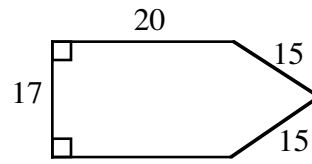
1. a rectangle with base = 7 and height = 13
2. a square with sides of length 13
3. a parallelogram with base = 28 and side = 15
4. a triangle with sides 7, 10, and 12
5. a parallelogram
- 6.



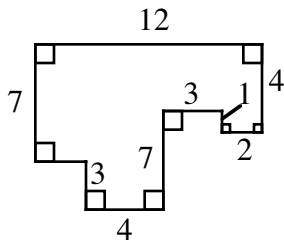
7.



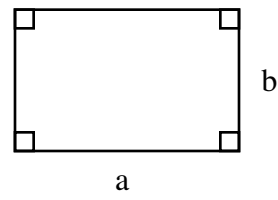
8.



9.



10.



Answers

1. 40 units

2. 52 units

3. 86 units

4. 29 units

5. 100 units

6. 27 units

7. 52 units

8. 87 units

9. 46 units

10. $a + b + a + b = 2a + 2b$

PROBABILITY

#23

PROBABILITY is the likelihood that a specific outcome will occur, represented by a number between 0 and 1.

There are two categories of probability.

THEORETICAL PROBABILITY is calculated probability. If every outcome is equally likely, it is the ratio of outcomes in an event to all possible outcomes.

$$\text{Theoretical probability} = \frac{\text{number of outcomes in the specified event}}{\text{total number of possible outcomes}}$$

EXPERIMENTAL PROBABILITY is the probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of outcomes in the specified event}}{\text{total number of possible outcomes}}$$

Example 1

There are three pink pencils, two blue pencils, and one green pencil. If one pencil is picked randomly, what is the theoretical probability it will be blue?

- Find the total number of possible outcomes, that is, the total number of pencils. $3 + 2 + 1 = 6$
- Find the number of specified outcomes, that is, how many pencils are blue? 2
- Find the theoretical probability. $P(\text{blue pencil}) = \frac{2}{6} = \frac{1}{3}$. (You may reduce your answer.)

Note that $P(\text{blue pencil})$ means "The probability of picking a blue pencil."

Example 2

Jayson rolled a die twelve times. He noticed that three of his rolls were fours.

- a) What is the theoretical probability of rolling a four?

Because the six sides are equally likely and there is only one four, $P(4) = \frac{1}{6}$.

- b) What is the experimental probability of rolling a four?

There were three fours in twelve rolls. The experimental probability is: $P(4) = \frac{3}{12} = \frac{1}{4}$.

Note that $P(4)$ means "The probability of rolling a 4."

Problems

1. There are five balls in a bag: 2 red, 2 blue, and 1 white. What is the probability of randomly choosing a red ball?
2. In a standard deck of cards, what is the probability of drawing an ace?
3. A fair die numbered 1, 2, 3, 4, 5, 6 is rolled. What is the probability of rolling an odd number?
4. In the word "probability," what is the probability of selecting a vowel?
5. Anna has some coins in her purse: 5 quarters, 3 dimes, 2 nickels, and 4 pennies.
 - a) What is the probability of selecting a quarter?
 - b) What is the probability she will select a dime or a penny?
6. Tim has some gum drops in a bag: 20 red, 5 green, and 12 yellow.
 - a) What is the probability of selecting a green?
 - b) What is the probability of **not** selecting a red?

Answers

1. $\frac{2}{5}$
2. $\frac{4}{52} = \frac{1}{13}$
3. There are 3 odd numbers.
 $\frac{3}{6} = \frac{1}{2}$
4. $\frac{4}{11}$
5. a) $\frac{5}{14}$ b) $\frac{7}{14} = \frac{1}{2}$
6. a) $\frac{5}{37}$ b) $\frac{17}{37}$

COMPOUND PROBABILITY

When multiple outcomes are specified (as in Example 1 below, pink or blue), and either may happen but not both, find the probability of each specified outcome and add their probabilities.

If the desired outcome is a compound event, that is, it has more than one characteristic (as in Example 3 on the next page, a red car), find the probability of each outcome (as in Example 3, the probability of "red," then the probability of "car") and multiply the probabilities.

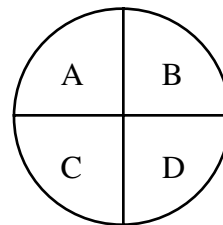
Example 1

On the spinner, what is the probability of spinning an A or a B?

The probability of an A is $\frac{1}{4}$. The probability of a B is $\frac{1}{4}$. Add the two probabilities for the combined total.

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \quad P(\text{A or B}) = \frac{1}{2}$$

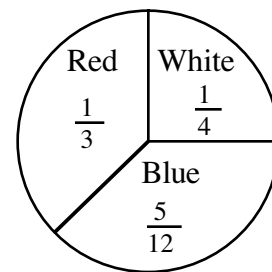
Note that P(A or B) means "The probability of spinning an A or a B."



Example 2

What is the probability of spinning red or white?

- We know that $P(R) = \frac{1}{3}$ and $P(W) = \frac{1}{4}$.
- Add the probabilities together. $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$
 $P(R \text{ or } W) = \frac{7}{12}$

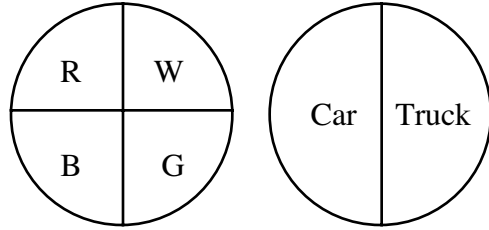


Note that P(R or W) means "The probability of spinning red or white."

Example 3

To find the P(Red and Car):

- Find the probability of Red: $\frac{1}{4}$.
- Find the probability of Car: $\frac{1}{2}$.
- Multiply them together.
 $\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$ P(Red and Car) = $\frac{1}{8}$



Note that P(Red and Car) means "The probability that the first spinner result is red and the second spinner result is a car."

This can also be shown with a probability rectangle.

- There are four equally likely choices of color on the first spinner. The rectangle is divided vertically into four equal parts, each labeled with its probability and color.
- There are two equally likely choices of vehicle on the second spinner. The rectangle is divided horizontally into two equal parts, each labeled with its probability and vehicle.
- Write the probability of spinning each combination in its section of the rectangle, multiplying the probability to get the area of the rectangular subproblem as in a multiplication table.
- The P(Red and Car) = $\frac{1}{8}$.

	Red $\frac{1}{4}$	White $\frac{1}{4}$	Blue $\frac{1}{4}$	Green $\frac{1}{4}$
Car $\frac{1}{2}$	RC $\frac{1}{8}$	WC $\frac{1}{8}$	BC $\frac{1}{8}$	GC $\frac{1}{8}$
Truck $\frac{1}{2}$	RC $\frac{1}{8}$	WC $\frac{1}{8}$	BC $\frac{1}{8}$	GC $\frac{1}{8}$

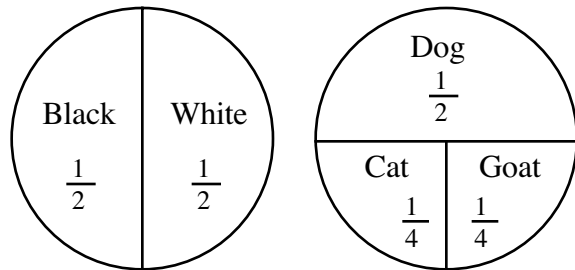
Example 4

What is the probability of spinning a black dog or a black cat?

$$P(\text{BCat}) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

$$P(\text{BDog}) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(\text{BCat or BDog}) = \frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$$

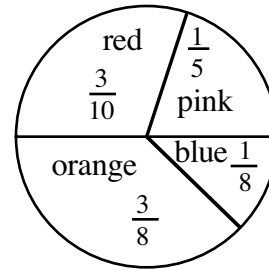


Note that P(B Cat or B Dog) means "The probability that the outcome of the two spins is either a black cat or a black dog."

Problems

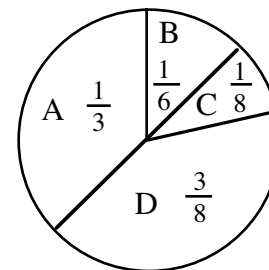
1. What is the probability of spinning:

- a) pink or blue? b) orange or pink?
 c) red or orange? d) red or blue?

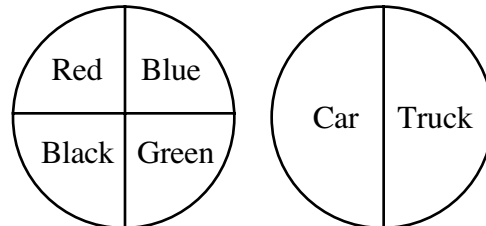


2. What is the probability of spinning:

- a) A or C? b) B or C?
 c) A or D? d) B or D?
 e) A, B, or C?



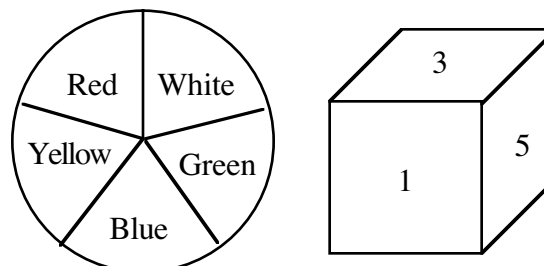
3. If each section in each spinner is the same size, what is the probability of getting a Black Truck?



4. Bipasha loves purple, pink, turquoise and black, and has a blouse in each color. She has two pairs of black pants and a pair of khaki pants. If she randomly chooses one blouse and one pair of pants, what is the probability she will wear a purple blouse with black pants?

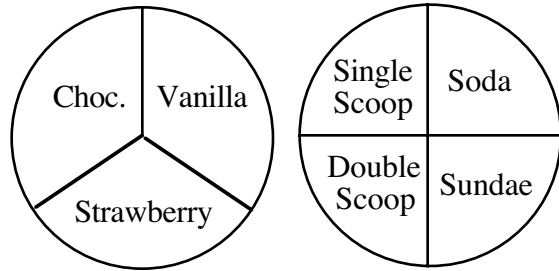
5. The spinner at right is divided into five equal regions. The die is numbered from 1 to 6. What is the probability of:

- a) rolling a red 5?
 b) a white or blue even number?



6. What is the probability Joanne will win a:

- a) chocolate double scoop?
- b) chocolate or strawberry sundae?
- c) chocolate double scoop or chocolate sundae?

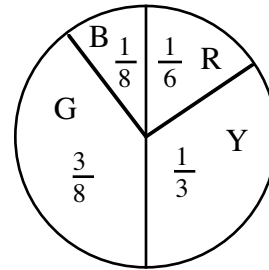


7. Jay is looking in his closet, trying to decide what to wear. He has 2 red t-shirts, 2 black t-shirts, and 3 white t-shirts. He has 3 pairs of blue jeans and 2 pairs of black pants.

- a) What is the probability of his randomly choosing a red shirt with jeans?
- b) What is the probability of his randomly choosing an all black outfit?
- c) Which combination out of all his possible choices has the greatest probability of being randomly picked? How can you tell?

8. What is the probability of spinning a:

- a) Red or Green?
- b) Blue or Yellow?
- c) Yellow or Green?



Answers

1. a) $\frac{13}{40}$ b) $\frac{23}{40}$ c) $\frac{27}{40}$ d) $\frac{17}{40}$

2. a) $\frac{11}{24}$ b) $\frac{7}{24}$ c) $\frac{17}{24}$ d) $\frac{13}{24}$ e) $\frac{5}{8}$

3. $\frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

4. $\frac{1}{4} \cdot \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$

5. a) $\frac{1}{5} \cdot \frac{1}{6} = \frac{1}{30}$

b) $(\frac{1}{5} + \frac{1}{5}) \cdot \frac{3}{6} = \frac{1}{5}$

6. a) $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

7. a) $\frac{2}{7} \cdot \frac{3}{5} = \frac{6}{35}$

8. a) $\frac{13}{24}$ c) $\frac{17}{24}$

b) $(\frac{1}{3} + \frac{1}{3}) \cdot \frac{1}{4} = \frac{1}{6}$

b) $\frac{2}{7} \cdot \frac{2}{5} = \frac{4}{35}$

b) $\frac{11}{24}$

c) $\frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{6}$

c) white shirt, blue jeans; n

PROBABILITY: DEPENDENT AND INDEPENDENT EVENTS

Two events are **DEPENDENT** if the outcome of the first event affects the outcome of the second event. For example, if you draw a card from a deck and do not replace it for the next draw, the two events – drawing one card without replacing it, then drawing a second card – are dependent.

Two events are **INDEPENDENT** if the outcome of the first event does not affect the outcome of the second event. For example, if you draw a card from a deck but replace it before you draw again, the two events are independent.

Example 1

Aiden pulls an ace from a deck of regular playing cards. He does not replace the card. What is the probability of pulling out a second ace?

$$\text{First draw: } \frac{4}{52} \quad \text{Second draw: } \frac{3}{51} \text{ aces left cards left to pull from}$$

This is an example of a dependent event – the probability of the second draw has been affected by the first draw.

Example 2

Jayson was tossing a coin. He tossed a head. What is the probability of tossing a second head on his next flip? It is still one-half. The probability for the second event has not changed. This is an independent event.

Problems

1. You throw a die twice. What is the probability of throwing a six and then a second six? Is this an independent or dependent event?
2. You have a bag of candy filled with pieces which are all the same size and shape. Four are gumballs and six are sweet and sour. You draw a gumball out, decide you don't like it, put it back, and select another piece of candy. What is the probability of selecting another gumball? Are these independent or dependent events?
3. Joey has a box of blocks with eight alphabet blocks and four plain red blocks. He gave an alphabet block to his sister. What is the probability his next selection will be another alphabet block? Are these independent or dependent events?

