

## OPTIMIZATION PROBLEMS

### MAXIMUM AND MINIMUM OF QUADRATIC FUNCTIONS

The graph of the quadratic function  $y = ax^2 + bx + c$  is a parabola. If  $a > 0$ , the parabola is oriented upward and the vertex is the minimum point of the function. If  $a < 0$ , the parabola is oriented downward and the vertex is the maximum point of the function. In either case the vertex of the parabola is located halfway between the roots (the zeros or  $x$ -intercepts.) The  $x$ -coordinate of the vertex can be found by:  $x = \frac{-b}{2a}$ . Substituting the  $x$ -value of the vertex into the equation of the function yields the  $y$ -value of the vertex.

The quadratic function may be given or it may need to be created based on the given information of the situation.

### Examples of Optimization Problems

#### Example 1

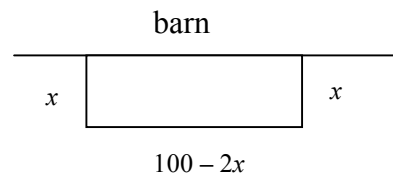
What is the minimum value of the function  $y = 2x^2 - 8x - 5$ ?

**Solution:** The  $x$ -coordinate of the vertex is  $x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = 2$ . The  $y$ -coordinate of the vertex is  $y = 2(2)^2 - 8(2) - 5 = -13$ . The coordinates of the vertex are  $(2, -13)$  and the minimum value of the function is  $-13$ .

#### Example 2

With 100 feet of fence, what are the dimensions of the corral of largest area that a farmer can create if his barn will provide one side of the corral?

**Solution:** Let  $x$  = the width of the corral and then  $100 - 2x$  is the length. The function which represents the area is  $y = (100 - 2x)x = -2x^2 + 100x$ .



The vertex of this function is at  $x = \frac{-b}{2a} = \frac{-100}{2(-2)} = 25$  and  $y = -2(25)^2 + 100(25) = 1250$ .

The corral of maximum area has width  $x = 25$  feet, length 50 feet, and area 1250 ft<sup>2</sup>.

### Example 3

Last year the yearbook at Central High cost \$75 and only 500 were sold. A student survey found that for every \$5 reduction in price, 100 more students will buy yearbooks. What price should be charged to maximize the revenue from yearbook sales?

**Solution:** Let  $x$  = number of \$5 reductions so  $75 - 5x$  is the new cost and  $500 + 100x$  is the new number of books sold. Total Revenue = (cost of books)(number sold) so:  
 $y = (75 - 5x)(500 + 100x) = -500x^2 + 5000x + 37500$  represents the revenue.

The vertex of this function is  $x = \frac{-b}{2a} = \frac{-5000}{2(-500)} = 5$ . The best price to charge would be to have five, \$5 reductions giving a yearbook cost of  $75 - 5(5) = \$50$ .

### Problems

1. What is the maximum value of the function  $y = -x^2 + 2x + 5$  ?
2. What is the minimum value of the function  $y = \frac{1}{3}x^2 + 2x + 5$  ?
3. The height in feet of a rocket after  $x$  seconds is given by  $y = 128x - 16x^2$ . What is the maximum height reached by the rocket and how long does it take to reach that height?
4. The equation for the cost of manufacturing lawn mowers is  $y = 0.008x^2 - 0.04x + 75$ . What number of lawn mowers should be produced to minimize costs?
5. What are the dimensions of the rectangle of greatest area that can be formed with 200 feet of fence?
6. The farmer in Example 2 above realized that he needed to split his one corral into two with a common side. If he still has only 100 feet of fence, now what are the dimensions that will create a maximum area?
7. A tomato grower needs to ship early when prices are high and spoilage is low. She now has 25 tons on hand and can add two tons a week by waiting. The current profit is \$250 per ton but it will reduce by \$15 per ton for each week she delays. When should she ship to receive maximum profit?
8. A transit company charges \$1.25 per ride and currently averages 10,000 riders per day. The company needs to increase revenue but found that for each \$0.10 increase in fare the company would lose 500 riders. What should the company charge to maximize revenues?
9. A piece of wire 20 feet long is cut into two pieces and each piece is bent to form a square. Determine the length of the two pieces so that the sum of the areas of the two squares is a minimum.

## Answers

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|----------------------|---|------------------------------|
| 1. 6                 | 2. 2                                    | 3. 256 feet, 4 seconds       |
| 4. 25 lawn mowers    | 5. 50 X 50 square                       | 6. $16\frac{2}{3}$ X 50 feet |
| 7. In about 2 weeks. | 8. \$0.75 raise to a new fare of \$2.00 | 9. 10 feet each              |