

DIMENSIONAL ANALYSIS

UNIT ANALYSIS

Unit analysis (or dimensional analysis) means using the rules for multiplying and simplifying fractions to solve problems involving different units.

Unit conversion equations are written as a fraction (equivalent to multiplying by "one") so that the unwanted units are removed during simplification and the desired units remain.

Example 1

A driveway is $15\frac{3}{4}$ feet long. How long is this in inches? Use: 12 inches = 1 foot

We want the "foot" units to cancel so the fraction we want is: $\frac{12 \text{ inches}}{1 \text{ foot}}$.

$$15\frac{3}{4} \text{ feet} = \frac{63 \text{ feet}}{4} \cdot \frac{12 \text{ inches}}{1 \text{ foot}} = \frac{63 \cancel{\text{feet}} \cdot 12 \text{ inches}}{4 \cdot 1 \cancel{\text{foot}}} = \frac{756 \text{ inches}}{4} = 189 \text{ inches}$$

Example 2

An automobile web site on the Internet advertises that the new *Neptune Stratus* averages 15 kilometers per liter of petroleum. What is the equivalent in miles per gallon?

Use: 1 gallon = 3.79 liters 1 mile = 1.61 kilometers

We want the "kilometer" and "liter" units to cancel. Use the fractions: $\frac{1 \text{ mile}}{1.61 \text{ kilometers}}$ and $\frac{3.79 \text{ liters}}{1 \text{ gallon}}$.

$$\frac{15 \text{ kilometers}}{\text{liter}} = \frac{15 \cancel{\text{kilometers}}}{\cancel{\text{liter}}} \cdot \frac{3.79 \cancel{\text{liters}}}{1 \text{ gallon}} \cdot \frac{1 \text{ mile}}{1.61 \cancel{\text{kilometers}}} = \frac{15 \cdot 3.79 \cdot 1 \text{ miles}}{1 \text{ gallon} \cdot 1.61} = \frac{35.3 \text{ miles}}{\text{gallon}}$$

Example 3

A container is strong enough not to break under a weight of 40 pounds per square inch $\left(\frac{40 \text{ pounds}}{(\text{inch})^2}\right)$.

What is the equivalent in grams per square centimeter $\left(\frac{\text{grams}}{(\text{centimeter})^2}\right)$?

Use: 1 kilogram = 1000 grams = 2.2 pounds 1 inch = 2.54 centimeters

We want "pounds" and "inches" unit to cancel. Use the fractions: $\frac{1000 \text{ grams}}{2.2 \text{ pounds}}$ and $\frac{1 \text{ inch}}{2.54 \text{ centimeter}}$.

Notice that to cancel the $(\text{inch})^2$ we will need to multiply by the second fraction twice.

$$\frac{40 \text{ pounds}}{(\text{inch})^2} = \frac{40 \cancel{\text{pounds}}}{(\cancel{\text{inch}})^2} \cdot \frac{1000 \text{ grams}}{2.2 \cancel{\text{pounds}}} \cdot \frac{1 \cancel{\text{inch}}}{2.54 \text{ centimeter}} \cdot \frac{1 \cancel{\text{inch}}}{2.54 \text{ centimeter}} = \frac{40 \cdot 1000 \text{ grams} \cdot 1 \cdot 1}{2.2 \cdot 2.54 \text{ centimeter} \cdot 2.54 \text{ centimeter}} = 2818 \frac{\text{grams}}{(\text{centimeter})^2}$$

Use unit analysis and the following conversion equations to solve each problem.

1 hour = 60 minutes	7 days = 1 week	24 hours = 1 day
365 days = 1 year	1 liter = 1000 milliliters	1 liter = 1000 centimeter ³
20 drops = 1 milliliter	.625 mile = 1 kilometer	5280 feet = 1 mile
1 gallon = 3.79 liters	1 inch = 2.54 centimeters	1 gallon = 64 fluid ounces
4 quarts = 1 gallon	1 meter = 100 centimeters	1 foot = 12 inches
3 feet = 1 yard		

Problems

1. A 10-kilometer race is how many miles?
2. One year is how many hours?
3. The distance to the moon is about 250,000 miles. How many feet is this?
4. Traveling 50 miles per hour is the same as how many feet per second?
5. Two hundred fluid ounces is how many gallons?
6. Five hundred feet is the same as how many meters?
7. If tile costs \$8.50 per square foot, how much does it cost per square inch?
8. If carpet is advertised at \$20 per square yard, what is the cost per square centimeter?
9. A soda can holds 355 milliliters. How many drops is this? How many gallons is this?
10. A worm moved 8 inches in 5 seconds. How many miles per hour is this?
11. A chemistry experiment calls for 2 drops of acid for 100 milliliters of solution. How much acid should be used for one gallon of solution?
12. If I buy \$30 of gasoline at \$2.75 per gallon and my car gets 34.2 miles per gallon, how far will I travel on my \$30?
13. A swimming pool holds 10,000 gallons of water. How many cubic meters is this?
14. John's go-cart went 10 laps around a 1320-foot track in 12 minutes. How fast was he traveling in kilometers per hour?
15. In October 2007, one euro was worth \$1.42. The price of gasoline in Switzerland was 0.85 euros per liter. What was the cost in dollars per gallon?

Answers

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| 1. | 6.25 miles | 2. | 8760 hours | 3. | 1,320,000,000 feet |
| 4. | $\approx 73.3 \frac{\text{feet}}{\text{second}}$ | 5. | 3.125 gallons | 6. | ≈ 152.4 meters |
| 7. | $\approx \$0.06$ per (inch) ² | 8. | $\$0.20$ per (centimeter) ² | 9. | 7100 drops, $\approx .09$ gal. |
| 10. | $\approx 1.1 \frac{\text{miles}}{\text{hour}}$ | 11. | 75.8 drops ≈ 4 ml. | 12. | 373 miles |
| 13. | 3.79 (meters) ³ | 14. | $20 \frac{\text{kilometers}}{\text{hour}}$ | 15. | $\approx \$4.57$ per gallon |