CPM Geometry Connections
Mathematical Practices

Introduction

The CPM *Connections* curriculum, developed from 2003-10, mirrors the elements of the CCSS Mathematical Practices. The principles of CPM course design—problem-based lessons, collaborative student work, and spaced practice—are based on the methodological research for teaching mathematics that leads to conceptual understanding. As such, the mathematical practices, similar to previous "best practices" such as the Marzano Principles or CPM's "Ways of Thinking" (see below), are integral to the pedagogy used throughout all of the courses. Task designs ask students to create models, make connections and explain their work regularly. Students are held responsible for high academic rigor, analysis, and critical thinking, and communicate their mathematical findings in writing and/or oral presentations in a clear and convincing manner.

Contents of this resource:

Sample lesson (detailed)

Page 2 presents a detailed review of one lesson from *Geometry Connections* that shows how the eight Mathematical Practices are woven into it.

Selected lessons for review of embedded Mathematical Practices

Page 3 offers two-dozen lessons that the reader can review to see the embedded Mathematical Practices. CPM editors have created a table to indicate which practices are in the lesson and to what degree. The reader should examine the detailed sample lesson on page 2 before examining any of these lessons. Keep in mind that this list is a **sampling** of lessons where you will find the mathematical practices. Elements of the Mathematical Practices are present in most lessons.

CPM's "Ways of Thinking" mirror the Mathematical Practices

The CPM *Connections* courses each focus on five mathematical ways of thinking that are similar or identical to the CCSS Mathematical Practices. Page 4 explains the connection in detail.

Integrating each Mathematical Practice into CPM courses

The paragraphs on pages 5 and 6 discuss in detail how each Mathematical Practice is integrated into the structure of the CPM courses.

Additional resources:

CCSSM Content Standards, Supplemental Lessons, and Pacing Guides

(1) Correlations: There is a separate document that has correlations to the algebra content standards. Be sure to read the list of abbreviations for the coding used in the citations near the top of the first page. **Most references are to lessons in the student textbook.**
(2) Supplement: Note that the other references are all available in the *Geometry Connections CCSSM Supplement* booklet (available February 2011) or via download at the CPM website. These topics provide the additional content—beyond the textbook—required to meet the CCSSM content standards for this course.
(3) Pacing guide: There is also a CCSSM table of contents file for *Geometry Connections* that shows which lessons may be omitted and where the supplementary lessons should be inserted.

Example of How Practices are Integrated Throughout a Lesson in *Geometry Connections*

A typical CPM lesson exemplifies how deeply the CCSS *Standards for Mathematical Practice* are integrated into the course even though the course predates the CCSS practices. In Lessons 2.1.2 and 2.1.3 of CPM *Geometry Connections* ("Angles Formed by Transversals), students work in their collaborative teams conducting an inquiry into angle relationships. Problem 2-13 is a warmup that reviews previous angle relationships. In Problem 2-44 students use translations to repeatedly determine angles in the physical structure of a tiling (Practice 7). Problem 2-15 is the heart of this day's lesson: students make sense of transversals (Practice 1) by looking at the repeated patterns of the previous problem (Practice 8). In the course of the problem, students choose between pencil-and-paper, tracing paper, and a computer geometry tool to assist in making the translations (Practice 5), moving back and forth between the abstract and concrete (Practice 2). In Problem 2-16 students move from the physical translation to practice the abstract, and then in Problem 2-17 student move from quantitative to abstract (Practice 2). Problem 2-17 also focuses on justification (Practice 3), and in particular the teacher notes for the lesson suggest that a whole-class discussion is appropriate on justifying why Problem 2-17(b) cannot be solved. In addition, while students are working in collaborative teams for most of the lesson, they are continuously justifying and defending their insights and critiquing those of their teammates (Practice 3).

On the next day, in Problems 2-23, 2-24, and 2-25, other relationships on transversal lines are explored and formalized using the repeated thinking of translating parallelograms (Practice 8). Problem 2-26 and 2-27 ask students to model situations with the mathematical relationships (Practice 4). Again, students are continuously justifying and critiquing as they work in collaborative teams (Practice 3). They are explicitly reminded to justify their thinking on each problem today (Practice 3).

Other Examples for Review—A Partial List

The CCSS *Standards for Mathematical Practice* are regularly integrated into the course design of CPM *Connections*. **The list below is by no means exhaustive (the course has about 135 lessons); it illustrates typical lessons that demonstrate the practices in action. These citations are just a few examples of where the mathematical practices are integrated into the course.**

An "xx" in the table below represents a practice that is a **focus** of the lesson. An "x" represents a practice that is **present** in the lesson.

