CPM *Core Connections, Course 3*
Mathematical Practices

Introduction

The CPM *Connections* curriculum, developed from 2003-10, mirrors the elements of the CCSS Mathematical Practices. The principles of CPM course design—problem-based lessons, collaborative student work, and spaced practice—are based on the methodological research for teaching mathematics that leads to conceptual understanding. As such, the mathematical practices, similar to previous "best practices" such as the Marzano Principles or CPM's original *Connections series'* "Ways of Thinking," are integral to the pedagogy used throughout all of the courses. Task designs ask students to create models, make connections and explain their work regularly. Students are held responsible for high academic rigor, analysis, and critical thinking, and communicate their mathematical findings in writing and/or oral presentations in a clear and convincing manner.

Contents of this resource:

Sample lesson (detailed)

Page 2 presents a detailed review of one lesson from *Core Connections, Course 3* that shows how the eight Mathematical Practices are woven through it.

Selected lessons for review of embedded Mathematical Practices

Page 3 offers a list of sample lessons that the reader can review to see the embedded Mathematical Practices. CPM editors have created a table to indicate which practices are in the lesson and to what degree. The reader should examine the detailed sample lesson on page 2 before examining any of these lessons. Keep in mind that this list is a **sampling** of lessons where you will find the mathematical practices. Elements of the Mathematical Practices are present in most lessons.

Integrating each Mathematical Practice into CPM courses

The paragraphs on pages 4 and 5 discuss in detail how each Mathematical Practice is integrated into the structure of the CPM courses.

CCSSM Content Standards

Correlations: There is a separate document that has correlations to the content standards.

Example of how the Mathematical Practices are integrated throughout a lesson in *Core Connections: Course 3*

A typical CPM lesson exemplifies how deeply the CCSS *Standards for Mathematical Practice* are integrated into the course. In Lesson 3.1.2 in CPM's *Core Connections, Course 3*, "John's Giant Redwood," students work in their collaborative teams conducting an inquiry early in the course into multiple representations of a linear function. In Problem 3-10 students try to find a rule that represents an input-output function table in haphazard order. They do this in an interactive (and fun!) whole-class game (for about 10 minutes) that is set up in a way that when one student finds a solution it does not end the process for the rest of the students. During the game, students need to make sense of the pattern they see, look for additional patterns and structure, and look for consistency in repeated input-output combinations (Practices 1, 7, 8). In Problem 3-11 students reason quantitatively and abstractly (Practice 2) to justify (Practice 3) their finding of the growth rate and their prediction. Students have calculators readily available, but quickly determine that repeated addition on the calculator is not a very efficient process for solving this problem (Practice 5). In Problem 3-12, students model (Practice 4) the growth of the tree with a graph, and start an inquiry into the capabilities of their model; in Problem 3-12(d) students justify (Practice 3) predictions made with the model. Problem 3-13 moves students back and forth between the concrete and abstract (Practice 2) to create an equation that models the situation (Practice 4). Students apply the same skills they used in Problem 3-10 from the beginning of the day in their interactive game. In the "Closure" activity for the day (as described in the teacher notes), students discuss the advantages and disadvantages of each of the three representations of the growth function, during which the precision of answers (Practice 6) is discussed. The purpose of the lesson is to begin developing a deep conceptual understanding of rate of growth. Students make sense of growth (Practice 1) by constructing arguments and defending them in collaborative groups, and—toward the end of the day—doing so in a whole-class discussion (Practice 3).

Other Examples for Review—A Partial List

The CCSS *Standards for Mathematical Practice* are regularly integrated into the course design of CPM *Connections*. **The list below is by no means exhaustive (the course has about 100 lessons); it illustrates typical lessons that demonstrate the practices in action. These citations are just a few examples of where the mathematical practices are integrated into the course.**

An "xx" in the table below represents a practice that is a **focus** of the lesson. An "x" represents a practice that is **present** in the lesson.

