

LOGICAL STATEMENTS

CONDITIONAL STATEMENTS

A **conditional statement** is a statement with a hypothesis and a conclusion. When a conditional statement is written in **if-then** form, the hypothesis is the “if” part and the conclusion is the “then” part.

Examples

Write each statement as a conditional (if-then) statement.

Statement 1: The month after July is August.

Answer: If this month is July, then the next month is August.

Statement 2: Three points are collinear if they lie on the same line.

Answer: If three points lie on the same line, then they are collinear.

Problems

Write each statement as a conditional (if-then) statement.

1. Dogs love water.
2. The sum of the angles in a triangle equals 180° .
3. Vertical angles are congruent.
4. Quadrilaterals have four sides.

CONVERSE, INVERSE, AND CONTRAPOSITIVE

The **converse** of a conditional statement is found by switching the hypothesis and the conclusion. The **inverse** of a conditional statement is found by negating both the hypothesis and the conclusion of the statement. The **contrapositive** of a conditional statement is found by negating both the hypothesis and conclusion of the converse.

Each of these statements may be true or false. One **counterexample** is sufficient to show that a statement is false.

Example

For the given statement, determine the converse, inverse, and contrapositive. Then, assuming the given statement is true, determine if each new statement is true or false.

Statement: If it rained, then the ground is wet. (Assume that this is true.)

Converse: If the ground is wet, then it rained. (False, a hose could have made the ground wet.)

Inverse: If it did not rain, then the ground is not wet. (False, same reason as above.)

Contrapositive: If the ground is not get wet, then it did not rain. (True.)

Note: If the original statement is true, the contrapositive is also true. If the original statement is false, the contrapositive is also false. This means that a conditional statement and the contrapositive are **logically equivalent**.

In the same way, the converse of a statement and the inverse of a statement are logically equivalent.

Problems

For the given statement, determine the converse, inverse, and contrapositive. Assuming the given statement is true, determine if each new statement is true or false.

5. If today is Monday, then tomorrow is Tuesday.
6. If $m\angle A = 60^\circ$, then $\angle A$ is acute.
7. If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.
8. If a figure is a square, then it has four congruent sides.

CONJUNCTION, DISJUNCTION, AND BICONDITIONAL

A **conjunction** is a compound sentence formed by joining two statements with the word “and.” A conjunction is true only when both parts are true. A **disjunction** is a compound sentence formed by joining two statements with the word “or.” A disjunction is true when either part or both parts are true. A **biconditional** statement is a conditional statement and its converse. It typically includes the words “if and only if” as in a definition. For a biconditional to be a true statement, it must be true both “forward and backward.”

Examples

Determine the type of statement and whether or not it is true or false.

Statement: Triangles have four sides or pentagons have five sides.

Answer: This is a disjunction because of the word “or.” Since the second part of the statement is true, the statement is true.

Statement: A triangle is equilateral if and only if the three sides are congruent.

Answer: This is a biconditional. Since an equilateral triangle has three congruent sides and if a triangle has three congruent sides it is equilateral, the statement is true.

Statement: Fish swim in water and $3 + 3 = 5$.

Answer: This is a conjunction because of the word “and.” Since one part is false, the entire statement is false.

Problems

Determine the type of statement and whether or not it is true or false.

9. Perpendicular lines form right angles and right angles measure 90° .
10. Mice chase cats or $(2) \cdot (5) = 7$.
11. Alternating interior angles are congruent if and only if parallel lines are cut by a transversal.
12. Rectangles have opposite sides congruent and also have opposite sides parallel.
13. $\frac{1}{2} = 50\%$ or a straight angle contains 360° .
14. A triangle is a right triangle if and only if $(\text{leg \#1})^2 + (\text{leg \#2})^2 > (\text{hypotenuse})^2$.
15. The numeral 102 is read “one hundred and two” or fifty cents is written 0.50¢.

Answers

1. If an animal is a dog, then it loves water.
2. If a figure is a triangle, then the sum of the angles equals 180° .
3. If vertical angles are formed, then the angles are congruent.
4. If the figure is a quadrilateral, then it has four sides.
5. Converse: If tomorrow is Tuesday, then today is Monday. true
Inverse: If today is not Monday, the tomorrow is not Tuesday. true
Contrapositive: If tomorrow is not Tuesday, then today is not Monday. true
6. Converse: If $\angle A$ is acute, then $m\angle A = 60^\circ$. false
Inverse: If $m\angle A \neq 60^\circ$, then $\angle A$ is not acute. false
Contrapositive: If $\angle A$ is not acute, then $m\angle A \neq 60^\circ$. true
7. Converse: If alternating interior angles are congruent, then the two lines cut by the transversal are parallel. true
Inverse: If two non-parallel lines are cut by a transversal, then alternating interior angles are not congruent. true
Contrapositive: If alternating interior angles are not congruent, then the two lines cut by the transversal are not parallel. true
8. Converse: If a figure has four congruent sides, then the figure is a square. false
Inverse: If the figure is not a square, then it does not have four congruent sides. false
Contrapositive: If a figure does not have four congruent sides, then the figure is not a square. true
9. Conjunction-true
10. Disjunction-false
11. Biconditional-true
12. Conjunction-true
13. Disjunction-true
14. Biconditional-false
15. Disjunction-false