A FEW EXAMPLES OF THE INTEGRATION OF CCSS PRACTICES INTO CPM CURRICULUM

	CCSS Standards for Mathematical Practice							
	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
Chapter 2 - Angles and Measurement								
2.1.2, 2.1.3 Angles Formed by Transversals	x	x	xx	x	x		x	xx
2.2.2 How Can I find Area?	xx	x	xx				x	
2.3.2 Triangle Inequality	xx	xx	xx					
Chapter 3 - Justification and Similarity								
3.2.4 More Conditions for Triangle Similarity	xx		xx				x	xx
3.2.6 Applying Similarity	xx	x	x	xx	x	x		
Chapter 4 - Trigonometry and Probability								
4.1.1 Leaning Tower of Pisa	x	x	x	xx	x	x		
4.1.4 Will It Topple?	x		x	xx	xx	xx		
4.2.4 Choosing a Probability Model	xx	xx	x	xx	xx	x		
Chapter 5 - Trigonometry & Triangle Tool Kit								
5.1.2, 5.1.3, 5.1.4 Trig Ratios	xx	xx	x	xx	x	xx	x	
5.2.1 Special Right Triangles	x		x		x	x	x	xx
5.3.2 Law of Sines	x	x	x			xx	x	x
Chapter 6 - Congruent Triangles								
6.1.1 Conditions for Triangle Congruence	xx	x	x				xx	x
6.2.3 At Your Service	xx	xx	xx	xx	x	x		
Chapter 7 - Proof and Quadrilaterals								
7.1.3 Shortest Distance Problems	xx	xx	x	xx	x	x		
7.2.2 Properties of Rhombi	x		xx				xx	x
7.3.2 Coordinate Geometry	x		x		x	x	x	
Chapter 8 - Polygons and Circles								
8.1.2 Interior Angles of a Polygon	x	x	x				xx	xx
8.1.5 Area of Regular Polygons	x	xx	x	xx	x	x	x	
Chapter 9 - Solids and Construction								
9.1.3 Prisms and Cylinders	x	x	xx	x			x	xx
9.2.3 More Explorations with Constructions	xx		x		x	x		
Chapter 10 - Circles and Expected Value								
10.1.2 Angles and Arcs	xx	x	x	x	x		x	x
10.2.2 Expected Value	x		x	xx		x	x	
Chapter 11 - Solids and Circles								
11.1.3 Volume of a Pyramid	xx	xx	x	x			x	x
11.2.3 Secant and Tangent Relationships	x		x		xx	x	x	

CPM's "Ways of Thinking" mirror the Mathematical Practices

Rather than discretely introducing each mathematical practice as a topic to be learned, CPM integrates these practices throughout each lesson. One way these practices are threaded throughout the CPM *Connections* courses is through its focus on mathematical "Ways of Thinking." These Ways of Thinking represent common ways of working mathematically and thus are forms of mathematical practice. Ways of Thinking differ slightly per course due to the different nature of the content, but several are common across courses. For example, since an important mathematical practice is to regularly ask and answer questions such as "How do I know this is true?" and "Is this always true?", Reasoning and Justifying is one of the Ways of Thinking common to most *Connections* courses. Other Ways of Thinking found across multiple *Connections* courses include choosing a strategy, generalizing, visualizing and investigating. Ways of Thinking found in the *Connections* courses are listed below. (MC = *Making Connections* for middle grades, AC = *Algebra Connections,* GC = *Geometry Connections*, and A2C = *Algebra 2 Connections*.)

MC1: comparing, visualizing, describing and explaining, looking for multiple ways of seeing or doing, and sense making
MC2: generalizing, reasoning and justifying, reversing, choosing a strategy, visualizing
AC: justifying, generalizing, making connections, reversing thinking, and applying and extending
GC: investigating, examining, reasoning and justifying, visualizing, and choosing a strategy/tool
A2C: justifying, generalizing, choosing a strategy, investigating, and reversing

Specifically, many of the mathematical practices proscribed by the Core Content State Standards document directly relate to the Ways of Thinking. For example, "Make sense of problems and persevere in solving them" asks students to engage in a way of thinking captured by *sense making* (MC1), *making connections* (AC), and *investigating* (GC and A2C). The practice "reason abstractly and quantitatively" is represented by the Way of Thinking referred to as *generalizing* (MC2, AC, and A2C) and *comparing* (MC1). The practice "construct viable arguments and critique the reasoning of others" is emphasized with the Ways of Thinking *describing and explaining* (MC1), *reasoning and justifying* (MC2 and GC) and *justifying* (AC and A2C). Finally, the mathematical practice "use appropriate tools strategically" is addressed with the Ways of Thinking *looking for multiple ways of seeing or doing* (MC1), *choosing a strategy* (MC2 and A2C), and *choosing a strategy/tool* (GC).

In addition to encountering prompts in each closure section which require students to reflect on the different ways they used each Way of Thinking throughout the chapter, the text also highlights the regularity of each Way of Thinking by bolding the frequent instances where students are prompted for that form of thinking.

The CPM *Connections* series predates the CCSS *Standards for Mathematical Practice* by several years, yet the practices advocated by the *Standards* are naturally integrated as a core foundation of the CPM curriculum.