4. In your pocket you have three nickels and two dimes.
- a) What is the probability of selecting a nickel?
 - b) What is the probability of selecting a dime?
 - c) If you select a nickel and place it on a table, what is the probability the next coin selected is a dime? Is this an independent or dependent event?
 - d) If all the coins are back in your pocket, what is the probability that the next coin you take out is a dime? Is this an independent or dependent event?
5. How do you tell the difference between dependent and independent events?

Answers

- 1. $\frac{1}{36}$, independent (the probability doesn't change)
- 2. $\frac{4}{10}$ or $\frac{2}{5}$, independent
- 3. $\frac{7}{11}$, dependent
- 4. a) $\frac{3}{5}$ b) $\frac{2}{5}$ c) $\frac{2}{4}$, dependent d) $\frac{2}{5}$, independent
- 5. In dependent events, the second probability changes because there was no replacement.

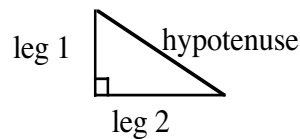
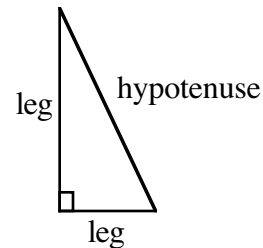
THE PYTHAGOREAN THEOREM

#24

A **right triangle** is a triangle in which the two shorter sides form a right angle. The shorter sides are called **legs**. Opposite the right angle is the third and longest side called the **hypotenuse**.

The **Pythagorean Theorem** states that for any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

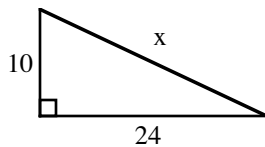
$$(\text{leg } 1)^2 + (\text{leg } 2)^2 = (\text{hypotenuse})^2$$



Example 1

Use the Pythagorean Theorem to find x .

a)



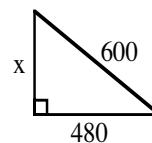
$$10^2 + 24^2 = x^2$$

$$100 + 576 = x^2$$

$$676 = x^2$$

$$26 = x$$

b)



$$x^2 + 480^2 = 600^2$$

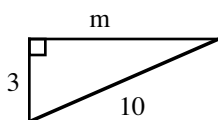
$$x^2 + 230,400 = 360,000$$

$$x^2 = 129,600$$

$$x = 360$$

Example 2

Not all problems will have exact answers. Use square root notation and your calculator. Round your answers to the nearest hundredth.



$$3^2 + m^2 = 10^2$$

$$9 + m^2 = 100$$

$$m^2 = 91$$

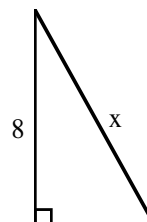
$$m = \sqrt{91} \approx 9.54$$

Example 3

A guy wire is needed to support a tower. The wire is attached to the ground five meters from the base of the tower. How long is the wire if the tower is 8 meters tall?

1. First draw a diagram to model the problem, then write an equation using the Pythagorean Theorem and

2.



3.

$$x^2 = 8^2 + 5^2$$

$$x^2 = 64 + 25$$

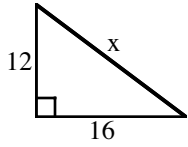
$$x^2 = 89$$

$$x = \sqrt{89} \approx 9.43$$

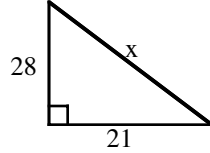
Problems

Write an equation and solve for each unknown side. Round to the nearest hundredth.

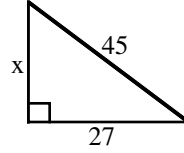
1.



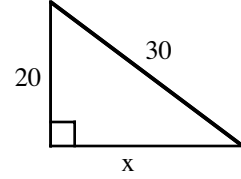
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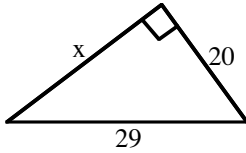
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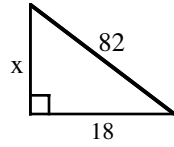
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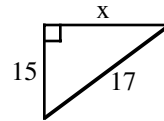
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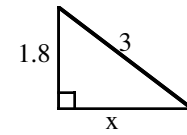
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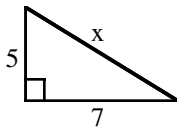
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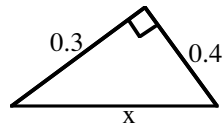
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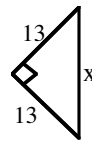
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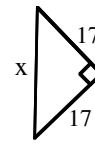
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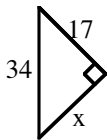
11.



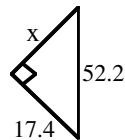
12.



13.

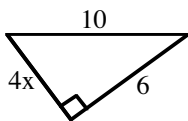


14.

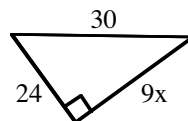


Be careful! Remember to square the whole side. For example, $(2x)^2 = (2x)(2x) = 4x^2$.

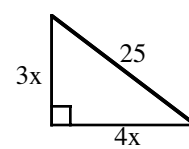
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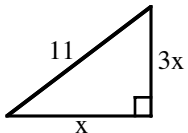
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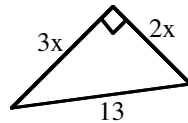
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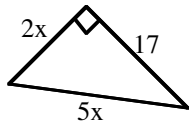
18.



19.



20.



For each of the following problems draw and label a diagram. Then write an equation using the Pythagorean Theorem and solve for the unknown. Round your answers to the nearest hundredth.

21. In a right triangle, the length of the hypotenuse is 13 inches. The length of one leg is 5 inches. Find the length of the other leg.
22. The length of the hypotenuse of a right triangle is 9 cm. The length of one leg is four cm. Find the length of the other leg.
23. Find the diagonal length of a television screen 20 inches wide by 17 inches long.
24. Find the length of a path that runs diagonally across a 75-yard by 120-yard soccer field.
25. A surveyor walked two miles north, then three miles west. How far was she from her starting point? Note: This question is different from the question, "How far did she walk?"
26. A 2.8 meter ladder is one meter from the base of a building. How high up the side of the building will the ladder reach?
27. What is the longest line you can draw on a paper that is 8.5 inches by 11 inches?
28. What is the longest length of an umbrella that will lay flat in the bottom of a backpack that is 12 inches by 17 inches?
29. Find the diagonal distance from one corner of a square classroom floor to the other corner of the floor if the length of the floor is 31 feet.
30. Mary can turn off her car alarm from up to 15 yards away. Will she be able to do it from the far corner of a 15-yard by 12-yard parking lot if her car is parked in the diagonally opposite corner from where she is standing?

Answers

- | | | | |
|-----------------------|--|-----------------------|-----------------------|
| 1. $x = 20$ | 2. $x = 35$ | 3. $x = 36$ | 4. $x \approx 22.36$ |
| 5. $x = 21$ | 6. $x = 80$ | 7. $x = 8$ | 8. $x = 2.4$ |
| 9. $x \approx 8.60$ | 10. $x = 0.5$ | 11. $x \approx 18.38$ | 12. $x \approx 24.04$ |
| 13. $x \approx 29.44$ | 14. $x \approx 49.21$ | 15. $x = 2$ | 16. $x = 2$ |
| 17. $x = 5$ | 18. $x \approx 3.48$ | 19. $x \approx 3.61$ | 20. $x \approx 3.71$ |
| 21. 12 inches | 22. 8.06 cm | 23. 26.25 inches | 24. 141.51 yards |
| 25. 3.61 mi | 26. 2.62 m | 27. 13.90 inches | 28. 20.81 inches |
| 29. 43.84 feet | 30. The corner is 19.21 yards away, so no! | | |

RATIO

#25

A **RATIO** is a comparison of two quantities by division. It can be written in several ways:

$$\frac{65 \text{ miles}}{1 \text{ hour}}, 65 \text{ miles} : 1 \text{ hour}, \text{ or } 65 \text{ miles to } 1 \text{ hour.}$$

Both quantities of a ratio (numerator and denominator) can be multiplied by the same number. We can use a **ratio table** to organize the multiples. Each ratio in the table will be equivalent to the others. Patterns in the ratio table can be used in problem solving.

Example 1

130 cups of coffee can be made from one pound of coffee beans. Doubling the amount of coffee beans will double the number of cups of coffee that can be made. Use the doubling pattern to complete the ratio table for different weights of coffee beans.

Pounds of coffee beans	1	2	4	8
Cups of coffee	130	260		

Doubling two pounds of beans doubles the number of cups of coffee made, so for 4 pounds of beans, $2 \cdot 260 = 520$ cups of coffee are made. Since 4 pounds of beans make 520 cups of coffee, 8 pounds of beans make $2 \cdot 520 = 1040$ cups of coffee.

Example 2

You can use the ratio table from Example 1 to determine how many cups of coffee you could make from 6 pounds of beans. Add another column to your ratio table.

Pounds of coffee beans	1	2	4	8	6
Cups of coffee	130	260	520	1040	

You know that the value of 6 is halfway between the values of 4 and 8. The value halfway between 520 and 1040 is 780, so 6 pounds of beans should make 780 cups of coffee.

Example 3

Jane's cookie recipe uses $3\frac{1}{2}$ cups of flour and 2 eggs. She needs to know how much flour she will need if she uses 9 eggs. Jane started a ratio table but discovered that she could not simply keep doubling because 9 is not a multiple of 2. She used other patterns to find her answer. Study the top row of Jane's table to find the patterns she used, then complete the table.

Number of eggs	2	4	12	36	9
Cups of flour	$3\frac{1}{2}$				

>>Example continues on the next page.>>

Jane doubled to get 4, tripled 4 to get 12, and tripled again to get 36. Then she divided 36 by four to get to 9. She followed the same steps to complete the “cups of flour” row.

Number of eggs	2	4	12	36	9
Cups of flour	$3\frac{1}{2}$	7	21	63	$15\frac{3}{4}$

Her last step was to divide 63 by 4. Jane will need $15\frac{3}{4}$ cups of flour.

In the examples above, the ratio tables have the ratios listed in the order in which they were calculated. When a row of a ratio table is provided, it is acceptable to complete a ratio table in the order that fits the patterns you see and use. In most cases it is easier to see patterns by listing the values in the first (or top) row in numerical order.

Problems

Complete each ratio table.

1.

2	4	6	8	10	12	16	20
6	12						

2.

3	6	9	12	15	18	21	30
5	10						

3.

5			20	30	45	60	100
3	6	9					

4.

7	14	35		49		700	
2			20		50		500

5.

4	8	16	12		32		80
11				66		99	

6.

6	12	18	30			360	
4.5				45	90		540

7.

9	36	45			180	900	300
34			85	340			

8.

10	15	20	25		35		4000
6.5				26		260	

Answers

1.

2	4	6	8	10	12	16	20
6	12	18	24	30	36	48	60

2.

3	6	9	12	15	18	21	30
5	10	15	20	25	30	35	50

3.

5	10	15	20	30	45	60	100
3	6	9	12	18	27	36	60

4.

7	14	35	70	49	175	700	1750
2	4	10	20	14	50	200	500

5.

4	8	16	12	24	32	36	80
11	22	44	33	66	88	99	220

6.

6	12	18	30	60	120	360	720
4.5	9	13.5	22.5	45	90	270	540

7.

9	36	45	22.5	90	180	900	300
34	136	170	85	340	680	3400	1133.3

8.

10	15	20	25	40	35	400	4000
6.5	9.75	13	16.25	26	22.75	260	2600

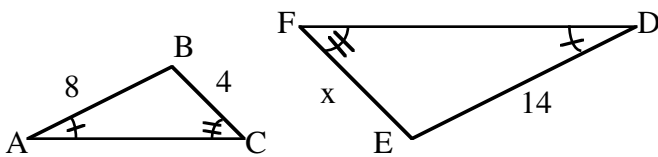
RATIO APPLICATIONS

#26

Ratios and proportions are used to solve problems involving similar figures, percents, and relationships that vary directly.

Example 1

$\triangle ABC$ is similar to $\triangle DEF$. Use ratios to find x .



Since the triangles are similar, the ratios of the corresponding sides are equal.

$$\frac{8}{14} = \frac{4}{x} \Rightarrow 8x = 56 \Rightarrow x = 7$$

Example 2

- a) What percent of 60 is 45?
b) Forty percent of what number is 45?

In percent problems use the following proportion: $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$.

- a) $\frac{45}{60} = \frac{x}{100}$ b) $\frac{40}{100} = \frac{45}{x}$
 $60x = 4500$ $40x = 4500$
 $x = 75$ (75%) $x = 112$

Example 3

Amy usually swims 20 laps in 30 minutes. How long will it take to swim 50 laps at the same rate?

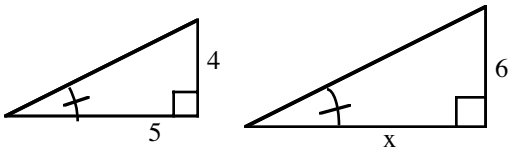
Since two units are being compared, set up a ratio using the unit words consistently. In this case, "laps" is on top (the numerator) and "minutes" is on the bottom (the denominator) in both ratios. Then solve as shown in Skill Builder #9.

$$\frac{\text{laps}}{\text{minutes}} : \frac{20}{30} = \frac{50}{x} \Rightarrow 20x = 1500 \Rightarrow x = 75 \text{ minutes}$$

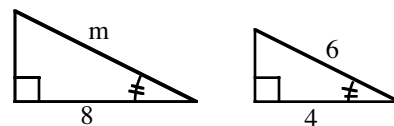
Problems

Each pair of figures is similar. Solve for the variable.