A FEW EXAMPLES OF THE INTEGRATION OF CCSS PRACTICES INTO CPM CORE CONNECTIONS 3

	CCSS Standards for Mathematical Practice							
	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
Chapter 2 - Simplifying with Variables								
2.1.7, 2.1.8 Recording Work and Solving for *x*		xx			x		x	
Chapter 3 - Graphs and Equations								
3.1.2 John's Giant Redwood	xx	x	x	xx	xx	x	x	x
3.1.3 The Big "C"s	x	x	x		xx		xx	xx
3.1.7 Completing Tables and Drawing Graphs	xx		xx	x		x		
Chapter 4 - Multiple Representations								
4.1.2 Tile Patterns	x		x	xx			x	
4.1.5 Checking the Connections	x	x	x				xx	
Chapter 5 - Systems of Equations								
5.2.2 Buying Bicycles	xx	xx	x	xx	x			
5.2.4 Extending the Web to New Situations		x	xx	xx	x	x	x	
Chapter 6 - Transformations and Similarity								
6.1.1 Key in the Lock Puzzles	x	x	x		x		xx	
6.2.1 Undoing Dilations		xx	x	x			x	x
Chapter 7 - Slope and Association								
7.1.2 Slope in Different Representations	x		x	x		x	xx	x
7.2.5 Setting Up Proportions	x	x	x	x	x		x	x
7.3.2 Describing Association	x	x		xx	x	x		
Chapter 8 - Exponents and Functions								
8.2.2 Exponent Rules		x	x				x	xx
8.3.2 The Cola Machine	xx		x	x			x	
Chapter 9 - Angles and the Pythagorean Theorem								
9.1.2 Tangled Triangles	x	xx	x	x	x	x		
9.2.1 Is it a Triangle?	x	xx	x	x	x	xx	x	
Chapter 10 - Surface Area and Volume								
10.2.2 Comparing the Gym Bags	x	x		xx			x	x

Each Standard of Mathematical Practice is Integrated into CPM

Standard 1 of the CCSS *Standards for Mathematical Practice* requires students to **"Make sense of problems and persevere in solving them."** The *Core Connections* courses have students solve realistic, non-routine problems that are rich in mathematics on a daily basis. These guided investigations are not mere "word problems" that mimic examples of rules. By having students make sense of the problem, rather than being told how to solve a particular kind of problem step-by-step, CPM problems develop deep conceptual understanding of the mathematics, procedural fluency, and perseverance on a daily basis, in addition to teaching and using problem-solving strategies. The curriculum fosters strategic competence and adaptive reasoning in students.

Standard 2 of the CCSS *Standards for Mathematical Practice* requires students to **"Reason abstractly and quantitatively."** In contrast to offering word problems at the end of each chapter, the CPM program generally presents mathematical ideas in contexts *first*, helping students make sense of otherwise abstract principles. Only then do students move on to abstraction and generalization using symbolic notation. Students are taught how to gather and organize information about these contextual problems, break them into smaller parts, look for connections to previous mathematics, and identify patterns and relationships that lead to solutions. Students are also asked to work in reverse, that is, create situations for abstract generalizations.

Standard 3 requires students to **"Construct viable arguments and critique the reasoning of others."** In CPM *Core Connections* courses, students regularly share information, opinions, and their expertise in collaborative study teams. They work at tables where they have room to manipulate their learning materials and tools. They take turns talking, listening, contributing, arguing, asking for help, checking for understanding, and keeping each other focused. More importantly, during this process students are using higher-order thinking: providing clarification, building on each other's ideas, analyzing and coming to consensus, and productively criticizing. Justifying and critiquing is a part of daily life in a CPM classroom, not an occasional assignment. For each problem, students are expected to communicate their mathematical findings in writing, in oral presentations, or in poster presentations in a clear and convincing manner. Teachers answer students' questions, but do so in a manner that challenges and motivates students to develop and test solutions themselves.

