

RIGID TRANSFORMATION

TRANSFORMATIONS THAT PRESERVE SIZE AND SHAPE

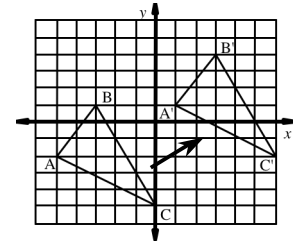
A **RIGID TRANSFORMATION** is a transformation of a figure that preserves size and shape. The three kinds of rigid transformations studied are translation (slide), reflection (flip), and rotation (turn). These transformations may also be combined together. The original figure is called the pre-image and the new figure is called the image. For

ABC it is common to say that the original points A , B , and C are mapped to the new points A' , B' , and C' respectively.

Example 1

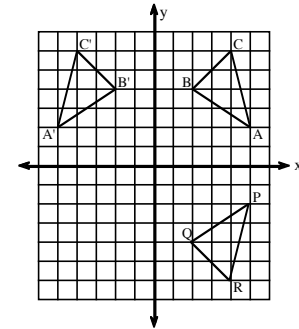
Translate (slide) ABC right six units and up three units. Give the coordinates of the image triangle.

The original vertices are $A(-5, -2)$, $B(-3, 1)$, and $C(0, -5)$. The new vertices are $A'(1, 1)$, $B'(3, 4)$, and $C'(6, -2)$. Notice that each point (x, y) is mapped to $(x + 6, y + 3)$.



Example 2

Reflect (flip) ABC with coordinates $A(5, 2)$, $B(2, 4)$, and $C(4, 6)$ across the y -axis to get $A'B'C'$. The key is that the reflection is the same distance from the axis as the original figure. The new points are $A'(-5, 2)$, $B'(-2, 4)$, and $C'(-4, 6)$. Notice that in reflecting across the y -axis, each point (x, y) is mapped to the point $(-x, y)$.

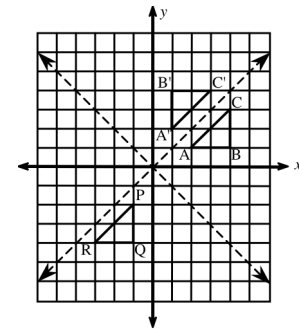


If you reflect ABC across the x -axis to get PQR , then the new points are $P(5, -2)$, $Q(2, -4)$, and $R(4, -6)$. In this case, reflecting across the x -axis, each point (x, y) is mapped to the point $(x, -y)$.

Example 3

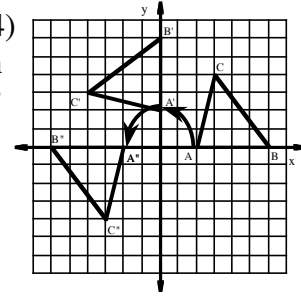
Reflect (flip) ABC with coordinates $A(2, 1)$, $B(4, 1)$, and $C(4, 3)$ across the line $y = x$ to get $A'B'C'$. Think of, or actually fold your paper along the line $y = x$ to see the image. The new points are $A'(1, 2)$, $B'(1, 4)$, and $C'(3, 4)$. Notice that in reflecting across the line $y = x$, each point (x, y) is mapped to the point (y, x) .

If you reflect ABC across the line $y = -x$ to get PQR , then the new points are $P(-1, -2)$, $Q(-1, -4)$, and $R(-3, -4)$. In this case, reflecting across the line $y = -x$, each point (x, y) is mapped to the point $(-y, -x)$.



Example 4

Rotate (turn) $\triangle ABC$ with coordinates $A(2, 0)$, $B(6, 0)$, and $C(3, 4)$ 90° counterclockwise about the origin $(0, 0)$ to get $\triangle A'B'C'$ with coordinates $A'(0, 2)$, $B'(0, 6)$, and $C'(-4, 3)$. Notice that this 90° counterclockwise rotation about the origin maps each point (x, y) to the point $(-y, x)$.