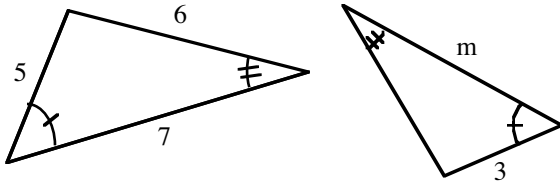
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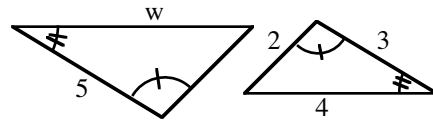
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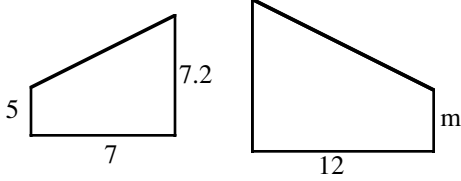
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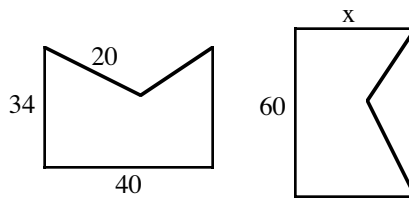
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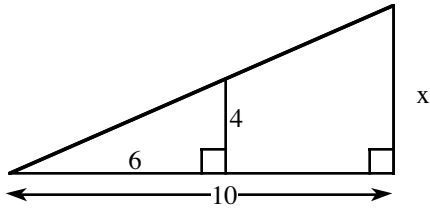
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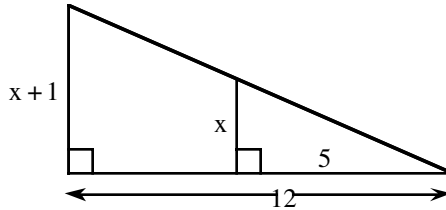
6.



7.



8.



Write and solve a proportion to find the missing part.

9. 45 is 25% of what?

10. 15 is 30% of what?

11. 45% of 300 is what?

12. 32% of 250 is what?

13. 18 is what percent of 30?

14. What percent of 400 is 250?

15. What is 12% of \$12.50?

16. What is 7.5% of \$425.75?

>>Problems continue on the next page.>>

Use ratios to solve each problem.

17. A rectangle has length 10 feet and width six feet. It is enlarged to a similar rectangle with length 16 feet. What is the new width?
18. If 200 vitamins cost \$8.75, what should 500 vitamins cost?
19. The tax on a \$200 painting is \$34. What should the tax be on a \$795 painting?
20. If a basketball player made 72 of 85 shots, how many shots could she expect to make in 300 shots?
21. A cookie recipe uses $\frac{1}{2}$ teaspoon of vanilla with $\frac{3}{4}$ cup of flour. How much vanilla should be used with eight cups of flour?
22. My brother grew $2\frac{3}{4}$ inches in $3\frac{1}{2}$ months. At that rate, how much would he grow in one year?
23. The length of a rectangle is four centimeters more than the width. If the ratio of the length to width is eight to five, find the dimensions of the rectangle.
24. A class has three fewer girls than boys. If the ratio of girls to boys is four to five, how many students are in the class?

Answers

- | | | | |
|----------------------------------|-----------------------|--|----------------------------------|
| 1. $\frac{30}{4} = 7\frac{1}{2}$ | 2. 12 | 3. $\frac{21}{5} = 4\frac{1}{5}$ | 4. $\frac{20}{3} = 6\frac{2}{3}$ |
| 5. $\frac{60}{7} = 8\frac{4}{7}$ | 6. 51 | 7. $\frac{20}{3} = 6\frac{2}{3}$ | 8. $\frac{5}{7}$ |
| 9. 180 | 10. 50 | 11. 135 | 12. 80 |
| 13. 60% | 14. $62\frac{1}{2}\%$ | 15. \$1.50 | 16. \$31.93 |
| 17. 9.6 ft. | 18. \$21.88 | 19. \$135.15 | 20. about 254 shots |
| 21. $5\frac{1}{3}$ teaspoons | 22. about 9.4 inches | 23. $6\frac{2}{3}$ cm x $10\frac{2}{3}$ cm | 24. 27 students |

SCIENTIFIC NOTATION

#27

SCIENTIFIC NOTATION is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when it is written as the product of two factors as described below.

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent (power of 10).
- The factors are separated by a multiplication sign.
- A positive exponent indicates a number whose absolute value is greater than one.
- A negative exponent indicates a number whose absolute value is less than one.

Scientific Notation	Standard Form
$3.51 \cdot 10^{10}$	35,100,000,000
$4.73 \cdot 10^{-13}$	0.000000000000473

It is important to note that the exponent does not necessarily mean to use that number of zeros.

The number $3.51 \cdot 10^{10}$ means $3.51 \cdot 10,000,000,000$. Thus, two of the 10 places in the standard form of the number are the 5 and the 1 in 3.51. Standard form in this case is 35,100,000,000. In this example you are moving the decimal point to the right 10 places to find standard form.

The number $4.73 \cdot 10^{-13}$ means $4.73 \cdot 0.0000000000001$. You are moving the decimal point to the left 13 places to find standard form. Here the standard form is 0.000000000000473.

Example 1

Write each number in standard form.

$$4.39 \cdot 10^9 \Rightarrow 4,390,000,000 \quad \text{and} \quad 5.17 \cdot 10^{-7} \Rightarrow 0.000000517$$

When taking a number in standard form and writing it in scientific notation, remember there is only one digit before the decimal point, that is, the number must be between 1 and 9, inclusive.

Example 2

$$43,207,000 \Rightarrow 4.3207 \cdot 10^7 \quad \text{and} \quad 0.0000857 \Rightarrow 8.57 \cdot 10^{-5}$$

The exponent denotes the number of places you have moved the decimal point in the standard form. In Example 2 above left, the decimal point is at the end of the number and it was moved 7 places. In the second part of the example above right, the exponent is negative because the original number is very small, that is, less than one.

Problems

Write each number in standard form.

- | | | | |
|-------------------------|--------------------------|-------------------------|-------------------------|
| 1. $9.43 \cdot 10^7$ | 2. $1.347 \cdot 10^{13}$ | 3. $4.38762 \cdot 10^4$ | 4. $5.10327 \cdot 10^2$ |
| 5. $6.84 \cdot 10^{-5}$ | 6. $7.03 \cdot 10^{-6}$ | | |

Write each number in scientific notation.

- | | | | |
|----------------|----------------|---------------|-----------------|
| 7. 723,000,000 | 8. 123,780,000 | 9. 0.0000576 | 10. 0.000000702 |
| 11. 48.68 | 12. 354.3 | 13. 8,500,000 | 14. 437,000,000 |
| 15. 0.0000004 | 16. 0.0000057 | 17. 0.00473 | 18. 0.00032 |
| 19. 12.37 | 20. 13,980,000 | | |

Note:

On your scientific calculator, numbers like 4.477^{10} and 3.45^{-04} are expressed in scientific notation. The first number means $4.477 \cdot 10^{10}$ and the second means $3.45 \cdot 10^{-4}$. The calculator does this because there is not enough room in its display window to show the whole number.

Answers

- | | | |
|--------------------------|--------------------------|-------------------------|
| 1. 94,300,000 | 2. 13,470,000,000,000 | 3. 43,876.2 |
| 4. 510.327 | 5. 0.0000684 | 6. 0.00000703 |
| 7. $7.23 \cdot 10^8$ | 8. $1.2378 \cdot 10^8$ | 9. $5.76 \cdot 10^{-5}$ |
| 10. $7.02 \cdot 10^{-7}$ | 11. $4.868 \cdot 10^1$ | 12. $3.543 \cdot 10^2$ |
| 13. $8.5 \cdot 10^6$ | 14. $4.37 \cdot 10^8$ | 15. $4 \cdot 10^{-7}$ |
| 16. $5.7 \cdot 10^{-6}$ | 17. $4.73 \cdot 10^{-3}$ | 18. $3.2 \cdot 10^{-4}$ |
| 19. $1.237 \cdot 10^1$ | 20. $1.398 \cdot 10^7$ | |

SIMILARITY OF LENGTH AND AREA

#28

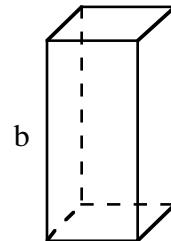
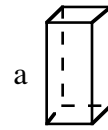
Once you know two figures are similar with a **SCALE FACTOR** or

RATIO OF SIMILARITY $\frac{a}{b}$, the following proportions for the SMALL (sm) and LARGE (lg) figures (which are enlargements or reductions of each other) are true:

$$\frac{\text{side}_{\text{sm}}}{\text{side}_{\text{lg}}} = \frac{a}{b} \quad \frac{P_{\text{sm}}}{P_{\text{lg}}} = \frac{a}{b} \quad \frac{A_{\text{sm}}}{A_{\text{lg}}} = \frac{a^2}{b^2}$$

where P = perimeter, A = area of a face or the total surface area.

In each proportion above, the data from the smaller figure is written on top (in the numerator) to help be consistent with correspondences. When working with area, the scale factor (ratio of similarity) is squared. NEVER square the actual areas themselves, only the scale factor.



Example

The two rectangular prisms above are similar. Suppose the ratio of their vertical edges is $\frac{3}{7}$.

- Find the ratio of their surface areas.
- Find the perimeter of the front face of the small prism if the perimeter of the front face of the large prism is 20 units.
- Find the area of the front face of the small prism if the area of the front face of the large prism is 24 square units.

Solutions

- a) The ratio of the surface areas is $\frac{A_{\text{sm}}}{A_{\text{lg}}} = \frac{3^2}{7^2} = \frac{9}{49}$. Use proportions to solve for the rest of the parts.

$$\begin{aligned} \text{b) } \quad \frac{3}{7} &= \frac{P}{20} \\ 7P &= 60 \\ P &= \frac{60}{7} \\ &\approx 8.57 \text{ units} \end{aligned}$$

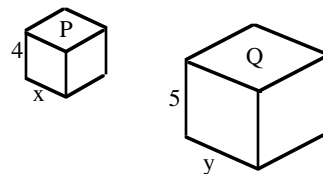
$$\begin{aligned} \text{c) } \quad \left(\frac{3}{7}\right)^2 &= \frac{A}{24} \\ \frac{9}{49} &= \frac{A}{24} \\ 49A &= 216 \end{aligned}$$

$$A = \frac{216}{49} \approx 4.41 \text{ units}$$

Note that the ratio of similarity must be squared when using it in area calculations.

Problems

- Two rectangular prisms are similar. The smaller, P, has a height of four units while the larger, Q, has a height of five units.
 - What is the scale factor from prism P to prism Q?
 - What is the ratio, small to large, of their surface areas?



figures are for problems 1 and 2

- What would be the ratio of the lengths of the edges labeled x and y in the prisms above?

For problems 3–16, you may want to draw two rectangles, label them A and B, and write in the specific data from each problem.

- If rectangle A and rectangle B have a ratio of similarity of $\frac{5}{3}$, what is the area of rectangle B if the area of rectangle A is 30 square units?
- If rectangle A and rectangle B have a ratio of similarity of $\frac{5}{3}$, what is the area of rectangle A if the area of rectangle B is 30 square units?
- If rectangle A and rectangle B have a ratio of similarity of $\frac{2}{5}$, what is the area of rectangle B if the area of rectangle A is 46 square units?
- If rectangle A and rectangle B have a ratio of similarity of $\frac{2}{5}$, what is the area of rectangle A if the area of rectangle B is 46 square units?
- If rectangle A and rectangle B have a ratio of similarity of $\frac{3}{7}$, what is the area of rectangle B if the area of rectangle A is 82 square units?
- If rectangle A and rectangle B have a ratio of similarity of $\frac{3}{7}$, what is the area of rectangle A if the area of rectangle B is 82 square units?
- If rectangle A and rectangle B have a ratio of similarity of $\frac{1}{9}$, what is the area of rectangle B if the area of rectangle A is 24 square units?
- If rectangle A and rectangle B have a ratio of similarity of $\frac{1}{9}$, what is the area of rectangle A if the area of rectangle B is 24 square units?
- Rectangle A is similar to rectangle B. The area of rectangle A is 121 square units while the area of rectangle B is 49 square units. What is the ratio of similarity between the two rectangles?
- Rectangle A is similar to rectangle B. The area of rectangle A is 36 square units while the area of rectangle B is 64 square units. What is the ratio of similarity between the two rectangles?
- Rectangle A is similar to rectangle B. The area of rectangle B is 81 square units while the area of rectangle A is 25 square units. What is the ratio of similarity between the two rectangles?

14. Rectangle A is similar to rectangle B. The area of rectangle B is 289 square units while the area of rectangle A is 121 square units. What is the ratio of similarity between the two rectangles?
15. Rectangle A is similar to rectangle B. The area of rectangle A is 324 square units while the area of rectangle B is 121 square units. If the perimeter of rectangle A is 12 units, what is the perimeter of rectangle B?
16. Rectangle A is similar to rectangle B. The area of rectangle A is 324 square units while the area of rectangle B is 121 square units. If the perimeter of rectangle B is 12 units, what is the perimeter of rectangle A?
17. The corresponding diagonals of two similar trapezoids are in the ratio of $\frac{11}{23}$. What is the ratio of their areas?
18. The corresponding diagonals of two similar trapezoids are in the ratio of $\frac{17}{19}$. What is the ratio of their areas?
19. The ratio of the perimeters of two similar parallelograms is $\frac{5}{13}$. What is the ratio of their areas?
20. The ratio of the perimeters of two similar parallelograms is $\frac{11}{15}$. What is the ratio of their areas?
21. The ratio of the areas of two similar trapezoids is $\frac{4}{81}$. What is the ratio of their heights?
22. The ratio of the areas of two similar trapezoids is $\frac{5}{9}$. What is the ratio of their heights?
23. The areas of two circles are in the ratio of $\frac{36}{1}$. What is the ratio of their radii?
24. The areas of two circles are in the ratio of $\frac{36}{49}$. What is the ratio of their radii?