Each Standard of Mathematical Practice is Integrated into CPM

Standard 1 of the CCSS *Standards for Mathematical Practice* requires students to **"Make sense of problems and persevere in solving them."** The *Connections* courses have students solve realistic, non-routine problems that are rich in mathematics on a daily basis. These guided investigations are not mere "word problems" that mimic examples of rules. By having students make sense of the problem, rather than being told how to solve a particular kind of problem step-by-step, CPM problems develop deep conceptual understanding of the mathematics, procedural fluency, and perseverance on a daily basis, in addition to teaching and using problem-solving strategies. The curriculum fosters strategic competence and adaptive reasoning in students.

Standard 2 of the CCSS *Standards for Mathematical Practice* requires students to **"Reason abstractly and quantitatively."** In contrast to offering word problems at the end of each chapter, the CPM program generally presents mathematical ideas in contexts *first*, helping students make sense of otherwise abstract principles. Only then do students move on to abstraction and generalization using symbolic notation. Students are taught how to gather and organize information about these contextual problems, break them into smaller parts, look for connections to previous mathematics, and identify patterns and relationships that lead to solutions. Students are also asked to work in reverse, that is, create situations for abstract generalizations.

Standard 3 requires students to **"Construct viable arguments and critique the reasoning of others."** In CPM *Connections* courses, students regularly share information, opinions, and their expertise in collaborative study teams. They work at tables where they have room to manipulate their learning materials and tools. They take turns talking, listening, contributing, arguing, asking for help, checking for understanding, and keeping each other focused. More importantly, during this process students are using higher-order thinking: providing clarification, building on each other's ideas, analyzing and coming to consensus, and productively criticizing. Justifying and critiquing is a part of daily life in a CPM classroom, not an occasional assignment. For each problem, students are expected to communicate their mathematical findings in writing, in oral presentations, or in poster presentations in a clear and convincing manner. Teachers answer students' questions, but do so in a manner that challenges and motivates students to develop and test solutions themselves.

Standard 4 has students **"Model with mathematics."** Modeling contextual situations with multiple representations is a recurring theme in the CPM *Connections* series. For example, from their earliest work with proportions and linear functions all the way through the more complex functions of later courses, students consistently model functions using tables, graphs, equations, and narrative or diagrams. In creating these models, students make assumptions, then predictions, and then check to see if their predictions make sense in the context of the problem. Students regularly use area models to multiply fractions, multiply and divide polynomials, factor, and solve probability problems. In contexts involving variability in data, students learn that a model may not be perfect, yet can be very useful for describing data and making predictions. CPM students find that a calculator or computer can help them model repeated probabilistic experiments much more efficiently than actually conducting the experiment.

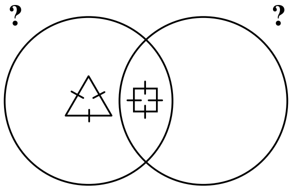
Standard 5 requires students to **"Use appropriate tools strategically."** In the typical CPM lesson, students have available to them a cornucopia of tools—from rulers and scissors, to tracing paper and graph paper, to blocks and tiles, to calculators—but are not typically told which specific tools to use to solve any particular problem. Indeed, a team of collaborating CPM students usually has a designated Resource Manager, whose task it is to ask the teacher for the tools their team needs for that lesson. It is not unusual for different teams to use different tools to solve a problem; during the course of the lesson students share with the whole class their solution strategies, and frequently this includes a lively discussion of which tools were most efficient and productive to solve a given problem. For problems where students are becoming fluent with algebraic procedures, the CPM *Connections* texts use an icon to indicate calculators should not be used. But during investigations students may choose to explore with their calculators to make sense of the mathematics without getting bogged down in computations. During various lessons, students might be exploring in a computer lab with programs provided by CPM, using motion detectors to determine rates, or using lasers or computer-based applets to demonstrate a point.

Standard 6 requires students **"Attend to precision."** Since they are solving contextual problems on a daily basis, the need for attending to precision soon becomes a natural consequence of being a CPM student. Whether they are converting the units in a problem to be consistent, or checking whether a numerical solution makes sense, dealing with precision in choosing units is inherent. Many CPM investigations make use of a calculator; using calculators extensively requires students to frequently attend to the precision of the results displayed. Since most problems are contextual, when students are symbolically solving problems, the mantra of "defining variables with units" becomes essential to coming up with a solution that makes sense. In the case of trigonometric or exponential situations, problems often require decimal approximations to make sense of the solution; CPM students find that approximations made in these situations may require higher levels of precision when evaluating expressions. They also determine that four decimal places of precision is useless when measuring angles in a garden plot.