Standard 4 has students **"Model with mathematics."** Modeling contextual situations with multiple representations is a recurring theme in the CPM *Core Connections* series. For example, from their earliest work with proportions and linear functions all the way through the more complex functions of later courses, students consistently model functions using tables, graphs, equations, and narrative or diagrams. In creating these models, students make assumptions, then predictions, and then check to see if their predictions make sense in the context of the problem. Students regularly use area models to multiply fractions, multiply and divide polynomials, factor, and solve probability problems. In contexts involving variability in data, students learn that a model may not be perfect, yet can be very useful for describing data and making predictions. CPM students find that a calculator or computer can help them model repeated probabilistic experiments much more efficiently than actually conducting the experiment.

Standard 5 requires students to **"Use appropriate tools strategically."** In the typical CPM lesson, students have available to them a cornucopia of tools—from rulers and scissors, to tracing paper and graph paper, to blocks and tiles, to calculators—but are not typically told which specific tools to use to solve any particular problem. Indeed, a team of collaborating CPM students usually has a designated Resource Manager, whose task it is to ask the teacher for the tools their team needs for that lesson. It is not unusual for different teams to use different tools to solve a problem; during the course of the lesson students share with the whole class their solution strategies, and frequently this includes a lively discussion of which tools were most efficient and productive to solve a given problem. For problems where students are becoming fluent with algebraic procedures, the CPM *Core Connections* texts use an icon to indicate calculators should not be used. But during investigations students may choose to explore with their calculators to make sense of the mathematics without getting bogged down in computations. During various lessons, students might be exploring in a computer lab with programs provided by CPM, using motion detectors to determine rates, or using lasers or computer-based applets to demonstrate a point.

Standard 6 requires students **"Attend to precision."** Since they are solving contextual problems on a daily basis, the need for attending to precision soon becomes a natural consequence of being a CPM student. Whether they are converting the units in a problem to be consistent, or checking whether a numerical solution makes sense, dealing with precision in choosing units is inherent. Many CPM investigations make use of a calculator; using calculators extensively requires students to frequently attend to the precision of the results displayed. Since most problems are contextual, when students are symbolically solving problems, the mantra of "defining variables with units" becomes essential to coming up with a solution that makes sense. In the case of trigonometric or exponential situations, problems often require decimal approximations to make sense of the solution; CPM students find that approximations made in these situations may require higher levels of precision when evaluating expressions. They also determine that four decimal places of precision is useless when measuring angles in a garden plot.

Standard 7 requires students to **"Look for and making use of structure."** Since CPM students are developing deep conceptual understanding of the underlying mathematics, they frequently use this practice to bring closure to investigations. For example, cross-multiplying to find equivalent fractions is not taught simply as a procedure to be practiced, but is developed from the underlying structure of a multiplication table. Students develop deep conceptual connections between proportions, growth, steepness, and slope by exploring different manifestations of the structure of rates. CPM students do not simplify rational expressions by "canceling;" instead they use the underlying structure of the "Giant One"—fractions where the numerator and denominator are equal. Theorems in geometry are developed from the structure of repeated translations, not just listed in isolation. Polynomials are not multiplied and divided by following an algorithm, but by looking at the underlying structure of an area model. Moreover, polynomials are not solved by just following algorithms, but by looking at the structure of the factored form and the different kinds of roots that structure leads to.

Standard 8 requires students to, **"Look for and express regularity in repeated reasoning."** When faced with a new investigation of a mathematical concept, CPM students often look for a simpler or analogous problem. By extending the structure of previous problems, students are continually expanding their ability to solve increasingly complex problems. At first students use repeated reasoning in multiplication tables to multiply fractions or find equivalent fractions. Students expand the reasoning of simpler intuitive probability problems into increasingly more complex probabilistic situations. CPM students observe repeated structure in area models and leverage that into the ability to multiply, factor, and eventually divide, polynomials. Students use repeated patterns to make sense of negative, zero, and fractional exponents, and to solve rational expressions. Repeated reasoning allows for increasingly complex geometric proofs to be developed from simpler ones and, more generally, by repeated building on conceptual understanding of previous underlying mathematics, make connections to continually and increasingly more complex situations.