Answers

- | | | | | |
|-----------------------------|---------------------------------------|-------------------------------------|---------------------------------------|------------------------------------|
| 1. a) $\frac{4}{5}$ | b) $\frac{16}{25}$ | 2. $\frac{x}{y} = \frac{4}{5}$ | 3. 10.8 units ² | 4. $83.\bar{3}$ units ² |
| 5. 287.5 units ² | 6. 7.36 units ² | 7. $446.\bar{4}$ units ² | 8. ≈ 15.06 units ² | |
| 9. 1944 units ² | 10. ≈ 0.30 units ² | 11. $\frac{11}{7}$ | 12. $\frac{6}{8}$ or $\frac{3}{4}$ | |
| 13. $\frac{5}{9}$ | 14. $\frac{11}{17}$ | 15. $7.\bar{3}$ units | 16. $19.\bar{63}$ units | |
| 17. $\frac{121}{529}$ | 18. $\frac{289}{361}$ | 19. $\frac{25}{169}$ | 20. $\frac{121}{225}$ | |
| 21. $\frac{2}{9}$ | 22. $\frac{\sqrt{5}}{3}$ | 23. $\frac{6}{1}$ | 24. $\frac{6}{7}$ | |

SIMILARITY OF VOLUME

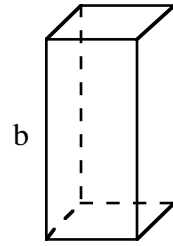
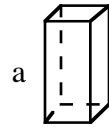
#29

Once you know two figures are similar with a **SCALE FACTOR** or **RATIO OF SIMILARITY** $\frac{a}{b}$, the following proportions for the **SMALL** (sm) and **LARGE** (lg) figures (which are enlargements or reductions of each other) are true:

$$\frac{\text{side}_{\text{sm}}}{\text{side}_{\text{lg}}} = \frac{a}{b} \qquad \frac{P_{\text{sm}}}{P_{\text{lg}}} = \frac{a}{b} \qquad \frac{A_{\text{sm}}}{A_{\text{lg}}} = \frac{a^2}{b^2} \qquad \frac{V_{\text{sm}}}{V_{\text{lg}}} = \frac{a^3}{b^3}$$

where P = perimeter, A = area of a face or the total surface area, and V = volume.

In each proportion above, the data from the smaller figure is written on top (in the numerator) to help be consistent with correspondences. When working with volume, the scale factor (ratio of similarity) is cubed. **NEVER** cube the actual volumes themselves, just the scale factor.



Example

The two rectangular prisms above are similar. Suppose the ratio of their vertical edges is $\frac{3}{7}$.

- Find the ratio of their surface areas.
- Find the ratio of their volumes.
- The volume of the small prism is 30 cubic units. Find the volume of the large prism.

Solutions

a) The ratio of the surface areas is $\frac{A_{\text{sm}}}{A_{\text{lg}}} = \frac{3^2}{7^2} = \frac{9}{49}$.

b) The ratio of the volumes is $\frac{V_{\text{sm}}}{V_{\text{lg}}} = \frac{3^3}{7^3} = \frac{27}{343}$. Use proportions to solve the next part.

c) $\left(\frac{3}{7}\right)^3 = \frac{30}{V}$ The ratio of similarity must be cubed when using it in volume calculations.

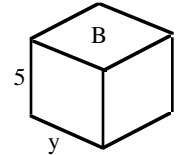
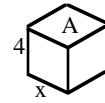
$$\frac{27}{343} = \frac{30}{V}$$

$$27V = 10,290$$

$$V = \frac{10290}{27} \approx 381.1$$

Problems

1. Two rectangular prisms are similar. The smaller, A, has a height of four units while the larger, B, has a height of five units.



- a) What is the scale factor from prism A to prism B?
- b) What is the ratio, small to large, of their volumes?
- c) A third prism, C, is similar to prisms A and B. Prism C's height is ten units. If the volume of prism A is 32 cubic units, what is the volume of prism C?
2. If prism A and prism B have a ratio of similarity of $\frac{2}{1}$, what is the volume of prism B if the volume of prism A is 36 cubic units?
3. If prism A and prism B have a ratio of similarity of $\frac{2}{1}$, what is the volume of prism A if the volume of prism B is 36 cubic units?
4. If prism A and prism B have a ratio of similarity of $\frac{3}{7}$, what is the volume of prism B if the volume of prism A is 83 cubic units?
5. If prism A and prism B have a ratio of similarity of $\frac{3}{7}$, what is the volume of prism A if the volume of prism B is 83 cubic units?
6. If prism A and prism B have a ratio of similarity of $\frac{7}{8}$, what is the volume of prism B if the volume of prism A is 96 cubic units?
7. If prism A and prism B have a ratio of similarity of $\frac{7}{8}$, what is the volume of prism A if the volume of prism B is 96 cubic units?
8. Prism A and prism B are similar. The volume of prism A is 64 cubic units while the volume of prism B is 125 cubic units. What is the ratio of similarity between these two prisms?
9. Prism A and prism B are similar. The volume of prism A is 512 cubic units while the volume of prism B is 125 cubic units. What is the ratio of similarity between these two prisms?
10. Prism A and prism B are similar. The volume of prism A is 8 cubic units while the volume of prism B is approximately 27 cubic units. If the surface area of prism B is 128 square units, what is the surface area of prism A?
11. Prism A and prism B are similar. The volume of prism A is 8 cubic units while the volume of prism B is approximately 27 cubic units. If the surface area of prism A is 128 square units, what is the surface area of prism B?
12. The ratio of the volumes of two similar circular cylinders is $\frac{125}{100}$. What is the ratio of the diameters of their similar bases?
13. The ratio of the volumes of two similar circular cylinders is $\frac{121}{49}$. What is the ratio of the diameters of their similar bases?

14. The surface areas of two cubes are in the ratio of $\frac{25}{49}$. What is the ratio of their volumes?
15. The surface areas of two cubes are in the ratio of $\frac{169}{196}$. What is the ratio of their volumes?
16. The ratio of the weights of two spherical steel balls is $\frac{27}{64}$. What is the ratio of the diameters of the two steel balls?
17. The ratio of the weights of two spherical steel balls is $\frac{64}{8}$. What is the ratio of the diameters of the two steel balls?

Answers

1. a) $\frac{4}{5}$ b) $\frac{64}{125}$ c) $\frac{4}{10} = \frac{2}{5}, \frac{2^3}{5^3} = \frac{32}{V} \Rightarrow 8V = 125 \cdot 32 \Rightarrow V = 500 \text{ u}^3$
2. 4.5 u^3 3. 288 u^3 4. $\approx 1054.41 \text{ u}^3$ 5. $\approx 6.53 \text{ u}^3$
6. $\approx 143.3 \text{ u}^3$ 7. $\approx 64.31 \text{ u}^3$ 8. $\frac{4}{5}$ 9. $\frac{8}{5}$
10. $\approx 56.89 \text{ u}^2$ 11. 288 u^2 12. $\approx \frac{5}{4.64}$ 13. $\approx \frac{4.95}{3.66}$
14. $\frac{125}{343}$ 15. $\frac{2197}{2744}$ 16. $\frac{3}{4}$ 17. $\frac{4}{2} = \frac{2}{1}$

SIMPLE AND COMPOUND INTEREST

#30

SIMPLE INTEREST is interest paid only on the original amount of the principal at each specified interval (such as annually). The formula is $I = Prt$, where I = Interest, P = Principal, r = rate, and t = time.

COMPOUND INTEREST is interest paid on both the original principal and the interest earned previously. The formula for compound interest is $A = P(1 + r)^t$, where A = total amount including previous interest earned, P = principal, r = interest rate, and t = time.

Example 1

Wayne earns 4.7% simple interest for 5 years on \$4500. How much interest does he earn?

Put the numbers in the formula $I = Prt$. $I = 4500(4.7\%)5$

Change the percent to a decimal. $= 4500(0.047)5$

Multiply. $= 1057.50$ Wayne would earn \$1057.50 interest.

Example 2

Use the numbers in Example 1 to find how much money Wayne would have if he earned 4.7% interest compounded annually.

Put the numbers in the formula $A = P(1 + r)^t$. $A = 4500(1 + 4.7\%)^5$

Change the percent to a decimal. $= 4500(1 + 0.047)^5$

Multiply. $= 5661.69$ Wayne would have \$5661.69.

Subtract the results of the two examples to compare the difference in earnings when Wayne is earning compound interest instead of simple interest. Wayne would earn \$1161.69 in compound interest: $\$5661.69 - \$4500 = \$1161.69$. $\$1161.69 - \$1057.50 = \$104.19$, so Wayne would have \$104.19 more with compound interest than he would have with simple interest.

Problems

Solve the following problems.

1. Tong loaned Jody \$70 for a month. He charged 5% simple interest for the month. How much did Jody have to pay Tong?
2. Jessica's grandparents gave her \$5000 for college to put in a savings account until she starts college in four years. Her grandparents agreed to pay her an additional 6.5% simple interest on the \$5000 for every year. How much extra money will her grandparents give her at the end of four years?
3. David read an ad offering $7\frac{1}{4}\%$ simple interest on accounts over \$1500 left for a minimum of 5 years. He has \$1500 and thinks this sounds like a great deal. How much money will he earn in the 5 years?

4. Javier's parents set an amount of money aside when he was born. They earned 5.25% simple interest on that money each year. When Javier was 15, the account had a total of \$1181.25 interest paid on it. How much did Javier's parents set aside when he was born?
5. Kristina received \$125 for her birthday. Her parents offered to pay her 4.25% simple interest per year if she would save it for at least one year. How much interest could Kristina earn in a year?
6. Kristina decided she would do better if she put her money in the bank for one year, which paid 3.75% interest compounded annually. Was she right?
7. Suppose Jessica (from problem 2) had put her \$5000 in the bank at 2.85% interest compounded annually. How much money would she have earned there at the end of the 4 years?
8. Mai put \$3750 in the bank at 3.87% interest compounded annually. How much was in her account after 7 years?
9. What is the difference in the amount of money in the bank after five years if \$3500 is invested at 3.4% interest compounded annually or at 2.8% interest compounded annually?
10. Elizabeth was listening to her parents talking about what a good deal compounded interest was for a retirement account. She wondered how much money she would have if she invested \$2000 at age 20 at 2.8% interest compounded quarterly and left it until she reached age 65. Determine what the value of the \$2000 would become.
11. Elizabeth decided to try another problem. If she invested \$2500 at age 25 at 2.65% interest compounded quarterly, how much would it be worth when she reached age 65?
12. Compare your answers for problems 10 and 11. What are your conclusions?

Answers

1. $I = 70(0.05)1 = \$3.50$; Jody paid back \$73.50.
2. $I = 5000(0.065)4 = \$1300$
3. $I = \$1500(0.0725)5 = \543.75
4. $\$1181.25 = x(0.0525)15$; $x = \$1500$
5. $I = 125(0.0425)1 = \$5.31$
6. $A = 125(1 + 0.0375)^1 = \129.69 ; no, for one year she needs to take the higher interest rate if the compounding is done annually. Only after one year will compounding outstrip simple interest.
7. $A = 5000(1 + 0.0285)^4 = \5594.83
8. $A = 3750(1 + 0.0387)^7 = \4891.73
Jessica would have earned \$594.83.
9. $A = 3500(1 + 0.034)^5 - 3500(1 + 0.028)^5 = \$4136.86 - \$4018.22 = \118.64
10. $A = 2000(1 + 0.028)^{180}$ (because $45 \cdot 4 = 180$ quarters) = \$288,264.15
11. $A = 2500(1 + 0.0265)^{160}$ (because $40 \cdot 4 = 160$ quarters) = \$164,199.79
12. You should see that the number of years is an important factor when interest is compounded.

SOLVING INEQUALITIES

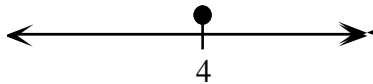
#31

To solve an inequality, first solve it as you would an equation. Use the solution as a **dividing point** of the line. Then test a value from each side of the dividing point on the number line. If the test number is true, then that part of the number line is part of the solution. In addition, if the inequality is \geq or \leq , then the dividing point is part of the solution and is indicated by a solid dot. If the inequality is $>$ or $<$, then the dividing point is not part of the solution, indicated by an open dot.

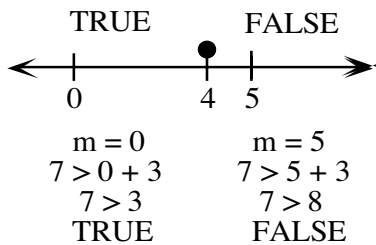
Example 1 $7 \geq m + 3$

Solve the equation: $7 = m + 3$
 $4 = m$

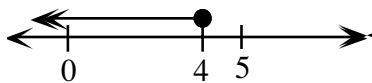
Draw a number line. Put a solid dot at 4.



Test a number on each side of 4 in the original inequality. We use 0 and 5.



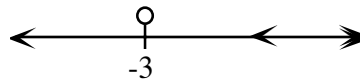
The solution is $m \leq 4$.



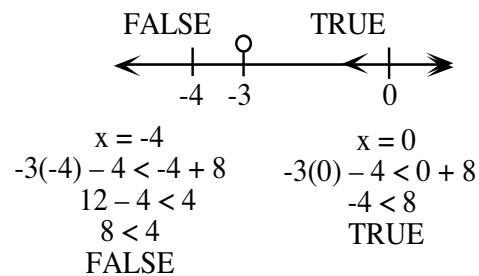
Example 2 $-3x - 4 < x + 8$

Solve the equation: $-3x - 4 = x + 8$
 $-3x = x + 12$
 $-4x = 12$
 $x = -3$

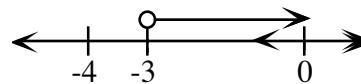
Draw a number line. Put an open dot at -3.



Test -4 and 0 in the original inequality.



The solution is $x > -3$.



Problems

Solve each inequality.

1. $x + 5 > 2$
2. $y - 5 \leq 8$
3. $-2x \leq -8$
4. $3m + 3 \geq -9$
5. $-5 < -2y + 3$
6. $7 \geq -3m + 4$
7. $3x - 2 < -x + 6$
8. $3(m + 2) \geq m - 6$
9. $2m + 3 \leq m + 6$
10. $4x + 3 \leq 2x - 7$

Answers

1. $x > -3$
2. $y \leq 13$
3. $x \geq 4$
4. $m \geq -4$
5. $y < 4$
6. $m \geq -1$
7. $x < 2$
8. $m \geq -6$
9. $m \leq 3$
10. $x \leq -5$

SOLVING LINEAR EQUATIONS

#32

Solving equations involves “undoing” what has been done to create the equation. By systematically working backward, the value of the variable can be found. Multiplication and division are inverse (opposite) operations, as are addition and subtraction. For example, to undo $x + 7 = 12$, that is, adding 7 to x , subtract 7 from both sides of the equation. The result is $x = 5$, which makes $x + 7 = 12$ true. For $3x = 24$, x is multiplied by 3, so divide both sides by 3 and the result is $x = 8$. Always remember to follow the correct order of operations.