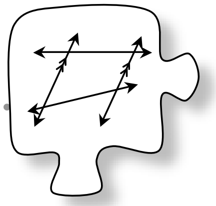
Standard 7 requires students to **"Look for and making use of structure."** Since CPM students are developing deep conceptual understanding of the underlying mathematics, they frequently use this practice to bring closure to investigations. For example, cross-multiplying to find equivalent fractions is not taught simply as a procedure to be practiced, but is developed from the underlying structure of a multiplication table. Students develop deep conceptual connections between proportions, growth, steepness, and slope by exploring different manifestations of the structure of rates. CPM students do not simplify rational expressions by "canceling;" instead they use the underlying structure of the "Giant One"—fractions where the numerator and denominator are equal. Theorems in geometry are developed from the structure of repeated translations, not just listed in isolation. Polynomials are not multiplied and divided by following an algorithm, but by looking at the underlying structure of an area model. Moreover, polynomials are not solved by just following algorithms, but by looking at the structure of the factored form and the different kinds of roots that structure leads to.

Standard 8 requires students to, **"Look for and express regularity in repeated reasoning."** When faced with a new investigation of a mathematical concept, CPM students often look for a simpler or analogous problem. By extending the structure of previous problems, students are continually expanding their ability to solve increasingly complex problems. At first students use repeated reasoning in multiplication tables to multiply fractions or find equivalent fractions. Students expand the reasoning of simpler intuitive probability problems into increasingly more complex probabilistic situations. CPM students observe repeated structure in area models and leverage that into the ability to multiply, factor, and eventually divide, polynomials. Students use repeated patterns to make sense of negative, zero, and fractional exponents, and to solve rational expressions. Repeated reasoning allows for increasingly complex geometric proofs to be developed from simpler ones and, more generally, by repeated building on conceptual understanding of previous underlying mathematics, make connections to continually and increasingly more complex situations.

2-12. Larry saw Javon's incomplete Venn diagram at right, and he wants to finish it. However, he does not know the condition that each circle represents. Find a possible label for each circle, and place two more shapes into the diagram.



2.1.2 What's the relationship?

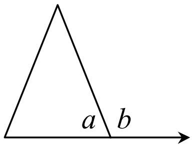


Angles Formed by Transversals

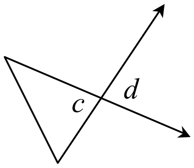
In Lesson 2.1.1, you **examined** vertical angles and found that vertical angles are always equal. Today you will look at another special relationship that guarantees angles are equal.

2-13. **Examine** the diagrams below. For each pair of angles marked on the diagram, quickly decide what relationship their measures have. Your responses should be limited to one of three relationships: congruent (equal measures), complementary (have a sum of 90°), and supplementary (have a sum of 180°).

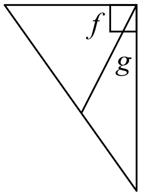
a.



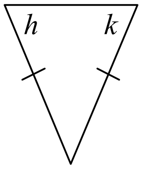
b.



c.

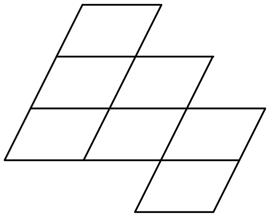


d.



2-14. Marcos was walking home after school thinking about special angle relationships when he happened to notice a pattern of parallelogram tiles on the wall of a building. Marcos saw lots of special angle relationships in this pattern, so he decided to copy the pattern into his notebook.

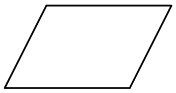
The beginning of Marcos's diagram is shown at right and provided on the Lesson 2.1.2 Resource Page. This type of pattern is sometimes called a **tiling**. In a tiling, a shape is repeated without gaps or overlaps to fill an entire page. In this case, the shape being tiled is a parallelogram.



Problem continues on next page →

2-14. *Problem continued from previous page.*

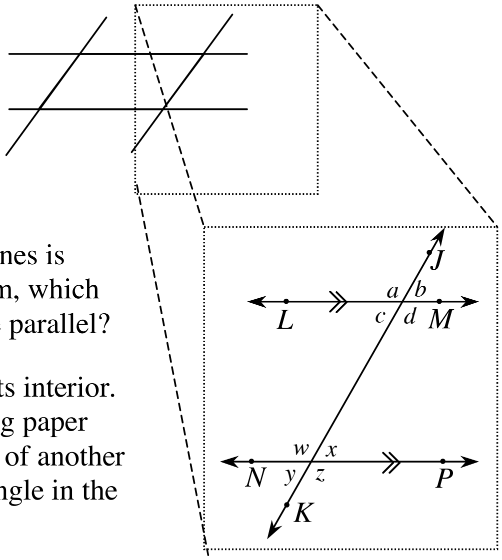
a. Consider the angles inside a single parallelogram. Are any angles congruent? On your resource page, use color to show which angles must have equal measure. If two angles are not equal, make sure they are shaded with different colors.



b. Since each parallelogram is a translation of another, what can be stated about the angles in the rest of Marcos' tiling? Use a dynamic geometry tool, transparencies on an overhead, or tracing paper to determine which angles must be congruent. Color all angles that must be equal the same color.

c. What about relationships between lines? Can you identify any lines that must be parallel? Mark all the lines on your diagram with the same number of arrows to show which lines are parallel.