Rotating another 90° (180° from the starting location) yields

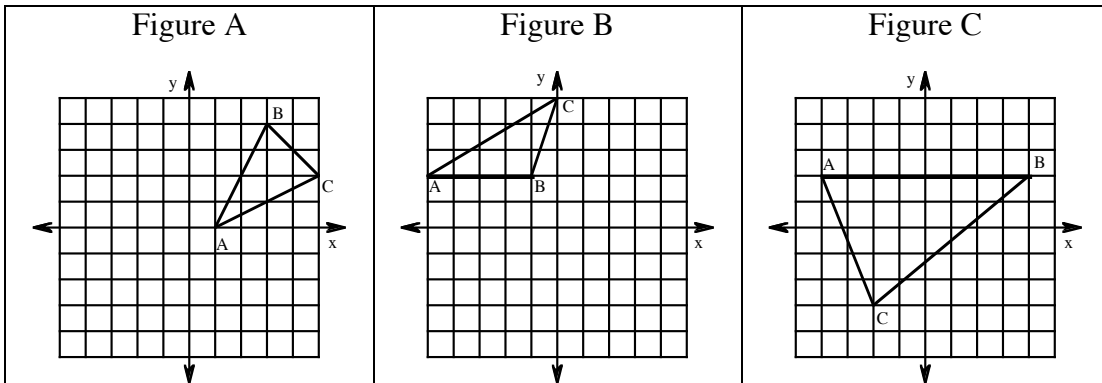
$\triangle A''B''C''$ with coordinates $A''(-2, 0)$, $B''(-6, 0)$, and $C''(-3, -4)$.

This 180° counterclockwise rotation about the origin maps each point (x, y) to the point $(-x, -y)$.

Similarly a 270° counterclockwise or 90° clockwise rotation about the origin maps each point (x, y) to the point $(y, -x)$.

Problems

For the following problems, refer to the figures below:



Tell the new coordinates after each transformation.

1. Slide figure A left 2 units and down 3 units.
2. Slide figure B right 3 units and down 5 units.
3. Slide figure C left 1 unit and up 2 units.
4. Flip figure A across the x -axis.
5. Flip figure B across the x -axis.
6. Flip figure C across the x -axis.
7. Flip figure A across the y -axis.
8. Flip figure B across the y -axis.
9. Flip figure C across the y -axis.
10. Flip figure A across the line $y = x$.
11. Flip figure A across the line $y = -x$.
12. Flip figure B across the line $y = x$.
13. Flip figure B across the line $y = -x$.
14. Flip figure C across the line $y = x$.
15. Flip figure C across the line $y = -x$.

16. Rotate figure A 90° counterclockwise about the origin.
17. Rotate figure B 90° counterclockwise about the origin.
18. Rotate figure C 90° counterclockwise about the origin.
19. Rotate figure A 180° counterclockwise about the origin.
20. Rotate figure C 180° counterclockwise about the origin.
21. Rotate figure B 270° counterclockwise about the origin.
22. Rotate figure C 90° clockwise about the origin.

Answers (given in order A', B', C')

- | | |
|-------------------------------|------------------------------|
| 1. (-1, -3) (1, 2) (3, -1) | 2. (-2, -3) (2, -3) (3, 0) |
| 3. (-5, 4) (3, 4) (-3, -1) | 4. (1, 0) (3, -4) (5, -2) |
| 5. (-5, -2) (-1, -2) (0, -5) | 6. (-4, -2) (4, -2) (-2, 3) |
| 7. (-1, 0) (-3, 4) (-5, 2) | 8. (5, 2) (1, 2) (0, 5) |
| 9. (4, 2) (-4, 2) (2, -3) | 10. (0, 1) (4, 3) (2, 5) |
| 11. (0, -1) (-4, -3) (-2, -5) | 12. (2, -5) (2, -1) (5, 0) |
| 13. (-2, 5) (-2, 1) (-5, 0) | 14. (2, -4) (2, 4) (-3, -1) |
| 15. (-2, 4) (-2, -4) (3, 1) | 16. (0, 1) (-4, 3) (-2, 5) |
| 17. (-2, -5) (-5, 0) (-2, -1) | 18. (-2, -4) (-2, 4) (3, -2) |
| 19. (-1, 0) (-3, -4) (-5, -2) | 20. (4, -2) (-4, -2) (2, 3) |
| 21. (2, 5) (2, 1) (5, 0) | 22. (2, 4) (2, -4) (-3, 2) |