Example 1

$4x$ means 4 times some number x . For example, suppose $4x = 20$. To solve, we do the inverse of multiplying by 4, which is dividing by 4. So $20 \div 4 = 5$, $x = 5$.

Example 2

$$\frac{x}{5} = 30$$

To solve, do the inverse of dividing by 5; that is, multiply by 5. The result is $x = 150$.

Example 3

$$\begin{array}{r} 4x + 6 = 18 \\ 4x = 12 \\ x = 3 \\ 4(3) + 6 = 18 \end{array}$$

Subtract 6.
Divide by 4.

Substitute 3 for x to check your answer.

Example 4

$$\frac{n}{2} + 8 = 14$$

Subtract 8.

$$\frac{n}{2} = 6$$

Multiply by 2.

$$n = 12$$

$$\frac{(12)}{2} + 8 = 14$$

Substitute to check.

Example 5

Solve the equation $2(3x + 3) = 4x + 18$.

$$\begin{array}{r} \overbrace{2(3x + 3)} \\ 2 \cdot 3x + 2 \cdot 3 = 4x + 18 \end{array}$$

Use the Distributive Property to eliminate the parentheses.

$$\begin{array}{r} 6x + 6 = 4x + 18 \\ -6 \quad -6 \end{array}$$

Subtract 6 from both sides of the equation.

$$\begin{array}{r} 6x = 4x + 12 \\ -4x \quad -4x \end{array}$$

Subtract $4x$ from both sides of the equation.

$$\frac{2x}{2} = \frac{12}{2}$$

Divide both sides by 2 to get one x .

$$x = 6$$

Substitute the solution into the original equation to check your answer.

$$2(3 \cdot 6 + 3) = 4 \cdot 6 + 18$$

Use the order of operations and simplify.

$$2(21) = 42$$

✓

Example 6

Solve the equation:

$$-\frac{2}{3}x - 1 = -3$$

OR

Add 1 to both sides of the equation.

$$-\frac{2}{3}x = -2$$

$$-\frac{2}{3}x = -2$$

Use reciprocals or division to solve.

$$\left(-\frac{3}{2}\right)\left(-\frac{2}{3}\right)x = -2\left(-\frac{3}{2}\right)$$
$$x = 3$$

$$\left(-\frac{2}{3}\right)x \div \left(-\frac{2}{3}\right) = -2 \div \left(-\frac{2}{3}\right)$$
$$x = 3$$

Problems

Solve these equations. Remember to check your answers.

1. $4x + 3 = 7$
2. $2x - 3 = 7$
3. $x - 10 = 35$
4. $2x - 10 = -4$
5. $\frac{x}{4} + 2 = 6$
6. $\frac{n}{3} - 6 = 2$
7. $8x + 12 = 132$
8. $14x - 9 = 75$
9. $21 = 4y - 7$
10. $16 + 6y = 82$
11. $\frac{x}{3} - 8 = -15$
12. $\frac{x}{15} - 7 = -2$
13. $3(4x - 3) = 63$
14. $2(2x + 5) = 33$
15. $-3(2x + 4) = -5(x - 3)$
16. $-4(-5x - 2) = 16$
17. $4x + 2x + 9 = -x + 12$
18. $x + 3(x - 1) = x + 18$
19. $2x + 4 + x - 7 = 3(x - 7)$
20. $4(m - 2) = -3(m - 16)$
21. $\frac{1}{4}x = 4$
22. $\frac{2}{3}x = 18$
23. $-\frac{1}{4}x = 5$
24. $-\frac{5}{2}x = -15$
25. $\frac{1}{3}x - 5 = -3$
26. $-\frac{2}{3}x - 14 = -10$

Solve the remaining equations for the indicated variable.

27. $2y + b = 9$ for b
28. $rt = 7$ for t
29. $5x - 2d = m$ for x
30. $3x + 2y = 4$ for x

Answers

1. $x = 1$
2. $x = 5$
3. $x = 45$
4. $x = 3$
5. $x = 16$
6. $n = 24$
7. $x = 15$
8. $x = 6$
9. $y = 7$
10. $y = 11$
11. $x = -21$
12. $x = 75$
13. $x = 6$
14. $x = \frac{23}{4} = 5\frac{3}{4}$
15. $x = -27$
16. $x = \frac{2}{5}$
17. $x = \frac{3}{7}$
18. $x = 7$
19. $x = -9$
20. $m = 8$
21. $x = 16$
22. $x = 27$
23. $x = -20$
24. $x = 6$
25. $x = 6$
26. $x = -6$
27. $b = 9 - 2y$
28. $t = \frac{7}{r}$
29. $x = \frac{m+2d}{5}$
30. $x = \frac{4-2y}{3}$

SUBSTITUTION AND EVALUATION

#33

SUBSTITUTION is replacing one symbol with an equivalent symbol (a number, a variable, or an expression). One application of the substitution property is replacing a variable name with a number in an expression or equation. A **VARIABLE** is a letter or symbol used to represent one or more numbers (or other algebraic expression). The numbers are the values of the variable. A variable expression has numbers and variables with arithmetic operations performed on it.

In general, if $a = b$, then a may replace b and b may replace a .

Examples

Evaluate each variable expression for $x = 3$.

a) $5x \Rightarrow 5(3) \Rightarrow 15$

b) $x + 10 \Rightarrow (3) + 10 \Rightarrow 13$

c) $\frac{18}{x} \Rightarrow \frac{18}{3} \Rightarrow 6$

d) $\frac{x}{3} \Rightarrow \frac{3}{3} \Rightarrow 1$

e) $3x - 5 \Rightarrow 3(3) - 5 \Rightarrow 9 - 5 \Rightarrow 4$

f) $5x + 3x \Rightarrow 5(3) + 3(3) \Rightarrow 15 + 9 \Rightarrow 24$

Problems

Evaluate each of the variable expressions below for the values $x = -4$ and $y = 3$. Be sure to follow the order of operations as you simplify each expression.

1. $x + 4$

2. $x - 1$

3. $x + y + 3$

4. $y - 3 + x$

5. $x^2 - 5$

6. $-x^2 + 5$

7. $x^2 + 4x - 3$

8. $-3x^2 + 2x$

9. $x + 3 + 2y$

10. $y^2 + 3x - 2$

11. $x^2 + y^2 + 3^2$

12. $2^2 + y^2 - 2x^2$

Evaluate the expressions below using the values of the variables in each problem. These problems ask you to evaluate each expression twice, once with each of the values.

13. $3x^2 - 2x + 5$ for $x = -3$ and $x = 4$

14. $2x^2 - 3x + 6$ for $x = -2$ and $x = 5$

15. $-3x^2 + 7$ for $x = -3$ and $x = 2$

16. $-2x^2 + 5$ for $x = -4$ and $x = 5$

Evaluate the variable expressions for $x = -3$ and $y = 4$.

17. $x(x + 3x)$

18. $2(x + 2x)$

19. $2(x + y) + 4\left(\frac{y+2}{x}\right)$

20. $3\left(y^2 + 2\left(\frac{x+7}{2}\right)\right)$

21. $2y(x + x^2 - 2y)$

22. $(3x + y)(2x + 4y)$

Answers

1. 0

2. -5

3. 2

4. -4

5. 11

6. -11

7. -3

8. -56

9. 5

10. -5

11. 34

12. -19

13. 38; 45

14. 20; 41

15. -20; -5

16. -27; -45

17. 36

18. -18

19. 20

20. 36

21. -16

22. -50

SOLVING PROPORTIONS

#34

A **proportion** is an equation stating that two ratios (fractions) are equal. To solve a proportion, use cross multiplication to remove the fractions, then solve in the usual way. Cross multiplication may only be used to solve proportions, that is, when each side of the equation is a ratio. Multiply the numerator of the left ratio by the denominator of the right ratio, write an equal sign, then multiply the denominator of the left ratio by the numerator of the right ratio.

Example 1

$$\frac{m}{6} = \frac{12}{5}$$

$$5 \cdot m = 6 \cdot 12$$

$$5m = 72$$

$$m = 14.4$$

Example 2

$$\frac{6}{\$9.75} = \frac{6.5}{w}$$

$$6 \cdot w = \$9.75 \cdot 6.5$$

$$6w = \$63.375$$

$$w = \$10.56$$

Example 3

$$\frac{x+2}{3} = \frac{x-2}{7}$$

$$7(x+2) = 3(x-2)$$

$$7x+14 = 3x-6$$

$$4x = -20$$

$$x = -5$$

Problems

Solve these equations. Remember to check your answers.

1. $\frac{4}{5} = \frac{y}{15}$

2. $\frac{x}{32} = \frac{4}{8}$

3. $\frac{3}{200} = \frac{6}{m}$

4. $\frac{3}{8} = \frac{x}{50}$

5. $\frac{8}{18} = \frac{3}{m}$

6. $\frac{8}{500} = \frac{m}{1500}$

7. $\frac{6}{\$10.75} = \frac{x}{\$24.95}$

8. $\frac{6}{\$4.35} = \frac{7}{x}$

9. $\frac{20}{40} = \frac{45}{x}$

10. $\frac{40}{70} = \frac{35}{x}$

11. $\frac{20}{40} = \frac{45}{x}$

12. $\frac{2y}{5} = \frac{16}{10}$

13. $\frac{x}{x+3} = \frac{3}{4}$

14. $\frac{2}{3x} = \frac{5}{x+1}$

15. $\frac{3}{2y} = \frac{8}{y-2}$

16. $\frac{2x}{5} = \frac{x-2}{76}$

17. $\frac{4x}{5} = \frac{x+2}{6}$

18. $\frac{7+x}{5} = \frac{12}{3}$

19. $\frac{6-y}{4} = \frac{3y}{4}$

20. $\frac{x-2}{3} = \frac{8}{9}$

21. $\frac{1}{2x} = \frac{7}{x+1}$

22. $\frac{4}{2y} = \frac{5}{y-3}$

23. $\frac{7}{2x+3} = \frac{3}{x}$

24. $\frac{3y-4}{12} = \frac{2y}{4}$

Answers

1. $y = 12$

2. $x = 16$

3. $m = 400$

4. $x = 18.75$

5. $x = 6.75$

6. $m = 24$

7. $x = 13.93$

8. $m = \$5.08$

9. $x = 90$

10. $x = 61.25$

11. $x = 90$

12. $y = 4$

13. $x = 9$

14. $x = 0.15$

15. $y \approx -0.46$

16. $x = -\frac{10}{7} = -1\frac{3}{7}$

17. $x = \frac{10}{19}$

18. $x = 13$

19. $x = \frac{3}{2} = 1\frac{1}{2}$

20. $x = \frac{14}{3} = 4\frac{2}{3}$

21. $x = \frac{1}{13}$

22. $y = -2$

23. $x = 9$

24. $y = -\frac{4}{3} = -1\frac{1}{3}$

STEM-AND-LEAF PLOT

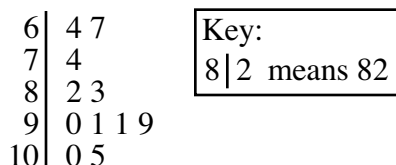
#35

A **STEM-AND-LEAF-PLOT** shows the individual values from a set of data and how the values are distributed. This type of display clearly shows **median, mode, range, and outliers**. The “stem” part on the graph represents the leading digit(s) of the number. The “leaf” part of the graph represents the remaining digit. A stem-and-leaf plot includes a key to make the arrangement of the plot clear.

Put the values in order, from lowest to highest. The stem represents the leading digit(s). Place the “leaf” part in numerical order, lowest to highest.

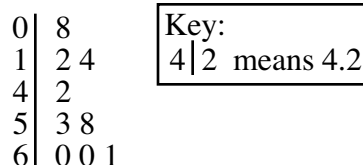
Example 1

Make a stem-and-leaf plot of this set of data:
83, 90, 82, 74, 91, 91, 99, 105, 100, 64, and 67.



Example 2

Make a stem-and-leaf plot of this set of data:
5.3, 5.8, 4.2, 6, 6.1, 6, 0.8, 1.2, and 1.4.



Problems

Make a stem-and-leaf plot of each set of data.