2-15. Julia wants to learn more about the angles in Marcos's diagram and has decided to focus on just a part of his tiling. An enlarged view of that section is shown in the image below right, with some points and angles labeled.



a. A line that crosses two or more other lines is called a **transversal**. In Julia's diagram, which line is the transversal? Which lines are parallel?

b. Trace $\measuredangle x$ on tracing paper and shade its interior. Then translate $\measuredangle x$ by sliding the tracing paper along the transversal until it lies on top of another angle and matches it exactly. Which angle in the diagram corresponds with x?

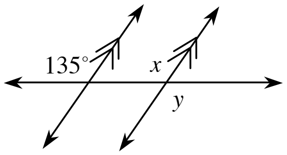
c. What is the relationship between the measures of angles x and b? Must one be greater than the other, or must they be equal? Explain how you know.

d. In this diagram, $\measuredangle x$ and $\measuredangle b$ are called **corresponding angles** because they are in the same position at two different intersections of the transversal. The corresponding angles in this diagram are equal because they were formed by translating a parallelogram. Name all the other pairs of equal corresponding angles you can find in Julia's diagram.

e. Suppose $b = 60^\circ$. Use what you know about vertical, supplementary, and corresponding angle relationships to find the measures of all the other angles in Julia's diagram.

2-16. Frank wonders whether corresponding angles are *always* equal. For parts (a) through (d) below, decide whether you have enough information to find the measures of x and y. If you do, find the angle measures and state the relationship. Use tracing paper to help you find corresponding angles.

a.



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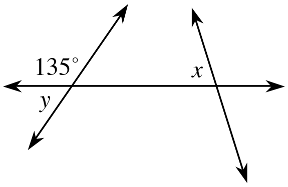

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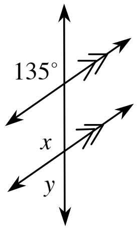

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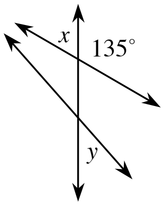
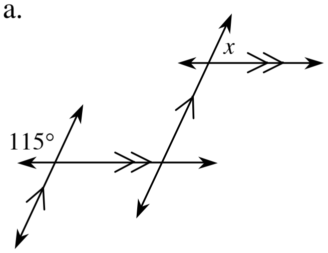


e. Answer Frank's question: Are corresponding angles always equal?

f. Conjectures are often written in the form, "*If…, then…*". A statement in if-then form is called a **conditional statement**. Make a conjecture about corresponding angles by completing this conditional statement: "If …, then corresponding angles are equal."

2-17. For each diagram below, find the value of x *if possible*. If it is not possible, explain how you know. State the relationships you use. Be prepared to **justify** every measurement you find to other members of your team.

a.



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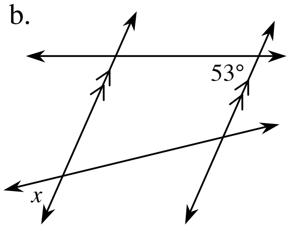

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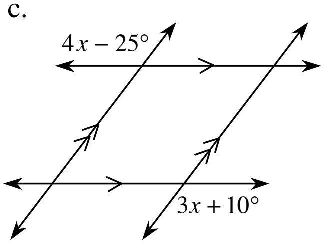


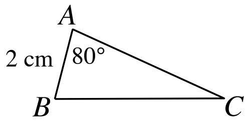
MATH NOTES

METHODS AND MEANINGS

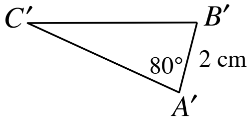
Naming Parts of Shapes

Part of geometry is the study of parts of shapes, such as points, line segments, and angles. To avoid confusion, standard notation is used to name these parts.

A point is named using a single capital letter. For example, the vertices (corners) of the triangle at right are named *A*, *B*, and *C*.

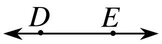


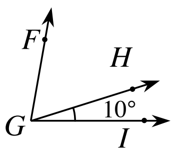
If a shape is transformed, the image shape is often named using **prime notation**. The image of point *A* is labeled A' (read as "A prime"), the image of *B* is labeled B' (read as "B prime"), etc. At right, $\triangle A'B'C'$ is the image of $\triangle ABC$.



The side of a polygon is a **line segment**. A line segment is named by naming its endpoints and placing a bar above them. For example, one side of the first triangle above is named $\overline{AB}$. When referring to the length of a segment, the bar is omitted. In $\triangle ABC$ above, $AB = 2$ cm.

A **line**, which differs from a segment in that it extends infinitely in either direction, is named by naming two points on the line and placing a bar with arrows above them. For example, the line below is named $\overleftrightarrow{DE}$. When naming a segment or line, the order of the letters is unimportant. The line below could also be named $\overleftrightarrow{ED}$.





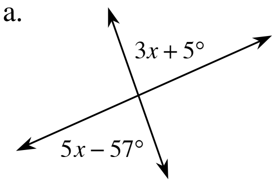
An angle can be named by putting an angle symbol in front of the name of the angle's vertex. For example, the angle measuring 80° in $\triangle ABC$ above is named $\measuredangle A$. Sometimes using a single letter makes it unclear which angle is being referenced. For example, in the diagram at right, it is unclear which angle is referred to by $\measuredangle G$. When this happens, the angle is named with three letters. For example, the angle measuring 10° is called $\measuredangle HGI$ or $\measuredangle IGH$. Note that the name of the vertex must be the second letter in the name; the order of the other two letters is unimportant.

To refer to an angle's measure, an *m* is placed in front of the angle's name. For example, $m\measuredangle HGI = 10^\circ$ means "the measure of $\measuredangle HGI$ is 10°."

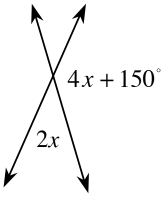
Review & Preview

2-18. **Examine** the diagrams below. What is the geometric relationship between the labeled angles? What is the relationship of their measures? Then, use the relationship to write an equation and solve for x.

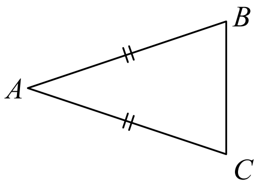
a.



b.

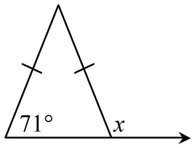


2-19. In problem 2-11, you determined that because an isosceles triangle has reflection symmetry, then it must have two angles that are congruent.



a. How can you tell which angles are congruent? For example, in the diagram at right, which angles must have equal measure? Name the angles and explain how you know.

b. **Examine** the diagram for part (a). If you know that $m\angle B + m\angle C = 124°$, then what is the measure of $\angle B$? Explain how you know.



c. Use this idea to find the value of x in the diagram at right. Be sure to show all work.

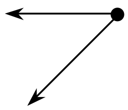
2-20. On graph paper, draw the quadrilateral with vertices (–1, 3), (4, 3), (–1, –2), and (4, –2).

a. What kind of quadrilateral is this?

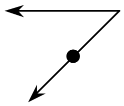
b. Translate the quadrilateral 3 units to the left and 2 units up. What are the new coordinates of the vertices?

2-21. Find the equation for the line that passes through (–1, –2) and (4, 3). Is the point (3, 1) on this line? Be sure to **justify** your answer.

2-22. Juan decided to test what would happen if he rotated an angle.



a. He copied the angle at right on tracing paper and rotated it 180° about its vertex. What type of angle pair did he create? What is the relationship of these angles?



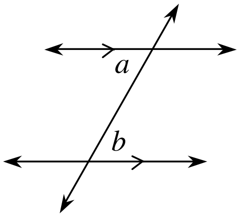
b. Juan then rotated the same angle 180° through a different point (see the diagram at right). On your paper, draw Juan's angle and the rotated image. Describe the overall shape formed by the two angles.

2.1.3 What's the relationship?

More Angles Formed by Transversals

In Lesson 2.1.2, you looked at corresponding angles formed when a transversal intersects two parallel lines. Today you will **investigate** other special angle relationships formed in this situation.

2-23. Suppose $\measuredangle a$ in the diagram at right measures 48°.

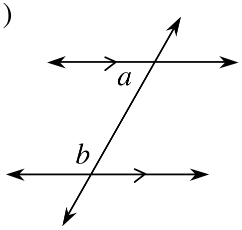


a. Use what you know about vertical, corresponding, and supplementary angle relationships to find the measure of $\measuredangle b$.

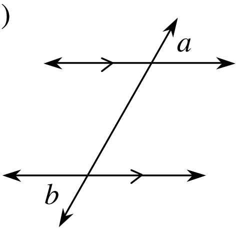
b. Julia is still having trouble seeing the angle relationships clearly in this diagram. Her teammate, Althea explains, "When I translate one of the angles along the transversal, I notice its image and the other given angle are a pair of vertical angles. That way, I know that *a* and *b* must be congruent."

Use Althea's method and tracing paper to determine if the following angle pairs are congruent or supplementary. Be sure to state whether the pair of angles created after the translation is a vertical pair or forms a straight angle. Be ready to **justify** your answer for the class.

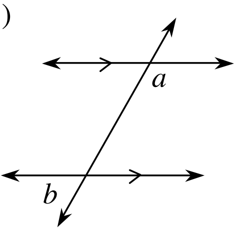
(1)



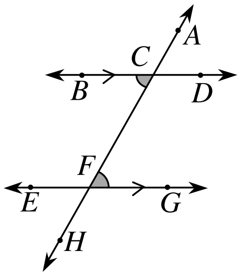
(2)



(3)



2-24. In problem 2-23, Althea showed that the shaded angles in the diagram are congruent. However, these angles also have a name for their geometric relationship (their relative positions on the diagram). These angles are called **alternate interior** angles. They are called "alternate" because they are on opposite sides of the transversal, and "interior" because they are both inside the parallel lines.



a. On tracing paper, trace and shade in $\measuredangle CFG$. How can you transform $\measuredangle CFG$ so that it lands on $\measuredangle BCF$? Be sure your team agrees.

b. Find another pair of alternate interior angles in this diagram.