- 16, 22, 33, 41, 42, 41, 41, 40, 27, and 18
- 121, 90, 83, 97, 96, 84, 110, 118, 128, and 120
- 41, 38, 47, 48, 22, 28, 25, 40, and 30
- 7, 10, 11, 12, 11, 11, 13, 8, 22, and 23
- 1.6, 2.4, 1.8, 2.3, 1.9, 1.6, 2, 3.1, 3.8, 4.1, and 4
- 10.8, 9.7, 6.1, 8.1, 8.3, 8.5, 8.5, 9.2, 9.1, and 9.1
- 76, 83, 54, 43, 44, 44, 103, 99, and 94
- 8, 12, 14, 11, 11, 15, 9, 21, 25, and 27

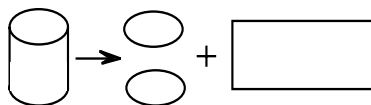
Answers

- | | | | |
|---|--|-----------|----------------------|
| 1 | | 6 8 | Key:
2 2 means 22 |
| 2 | | 2 7 | |
| 3 | | 3 | |
| 4 | | 0 1 1 1 2 | |
| | | | |
- | | | | |
|----|--|-------|----------------------|
| 8 | | 3 4 | Key:
9 6 means 96 |
| 9 | | 0 6 7 | |
| 10 | | | |
| 11 | | 0 8 | |
| 12 | | 0 1 8 | |
- | | | | |
|---|--|---------|----------------------|
| 2 | | 2 5 8 | Key:
2 2 means 22 |
| 3 | | 0 8 | |
| 4 | | 0 1 7 8 | |
| | | | |
- | | | | |
|---|--|-------------|---------------------|
| 0 | | 7 8 | Key:
0 7 means 7 |
| 1 | | 0 1 1 1 2 3 | |
| 2 | | 2 3 | |
- | | | | |
|---|--|---------|-----------------------|
| 1 | | 6 6 8 9 | Key:
2 3 means 2.3 |
| 2 | | 0 3 4 | |
| 3 | | 1 8 | |
| 4 | | 1 | |
- | | | | |
|----|--|---------|-----------------------|
| 6 | | 1 | Key:
8 1 means 8.1 |
| 7 | | | |
| 8 | | 1 3 5 5 | |
| 9 | | 1 1 2 7 | |
| 10 | | 8 | |
- | | | | |
|----|--|-------|----------------------|
| 4 | | 3 4 4 | Key:
5 4 means 54 |
| 5 | | 4 | |
| 6 | | | |
| 7 | | 6 | |
| 8 | | 3 | |
| 9 | | 4 9 | |
| 10 | | 3 | |
| | | | |
| | | | |
| | | | |
| | | | |
- | | | | |
|---|--|-----------|---------------------|
| 0 | | 8 9 | Key:
0 9 means 9 |
| 1 | | 1 1 2 4 5 | |
| 2 | | 1 5 7 | |

SURFACE AREA OF A CYLINDER

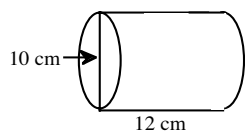
#36

The surface area of a cylinder is the sum of the two base areas and the lateral surface area. The lateral (side) surface is a rectangle when the cylinder's bases are removed, the lateral side is cut vertically, and the curved surface is laid flat. The formula for the surface area, where r = radius and h = height of the cylinder, is:



$$SA = 2\pi r^2 + 2\pi rh$$

Example 1



$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi (5)^2 + 2\pi \cdot 5 \cdot 12 \\ &= 50\pi + 120\pi \\ &= 170\pi \approx 534 \text{ cm}^2 \end{aligned}$$

Example 2



If the volume of the tank above is 2826 ft^3 , what is the surface area? (See page 97 for volume.)

$$\begin{aligned} V &= \pi r^2 h & SA &= 2\pi r^2 + 2\pi rh \\ 2826 &= \pi r^2 (4) & &= 2\pi(15)^2 + 2\pi(15)(4) \\ \frac{2826}{4\pi} &= r^2 & &= 450\pi + 120\pi \\ 225 &= r^2 & &= 570\pi \approx 1789.8 \text{ ft}^2 \\ 15 &= r & & \end{aligned}$$

Problems

Find the surface area of each cylinder.

- $r = 7 \text{ cm}$, $h = 7 \text{ cm}$
- $r = 8 \text{ cm}$, $h = 11 \text{ cm}$
- $r = 9 \text{ cm}$, $h = 13 \text{ cm}$
- $r = 10 \text{ cm}$, $h = 15 \text{ cm}$
- $r = 15 \text{ cm}$, $h = 9 \text{ cm}$
- $r = 16 \text{ cm}$, $h = 8 \text{ cm}$
- $r = 17 \text{ cm}$, $h = 6 \text{ cm}$
- $r = 18 \text{ cm}$, $h = 4 \text{ cm}$
- $r = 32 \text{ cm}$, $h = 2 \text{ cm}$
- $r = 10 \text{ cm}$, $h = 1 \text{ cm}$

Find the height of each cylinder.

- $r = 10 \text{ cm}$, $SA = 690.8 \text{ cm}^2$
- $r = 5 \text{ cm}$, $SA = 408.2 \text{ cm}^2$

Answers

- 615.44 cm^2
- 954.56 cm^2
- 1243.44 cm^2
- 1570.80 cm^2
- 2260.8 cm^2
- 2411.52 cm^2
- 2455.48 cm^2
- 2486.88 cm^2
- 6832.64 cm^2
- 690.8 cm^2
- 0.99 cm
- 7.99 cm

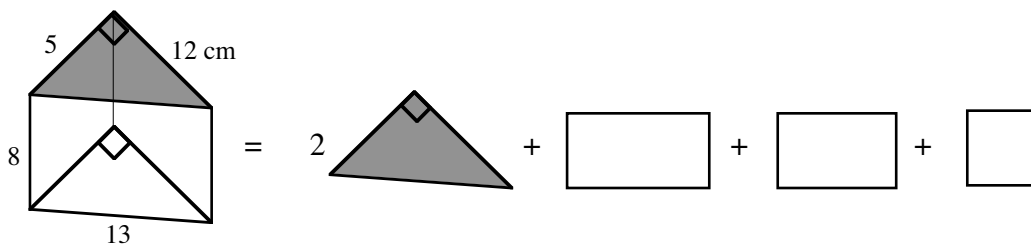
SURFACE AREA OF A PRISM

#37

The **SURFACE AREA OF A PRISM (SA)** is the sum of the areas of all of the faces, including the bases. Surface area is expressed in square units.

Example

Find the surface area of the triangular prism below.



Subproblem 1: Find the area of each face.

$$SA = 2\left(\frac{1}{2} \cdot 5 \cdot 12\right) + 8 \cdot 13 + 8 \cdot 12 + 8 \cdot 5$$

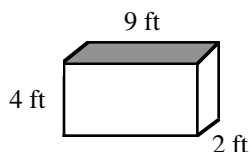
Subproblem 2: Find the sum of the areas.

$$SA = 60 + 104 + 96 + 40 \\ = 300 \text{ cm}^2$$

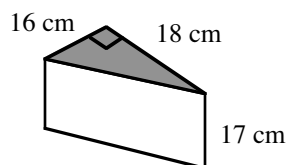
Problems

Calculate the surface area of each prism. One base of some figures is shaded.

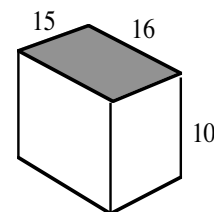
1. Rectangular prism



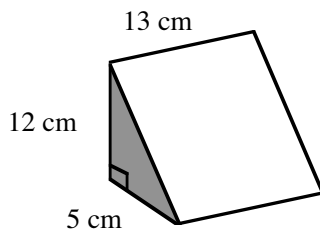
2. Right triangular prism



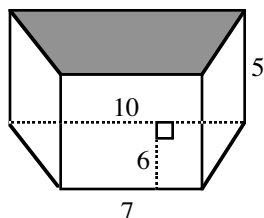
3. Rectangular prism



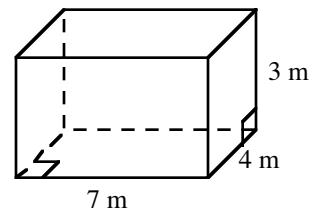
4. Right triangular prism



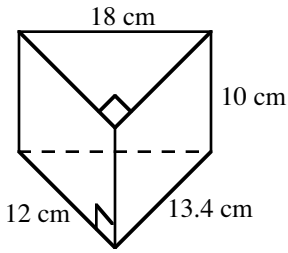
5. Trapezoidal prism



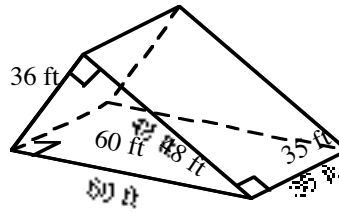
6. Rectangular prism



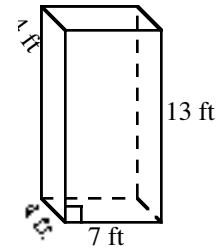
7. Right triangular prism



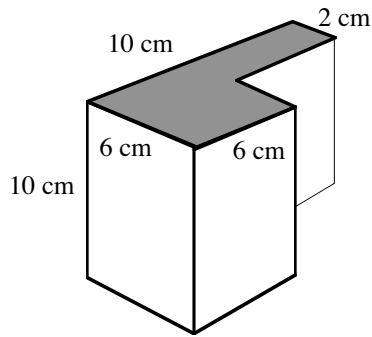
8. Right triangular prism



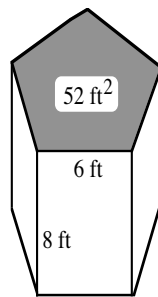
9. Rectangular prism



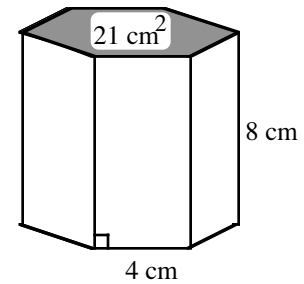
10. Figure is a prism. The base and face angles are all right angles.



11. Regular pentagonal prism

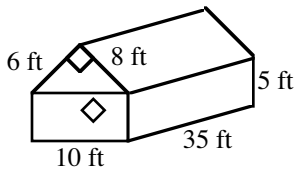


12. Regular hexagonal prism

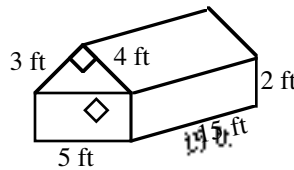


In figures 13-16, the sides and the tops are rectangles (except for the triangular regions).

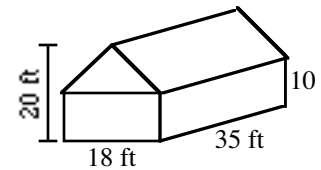
13.



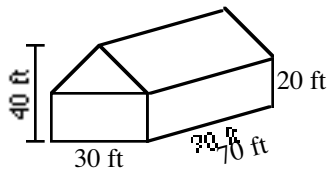
14.



15.



16.



Answers

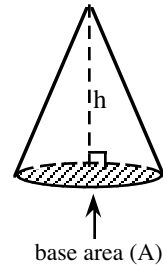
1. 124 feet² 2. 1274 cm² 3. 1100 units² 4. 450 cm² 5. 249 units²
 6. 122 m² 7. 594.8 cm² 8. 6768 feet² 9. 342 feet² 10. 408 cm²
 11. 344 feet² 12. 234 cm² 13. 1338 feet² 14. 272 feet² 15. 2342.5 feet²
 16. 10,225 feet²

VOLUME OF A CONE

#38

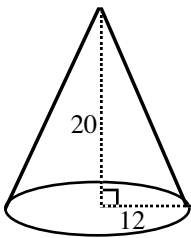
Every cone has a **VOLUME** that is one-third the volume of the cylinder with the same base and height. To find the volume of a cone, use the same formula as the volume of a cylinder and divide by three. The formula for the volume of a cone of base area A and height h is:

$$\text{volume} = \frac{1}{3} (\text{area of base})(\text{height}) \text{ or } V = \frac{A \cdot h}{3} = \frac{\pi r^2 h}{3} \text{ or } \frac{1}{3} \pi r^2 h$$



Example 1

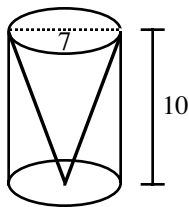
Find the volume of the cone below.



$$\begin{aligned} \text{Volume} &= \frac{1}{3} \pi (12)^2 \cdot 20 \\ &= \frac{2880\pi}{3} \\ &\approx 3014.4 \text{ units}^3 \end{aligned}$$

Example 2

Find the volume of the cone below.



$$\begin{aligned} \text{radius} &= 3.5 \\ \text{Volume} &= \frac{1}{3} \pi (3.5)^2 \cdot 10 \\ &= \frac{122.5}{3} \pi \\ &\approx 128.22 \text{ units}^3 \end{aligned}$$

Example 3

If the volume of a cone is 1441.26 cm^3 and its radius is 9 cm , find its height.

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \pi r^2 h \\ 1441.26 &= \frac{1}{3} \pi (9)^2 \cdot h \\ 4323.78 &= \pi (81) \cdot h \\ \frac{4323.78}{81\pi} &= h \\ 17 \text{ cm} &= h \end{aligned}$$

Problems

Find the volume of each cone. Use $\pi \approx 3.14$. Round to the cubic unit. Hint: first solve a subproblem in problems 9 and 10.

1. base area = 50 cm^2
h = 3 cm

2. base area = 17 cm^2
h = 10 cm

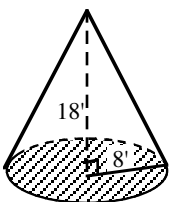
3. r = 5 cm
h = 12 cm

4. r = 7.5 in.
h = 3.6 in.

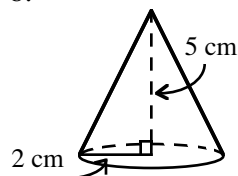
5. diameter = 28 cm
h = 5 cm

6. d = 29 cm
h = 41 cm

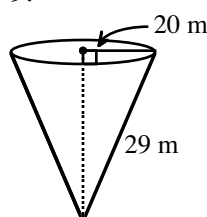
7.



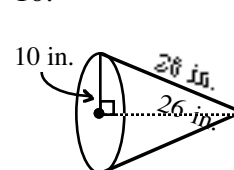
8.



9.



10.



Find the missing part of each cone. Use $\pi \approx 3.14$.

11. If the volume is 355 ft^3 and the height is 12 ft, find the radius of the base.
12. If the volume is 4000 ft^3 and the height is 25.4 ft, find the radius of the base.
13. If the volume is 864 ft^3 and the height is 9 ft, find the diameter of the base.
14. If the volume is 864 ft^3 and the height is 30 ft, find the diameter of the base.
15. If the volume is $14,736.02 \text{ inches}^3$ and the radius is 19 inches, find the height.
16. If the volume is $22,372.5 \text{ inches}^3$ and the radius is 25 inches, find the height.
17. If the circumference of the base is 25 cm and the height is 8 cm, find the volume. Round to the cubic cm.
18. If the circumference of the base is 14 cm and the height is 11 cm, find the volume. Round to the cubic cm.