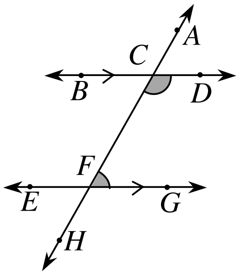
c. Think about the relationship between the measures of alternate interior angles. If the lines are parallel, are they always congruent? Are they always supplementary? Complete the conjecture, "*If lines are parallel, then alternate interior angles are…*".

d. Instead of writing conditional statements, Roxie likes to write **arrow diagrams** to express her conjectures. She expresses the conjecture from part (b) as

Lines are parallel → *alternate interior angles are congruent.*

This arrow diagram says the same thing as the conditional statement you wrote in part (c). How is it different from your conditional statement? What does the arrow mean?

2-25. The shaded angles in the diagram at right have another special angle relationship. They are called **same-side interior** angles.



a. Why do you think they have this name?

b. What is the relationship between the angle measures of same-side interior angles? Are they always congruent? Supplementary? Talk about this with your team.

Then write a conjecture about the relationship of the angle measures. Your conjecture can be in the form of a conditional statement or an arrow diagram. If you write a conditional statement, it should begin, "*If lines are parallel, then same-side interior angles are…*".

2-26. THE REFLECTION OF LIGHT

You know enough about angle relationships now to start analyzing how light bounces off mirrors. **Examine** the two diagrams below. Diagram A shows a beam of light emitted from a light source at A. In Diagram B, someone has placed a mirror across the light beam. The light beam hits the mirror and is reflected from its original path.

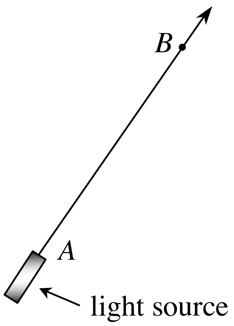


Diagram A

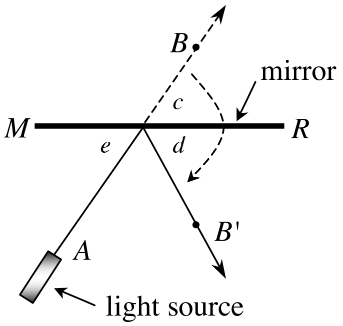


Diagram B

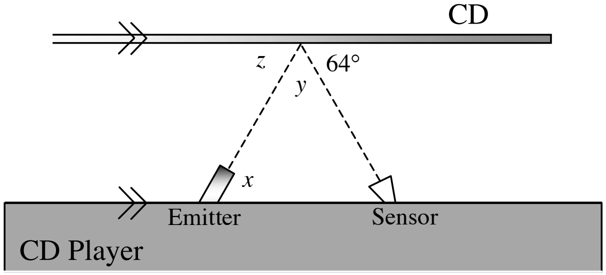
a. What is the relationship between angles *c* and *d*? Why?

b. What is the relationship between angles *c* and *e*? How do you know?

c. What is the relationship between angles *e* and *d*? How do you know?

d. Write a conjecture about the angle at which light hits a mirror and the angle at which it bounces off the mirror.

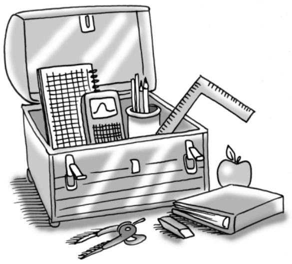
2-27. A CD player works by bouncing a laser off the surface of the CD, which acts like a mirror. An emitter sends out the light, which bounces off the CD and then is detected by a sensor. The diagram below shows a CD held parallel to the surface of the CD player, on which an emitter and a sensor are mounted.



a. The laser is supposed to bounce off the CD at a 64° angle as shown in the diagram above. For the laser to go directly into the sensor, at what angle does the emitter need to send the laser beam? In other words, what does the measure of angle x have to be? **Justify** your conclusion.

b. The diagram above shows two parts of the laser beam: the one coming out of the emitter and the one that has bounced off the CD. What is the angle ($\measuredangle y$) between these beams? How do you know?

2-28. ANGLE RELATIONSHIPS TOOLKIT



Obtain a Lesson 2.1.3 Resource Page ("Angle Relationships Toolkit") from your teacher. This will be a continuation of the Geometry Toolkit you started in Chapter 1. Think about the new angle relationships you have studied so far in Chapter 2. Then, in the space provided, add a diagram and a description of the relationship for each special angle relationship you know. Be sure to specify any relationship between the measures of the angle (such as whether or not they are always congruent). In later lessons, you will continue to add relationships to this toolkit, so be sure to keep this resource page in a safe place. At this point, your toolkit should include:

- Vertical angles
- Corresponding angles
- Same-side interior angles
- Straight angles
- Alternate interior angles

MATH NOTES

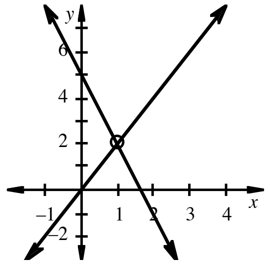
METHODS AND MEANINGS

Systems of Linear Equations

In a previous course, you learned that a **system of linear equations** is a set of two or more linear equations that are given together, such as the example at right. In a system, each variable represents the same quantity in both equations. For example, y represents the same quantity in *both* equations at right.

$$y = 2x$$
$$y = -3x + 5$$

To represent a system of equations graphically, you can simply graph each equation on the same set of axes. The graph may or may not have a **point of intersection**, as shown circled at right.



Sometimes two lines have *no* points of intersection. This happens when the two lines are parallel. It is also possible for two lines to have an *infinite* number of intersections. This happens when the graphs of two lines lie on top of each other. Such lines are said to **coincide**.

The **Substitution Method** is a way to change two equations with two variables into one equation with one variable. It is convenient to use when only one equation is solved for a variable. For example, to solve the system at right:

$$x = -3y + 1$$
$$4x - 3y = -11$$

- Use substitution to rewrite the two equations as one. In other words, replace x with $(-3y + 1)$ to get $4(-3y + 1) - 3y = -11$. This equation can then be solved to find y. In this case, $y = 1$.
- To find the point of intersection, substitute to find the other value.
- Substitute $y = 1$ into $x = -3y + 1$ and write the answer for x and y as an ordered pair.
- To test the solution, substitute $x = -2$ and $y = 1$ into $4x - 3y = -11$ to verify that it makes the equation true. Since $4(-2) - 3(1) = -11$, the solution (–2, 1) must be correct.

$$x = \boxed{-3y + 1}$$
$$4(\quad) - 3y = -11$$
$$4(-3y + 1) - 3y = -11$$
$$-12y + 4 - 3y = -11$$
$$-15y + 4 = -11$$
$$-15y = -15$$
$$y = 1$$

$$x = -3(1) + 1 = -2$$
$$(-2, 1)$$

Review & Preview

2-29. The set of equations at right is an example of a **system of equations**. Read the Math Notes box for this lesson on how to solve systems of equations. Then answer the questions below.

$$y = -x + 1$$
$$y = 2x + 7$$

a. Graph the system on graph paper. Then write its solution (the point of intersection) in (x, y) form.

b. Now solve the system using the Substitution Method. Did your solution match your result from part (a)? If not, check your work carefully and look for any mistakes in your algebraic process or on your graph.

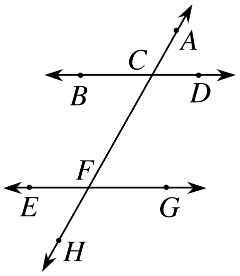
2-30. On graph paper, graph the rectangle with vertices at (2, 1), (2, 5), (7, 1), and (7, 5).

a. What is the area of this rectangle?

b. Shirley was given the following points and asked to find the area, but her graph paper is not big enough. Find the area of Shirley's rectangle, and explain to her how she can find the area without graphing the points.

Shirley's points: (352, 150), (352, 175), (456, 150), and (456, 175)

2-31. Looking at the diagram below, John says that $m\measuredangle BCF = m\measuredangle EFH$.



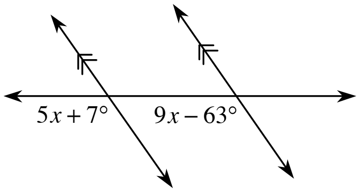
Note: This stoplight icon will appear periodically throughout the text. Problems with this icon display common errors that can be made. Be sure not to make the same mistakes yourself!

a. Do you agree with John? Why or why not?

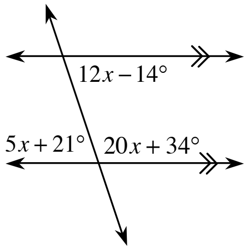
b. Jim says, "You can't be sure those angles are equal. An important piece of information is missing from the diagram!" What is Jim talking about?

2-32. Use your knowledge of angle relationships to solve for x in the diagrams below. **Justify** your solutions by naming the geometric relationship.

a.



b.



2-33. On graph paper, draw line segment $\overline{AB}$ if $A(6, 2)$ and $B(3, 5)$.

a. Reflect $\overline{AB}$ across the line $x = 3$ and connect points A and A'. What shape is created by this reflection? Be as specific as possible.

b. What polygon is created when $\overline{AB}$ is reflected across the line $y = -x + 6$ and all endpoints are connected to form a polygon?