Answers

- | | | | | |
|------------------------|------------------------|-----------------------|------------------------|--------------------------|
| 1. 50 cm^3 | 2. 57 cm^3 | 3. 314 cm^3 | 4. 212 in.^3 | 5. 1026 cm^3 |
| 6. 9023 cm^3 | 7. 1206 ft^3 | 8. 21 cm^3 | 9. 8796 m^3 | 10. 2513 in.^3 |
| 11. 5.3 ft | 12. 12.3 ft | 13. 19.2 ft | 14. 10.4 ft | 15. 39 in. |
| 16. 34.2 in. | 17. 133 cm^3 | 18. 57 cm^3 | | |

VOLUME OF A CYLINDER

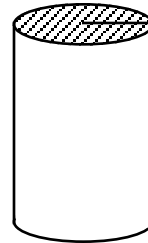
#39

The volume of a cylinder is the area of its base multiplied by its height:

$$\text{Volume} = (\text{Area of Base})(\text{height}) \text{ or } V = A \cdot h.$$

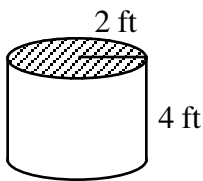
Since the base of a cylinder is a circle of area $A = \pi r^2$, we can write:

$$V = \pi r^2 h.$$



Example 1

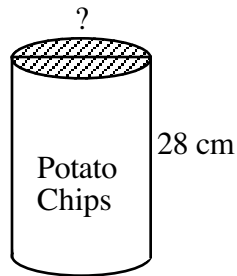
Find the volume of the cylinder.



$$\begin{aligned} \text{Volume} &= \pi r^2 h \\ &= \pi(2)^2(4) \\ &= 16\pi \\ &= 50.27 \text{ ft}^3 \end{aligned}$$

Example 2

The can has volume 3165 cm^3 and height 28 cm .
What is its diameter?



$$\begin{aligned} \text{Volume} &= \pi r^2 h \\ 3165 &= \pi r^2(28) \\ \frac{3165}{28\pi} &= r^2 \\ 36 &= r^2 \\ \text{radius} &= 6 \\ \text{diameter} &= 2(6) = 12 \text{ cm} \end{aligned}$$

Problems

Find the volume of each cylinder. Use 3.14 for π . Round your answer to the cubic unit.

1. base area = 50 cm^2
h = 7 cm

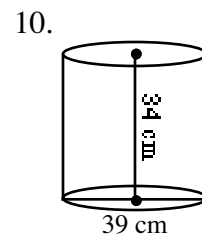
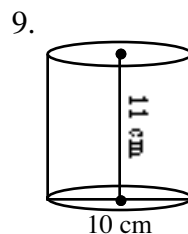
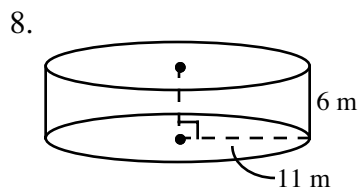
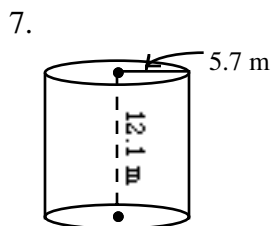
2. base area = 17 cm^2
h = 10 cm

3. r = 5 cm
h = 12 cm

4. r = 7.5 in.
h = 3.6 in.

5. diameter = 28 cm
h = 5 cm

6. d = 29 cm
h = 41 cm



Find the missing part of each cylinder.

11. If the volume is 355 ft^3 and the height is 12 ft, find the radius, to the nearest foot.
12. If the volume is 4000 ft^3 and the height is 25.4 ft, find the radius, to the nearest foot.
13. If the volume is 864 ft^3 and the height is 9 ft, find the diameter, to the nearest foot.
14. If the volume is 864 ft^3 and the height is 30 ft, find the diameter, to the nearest foot.
15. If the volume is $14,736.02 \text{ inches}^3$ and the radius is 19 inches, find the height, to the nearest inch.
16. If the volume is $22,372.5 \text{ inches}^3$ and the radius is 25 inches, find the height, to the nearest inch.
17. If the circumference is 25 cm and the height is 8 cm, find the volume. Round to the cubic cm.
18. If the circumference is 14 cm and the height is 11 cm, find the volume. Round to the cubic cm.

Answers

- | | | | | |
|--------------------------|------------------------|------------------------|------------------------|---------------------------|
| 1. 350 cm^3 | 2. 170 cm^3 | 3. 942 cm^3 | 4. 636 in.^3 | 5. 3077 cm^3 |
| 6. $27,068 \text{ cm}^3$ | 7. 1234 m^3 | 8. 2280 m^3 | 9. 864 cm^3 | 10. $40,595 \text{ cm}^3$ |
| 11. 3 ft | 12. 7 ft | 13. 11 ft | 14. 6 ft | 15. 13 in. |
| 16. 11 in. | 17. 398 cm^3 | 18. 172 cm^3 | | |

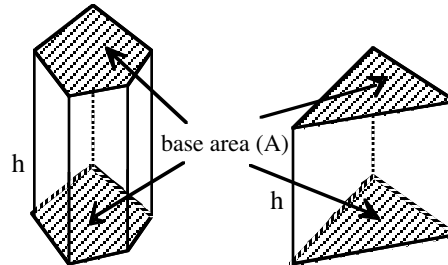
VOLUME OF A PRISM

#40

Volume is a three-dimensional concept. It measures the amount of interior space of a three-dimensional figure based on a cubic unit, that is, the number of 1 by 1 by 1 cubes that will fit inside a figure.

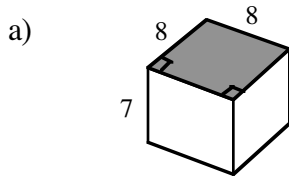
The volume of any prism is the area of either base (A) times the height (h) of the prism.

$$V = (\text{Area of base}) \cdot (\text{height}) \text{ or } V = Ah$$



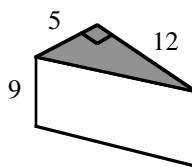
Example 1

Find the volume of each figure below. Show the steps you use in your subproblems.



This is a square prism.
The base is a square with area (A) $8 \cdot 8 = 64 \text{ units}^2$.

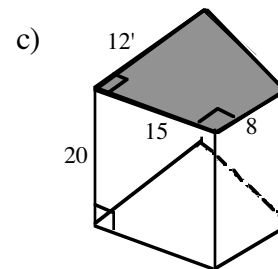
$$\begin{aligned} \text{Volume} &= A(h) \\ &= 64(7) \\ &= 448 \text{ units}^3 \end{aligned}$$



This is a triangular prism.
The base is a right triangle with area (A).

$$\frac{1}{2}(5)(12) = 30 \text{ units}^2.$$

$$\begin{aligned} \text{Volume} &= A(h) \\ &= 30(9) \\ &= 270 \text{ units}^3 \end{aligned}$$



This is a trapezoidal prism with area (A).

$$\frac{1}{2}(12 + 8)(15) = 150 \text{ ft}^2.$$

$$\begin{aligned} \text{Volume} &= A(h) \\ &= 150(20) \\ &= 3000 \text{ ft}^3 \end{aligned}$$

Example 2

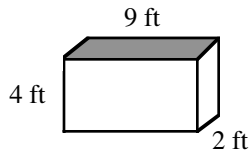
Find the height of the prism with a volume of 3240 cm^3 and base area of 40.5 cm^2 .

$$\begin{aligned} \text{Volume} &= A(h) \\ 3240 &= 40.5(h) \\ \frac{3240}{40.5} &= h \\ 80 \text{ cm} &= h \end{aligned}$$

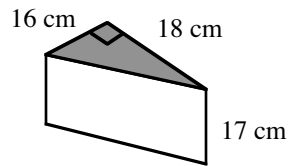
Problems

Calculate the volume of each prism. One base in some figures is shaded.

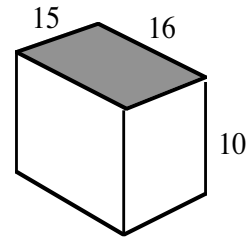
1. Rectangular prism



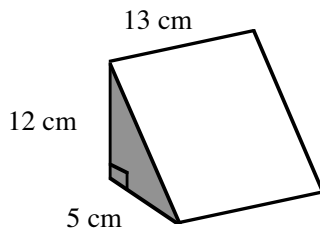
2. Right triangular prism



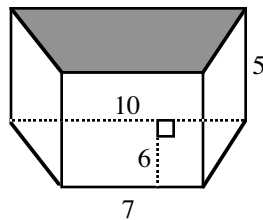
3. Rectangular prism



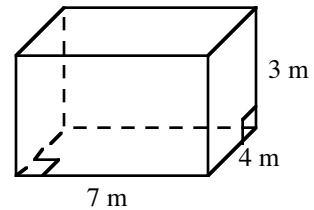
4. Right triangular prism



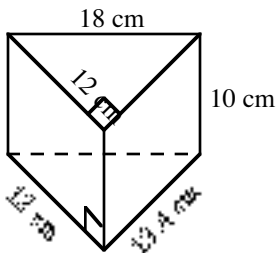
5. Trapezoidal prism



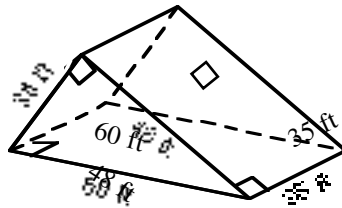
6. Rectangular prism



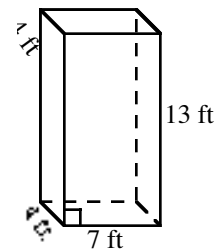
7. Right triangular prism



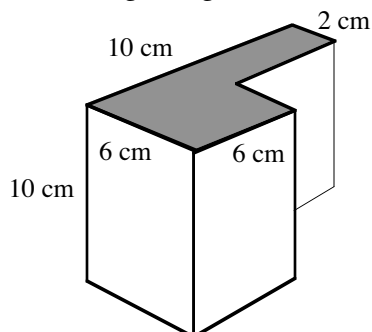
8. Right triangular prism



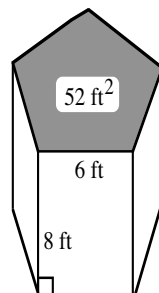
9. Rectangular prism



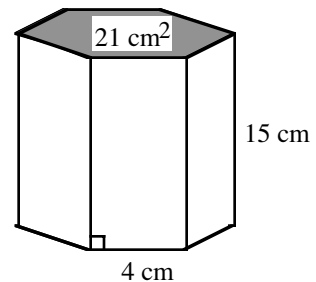
10. Figure is a prism. The base and face angles are all right angles.



11. Regular pentagonal prism

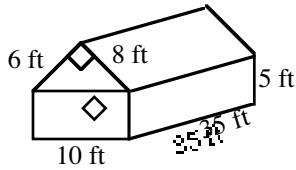


12. Regular hexagonal prism

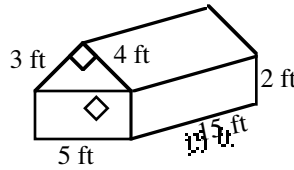


In figures 13-16, the sides and the tops are rectangles (except for the triangular regions).

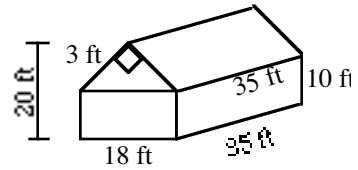
13.



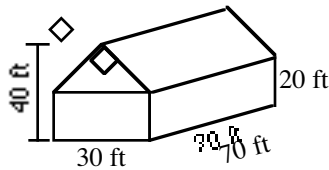
14.



15.



16.



17. Find the volume of a prism with base area 44 cm^2 and height 1.5 cm .
18. Find the volume of a prism with base area 75 cm^2 and height 26.2 cm .
19. Find the height of a prism with base area 32 cm^2 and volume 179 cm^3 .
20. Find the area of the base of a prism with volume 760.48 cm^3 and height 9.8 cm .

Answers

- | | | | | |
|---------------------------|------------------------|---------------------------|------------------------|--------------------------|
| 1. 72 ft^3 | 2. 2448 cm^3 | 3. 2400 units^3 | 4. 390 cm^3 | 5. 255 units^3 |
| 6. 84 m^3 | 7. 804 cm^3 | 8. $51,840 \text{ ft}^3$ | 9. 364 ft^3 | 10. 440 cm^3 |
| 11. 416 ft^3 | 12. 315 cm^3 | 13. 2590 ft^3 | 14. 240 ft^3 | 15. 9450 ft^3 |
| 16. $66,150 \text{ ft}^3$ | 17. 66 cm^3 | 18. 1965 cm^3 | 19. 5.6 cm | 20. 77.6 cm^2 |

WRITING EQUATIONS FROM A GUESS AND CHECK TABLE**#41**

You have used Guess and Check tables to solve problems. However, solving complicated problems with a Guess and Check table can be time consuming and it may be difficult to find the correct solution if it is not an integer. The patterns developed in the Guess and Check table can be generalized by using a variable to write an equation. Once you have an equation for the problem, it is often more efficient to solve the equation than to continue to guess and check. Most of the problems here will not be complex so that you can practice writing equations from Guess and Check tables.

Example 1

A box of fruit has 4 times as many oranges as grapefruit. Together there are 60 pieces of fruit. How many pieces of each type of fruit are there?

Guess Number of Grapefruit	Number of Oranges	Total Pieces of Fruit	Check 60
10	40	50	too low
15	60	75	too high

After several guesses and checks establish a pattern in the problem, you can generalize the pattern using a variable. Since we could guess any number of grapefruit, use x to represent it. The pattern for the number of oranges is four times the number of grapefruit, or $4x$. The total pieces of fruit is the sum of column one and column two, so our table becomes:

Guess Number of Grapefruit	Number of Oranges	Total Pieces of Fruit	Check 60
x	$4x$	$x + 4x$	$= 60$

Since we want the total to agree with the check, our equation is $x + 4x = 60$. Simplifying this yields $5x = 60$, so $x = 12$ (grapefruit) and then $4x = 48$ (oranges).

Example 2

The perimeter of a rectangle is 90 feet. If the length of the rectangle is 15 feet more than the width, what are the dimensions (length and width) of the rectangle?

Guess Width	Length	Perimeter	Check 90
10	25	$(10 + 25) \cdot 2 = 70$	too low
20	35	110	too high

Again, since we could guess any width, we put an x in this column. The pattern for the second column is that it is 15 more than the first: $x + 15$. Perimeter is found by multiplying the sum of the width and length by 2. Our table now becomes:

Guess Width	Length	Perimeter	Check 90
x	$x + 15$	$(x + x + 15) \cdot 2$	$= 90$

>>The example continues on the next page.>>

Solving the equation: $(x + x + 15) \cdot 2 = 90$
 $2x + 2x + 30 = 90$
 $4x + 30 = 90$
 $4x = 60$ So $x = 15$ (width) and $x + 15 = 30$ (length).

Example 3

Jorge has some dimes and quarters. He has 8 more dimes than quarters and the collections of coins is worth \$7.45. How many dimes and quarters does Jorge have?

Guess Number of Quarters	Number of Dimes	Value of Quarters	Value of Dimes	Total Value	Check \$7.45
10	18	2.50	1.80	4.30	too low
20	28	5.00	2.80	7.80	too high
x	$x + 8$	$0.25x$	$0.10(x + 8)$	7.45	

Since you need to know both the number of coins and their value, the equation is more complicated. The number of quarters becomes x , but then in the table the Value of Quarters column is $0.25x$. Thus the number of dimes is $x + 8$, but the value of dimes is $0.10(x + 8)$. Finally, to find the numbers, the equation becomes $0.25x + 0.10(x + 8) = 7.45$.

Solving the equation: $0.25x + 0.10x + 0.8 = 7.45$
 $0.35x + 0.8 = 7.45$
 $0.35x = 6.65$ So $x = 19$

Thus there are 19 quarters worth \$4.75 and 27 dimes worth \$2.70 for a total value of \$7.45.

Problems

Start the problems using a Guess and Check table. Then write an equation. Solve the equation.

1. A wooden board 100 centimeters long is cut into two pieces. One piece is 16 centimeters longer than the other. What are the lengths of the two pieces?
2. Thu is six years older than her brother Tuan. The sum of their ages is 48. What are their ages?
3. Tomas is thinking of a number. If he doubles his number and subtracts 27, the result is 407. Of what number is Tomas thinking?
4. Two consecutive numbers have a sum of 327. What are the two numbers?
5. Two consecutive even numbers have a sum of 186. What are the numbers?
6. Joanne's age is two times Devin's age and Devin is eight years older than Christena. If the sum of their ages is 76, what is Christena's age? Joanne's age? Devin's age?
7. Farmer Fran has 47 barnyard animals, consisting of only chickens and sheep. If these animals have 138 legs, how many of each type of animal are there?

8. A wooden board 228 centimeters long is cut into three parts. The two longer parts are the same length and are 18 centimeters longer than the shortest part. How long are the three parts?
9. Juan has 16 coins, all nickels and dimes. This collection of coins is worth \$1.15. How many nickels and dimes are there?
10. Tickets to the school play are \$7.00 for adults and \$5.50 for students. If the total value of all the tickets sold is \$3825 and 100 more students bought tickets than adults, how many adults and students bought tickets?
11. A wooden board 180 centimeters long is cut into six pieces: four short ones of equal length and two that are each 18 centimeters longer than the shorter ones. What are the lengths of the boards?
12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There are five more nickels than quarters but four times as many dimes as quarters. If the entire collection is worth \$5.85, how many nickels, dimes, and quarters are there?

Answers

- | | |
|---|---|
| 1. $x + (x + 16) = 100$
The lengths of the boards are 42 cm and 58 cm. | 2. $x + (x + 6) = 48$
Thu is 27 years old and her brother is 21 years old. |
| 3. $2x - 27 = 407$
Tomas is thinking of the number 217. | 4. $x + (x + 1) = 327$
The two consecutive numbers are 163 and 164. |
| 5. $x + (x + 2) = 186$
The two consecutive numbers are 92 and 94. | 6. $x + (x + 8) + 2(x + 8) = 76$
Christena is 13, Devin is 21, and Joanne is 42 years old. |
| 7. $2x + 4(47 - x) = 138$
Farmer Fran has 22 sheep and 25 chickens. | 8. $x + (x + 18) + (x + 18) = 228$
The lengths of the boards are 64, 82, and 82 cm. |
| 9. $0.05x + 0.10(16 - x) = 1.15$
Juan has 9 nickels and 7 dimes. | 10. $\$7x + \$5.50(x + 100) = \$3825$
There were 262 adult and 362 student tickets purchased for the play. |
| 11. $4x + 2(x + 18) = 180$
The lengths of the boards are 24 and 42 cm. | 12. $0.25x + 0.05(x + 5) + 0.10(4x) = 5.85$
Conrad has 8 quarters, 13 nickels, and 32 dimes. |

WRITING AND GRAPHING LINEAR EQUATIONS

#42

SLOPE (rate of change) is a number that indicates the steepness (or flatness) of a line, that is, its rate of change, as well as its direction (up or down) left to right.

SLOPE (rate of change) is determined by the ratio: $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x}$ between any two points on a line. Some books and teachers refer to this ratio as the rise (y) over the run (x).

For lines that go **up** (from left to right), the sign of the slope is **positive**. For lines that go **down** (left to right), the sign of the slope is **negative**.

Any linear equation written as $y = mx + b$, where m and b are any real numbers, is said to be in **SLOPE-INTERCEPT FORM**. m is the **SLOPE** of the line. b is the **Y-INTERCEPT**, that is, the point $(0, b)$ where the line intersects (crosses) the y-axis.

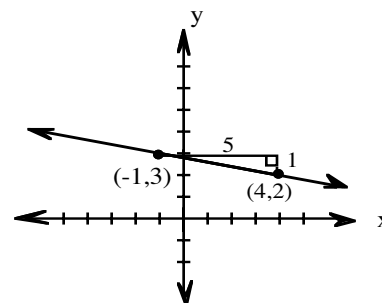
Example 1

Write the slope of the line containing the points $(-1, 3)$ and $(4, 2)$.

First graph the two points and draw the line through them.

Look for and draw a slope triangle using the two given points.

Write the ratio $\frac{\text{vertical change in } y}{\text{horizontal change in } x}$ using the legs of the right triangle: $\frac{1}{5}$.



Assign a positive or negative value to the slope depending on whether the line goes up (+) or down (-) from left to right. The slope is $-\frac{1}{5}$.

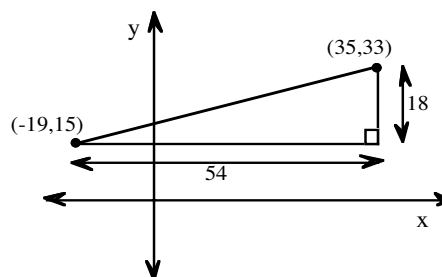
Example 2

Write the slope of the line containing the points $(-19, 15)$ and $(35, 33)$.

Since the points are inconvenient to graph, use a "Generic Slope Triangle," visualizing where the points lie with respect to each other and the axes.

Make a sketch of the points.

Draw a slope triangle and determine the length of each leg. Write the ratio of y to x : $\frac{18}{54} = \frac{1}{3}$. The slope is $\frac{1}{3}$.



Example 3

Given a table, determine the rate of change (slope) and the equation of the line.

		+2	+2	+2	
x	-2	0	2	4	
y	1	4	7	9	
		+3	+3	+3	

rate of change = $\frac{3}{2}$

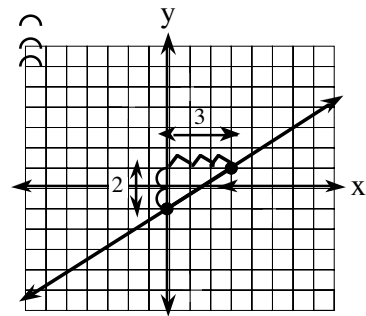
y - intercept = (0, 4)

so the equation of the line is $y = \frac{3}{2}x + 4$.

Example 4

Graph the linear equation $y = \frac{2}{3}x - 1$

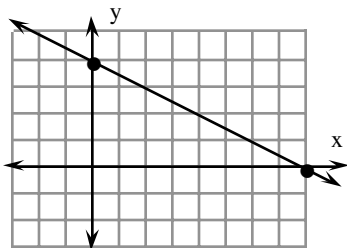
Using $y = mx + b$, the slope in $y = \frac{2}{3}x - 1$ is $\frac{2}{3}$ and the y-intercept is the point (0, -1). To graph, begin at the y-intercept (0, -1). Remember that slope is $\frac{\text{vertical change}}{\text{horizontal change}}$ so go up 2 units (since 2 is positive) from (0, -1) and then move right 3 units. This gives a second point on the graph. To create the graph, draw a straight line through the two points.



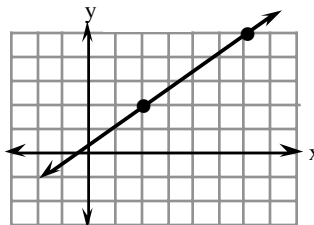
Problems

Determine the slope of each line using the highlighted points.

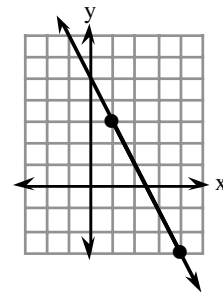
1.



2.



3.



Write the slope of the line containing each pair of points. Sketch a slope triangle to visualize the vertical and horizontal change.

4. (2, 3) and (5, 7)

5. (2, 5) and (9, 4)

6. (1, -3) and (7, -4)

7. (-2, 1) and (3, -3)

8. (-2, 5) and (4, 5)

9. (5, 8) and (3, 5)

Use a Generic Slope Triangle to write the slope of the line containing each pair of points:

10. (50, 40) and (30, 75)

11. (10, 39) and (44, 80)

12. (5, -13) and (-51, 10)

Identify the slope and y-intercept in each equation.

13. $y = \frac{1}{2}x - 2$

14. $y = -3x + 5$

15. $y = 4x$

16. $y = -\frac{2}{3}x + 1$

17. $y = x - 7$

18. $y = 5$

Draw a graph to find the equation of the line with:

19. slope = $\frac{1}{2}$ and passing through (2, 3).

20. slope = $\frac{2}{3}$ and passing through (3, -2).

21. slope = $-\frac{1}{3}$ and passing through (3, -1).

22. slope = -4 and passing through (-3, 8).

For each table, determine the rate of change and the equation. Be sure to record whether the rate is positive or negative for both x and y.

23.

x	-2	-1	0	1	2
y	-5	-2	1	4	7

24.

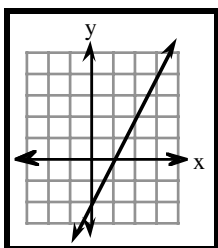
x	-2	0	2	4	6
y	7	3	-1	-5	-9

25.

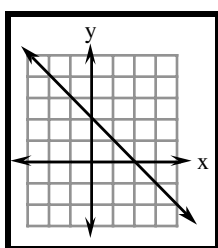
x	-6	-3	0	3	6
y	-3	-1	1	3	5

Using the slope and y-intercept, determine the equation of the line.

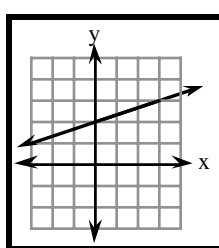
26.



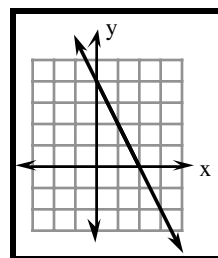
27.



28.



29.



Graph the following linear equations on graph paper.

30. $y = \frac{1}{2}x + 2$

31. $y = -\frac{3}{5}x + 1$

32. $y = -4x$

33. $y = -2x + \frac{1}{2}$

34. $3x + 2y = 12$

Answers

1. $-\frac{1}{2}$

2. $\frac{3}{4}$

3. -2

4. $\frac{4}{3}$

5. $-\frac{1}{7}$

6. $\frac{1}{6}$

7. $-\frac{4}{5}$

8. 0

9. $\frac{3}{2}$

10. $-\frac{35}{20} = -\frac{7}{4}$

11. $\frac{41}{34}$

12. $-\frac{33}{71}$

13. $\frac{1}{2}; (0, -2)$

14. $-3; (0, -5)$

15. $4; (0, 0)$

16. $-\frac{2}{3}; (0, 1)$

17. $1; (0, -7)$

18. $0; (0, 5)$

19. $y = \frac{1}{2}x + 2$

20. $y = \frac{2}{3}x - 4$

21. $y = -\frac{1}{3}x$

22. $y = -4x - 4$

23. $3; y = 3x + 1$

24. $-\frac{4}{2} = -2; y = -2x + 3$

25. $\frac{2}{3}; y = \frac{2}{3}x + 1$

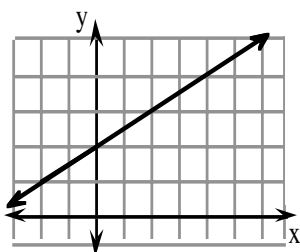
26. $y = 2x - 2$

27. $y = -x + 2$

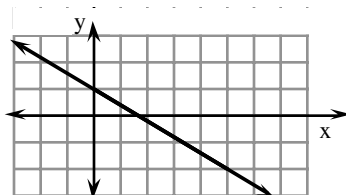
28. $y = \frac{1}{3}x + 2$

29. $y = -2x + 4$

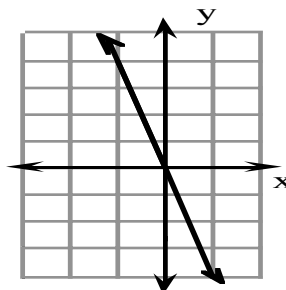
30. $y = \frac{1}{2}x + 2$



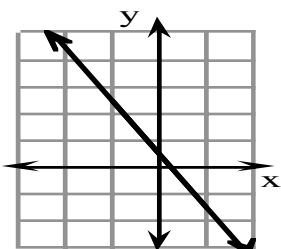
31. $y = -\frac{3}{5}x + 1$



32. $y = -4x$



33. $y = -2x + \frac{1}{2}$



34. $y = -\frac{3}{2}x + 6$

