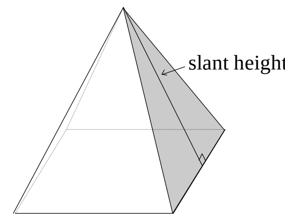
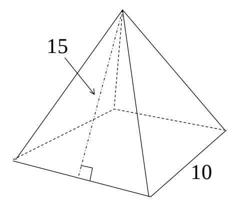
SURFACE AREA OF A PYRAMID

A **pyramid** is a geometrical solid made up of one base and lateral faces that are triangles. These triangles meet in a single vertex. The base can be any shape. To find the surface area of a pyramid, you must find the area of each triangular lateral face and the area of the base, and then sum these areas to get the surface area.



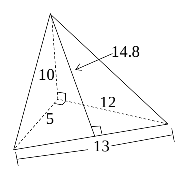
Because each lateral face is a triangle, we will find the area of each face using $A = \frac{1}{2}bh$. "*h*," the height of the triangular face, is called the **slant height** of the pyramid. To calculate the area of the base of the pyramid, use the formula that is appropriate for that shape.

Examples



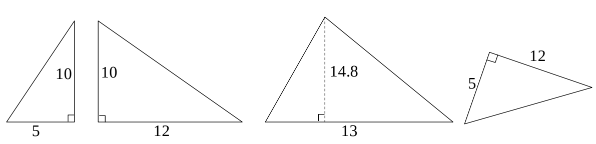
Example 1: The pyramid at right has a square base with sides of length 10 units. The lateral faces are four congruent triangles with a height of 15 units.

To find the surface area we need to find the area of the base and add it to the areas of each of the four lateral faces. Since these triangles are congruent, they all have the same area. The area of the square base is $A = 10^2 = 100$ square units. Each triangle has a base of length 10 and a height of 15. The area of one lateral face is $A = \frac{1}{2}(10)(15) = 75$ square units. Therefore the surface area is $A = 100 + 4(75) = 400$ square units.



Example 2: At right is a triangular pyramid in which the vertex is not centered over the base, but rather directly over a vertex of the base. The base is a right triangle.

To find the surface area of this solid, we have to find the area of each face, all of which are different triangles. Imagine if we could make a net of this solid, or unfold it into its pieces. We'd have four triangles: (Make sure you can find each triangle in the diagram above.)



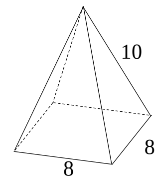
To find the area of each triangle, we will use the formula $A = \frac{1}{2}bh$.

$A(\Delta\#1) = \frac{1}{2}(5)(10) = 25$ square units $\qquad$ $A(\Delta\#2) = \frac{1}{2}(12)(10) = 60$ square units

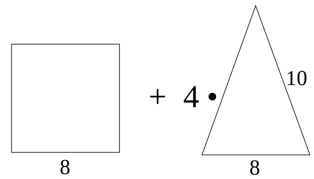
$A(\Delta\#3) = \frac{1}{2}(13)(14.8) = 96.2$ square units $\qquad$ $A(\Delta\#4) = \frac{1}{2}(5)(12) = 30$ square units

Therefore the surface area is: $S.A. = 25 + 60 + 96.2 + 30 = 211.2$ square units

Example 3: The pyramid at right has a square base with side length of 8 units. The four lateral faces are congruent isosceles triangles with lateral edges of length 10 units.

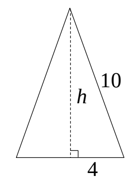


To find the surface area of this pyramid, we will find the area of the square base and the four congruent isosceles triangles. This can be represented as:

$A =$ 

Because the four triangles are congruent, we can find the area of one triangle and multiply by four to get the area of all the lateral faces.

The area of the square is $A = 8^2 = 64$ square units, but the area of the triangle will not come so easily. To find the area of the triangle we need to find the height, and we can use the Pythagorean theorem to do this.



$$4^2 + h^2 = 10^2$$
$$16 + h^2 = 100$$
$$h^2 = 84$$
$$h = \sqrt{84} = 2\sqrt{21}$$

Therefore the area of the triangle is:

$$A(\Delta) = \tfrac{1}{2}(8)(2\sqrt{21})$$
$$= 8\sqrt{21}$$

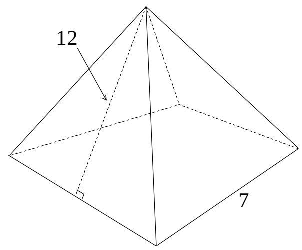
Now we can find the surface area of the pyramid.

$$\text{Surface Area } = 64 + 4\left(8\sqrt{21}\right)$$
$$= 64 + 32\sqrt{21} \text{ square units}$$

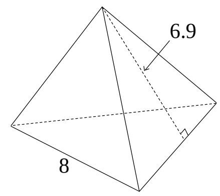
Problems

Find the surface area of each of the following solids.

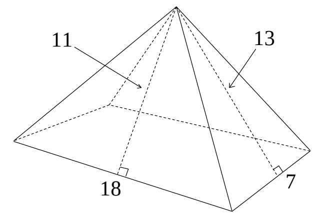
1. The base is a square, and all four lateral faces are congruent.



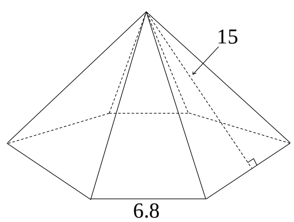
2. The lateral faces and the base are congruent.



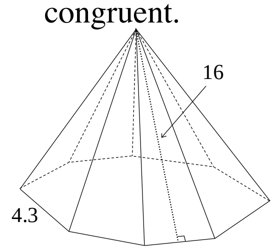
3. The base is a rectangle, and opposite lateral faces are congruent.



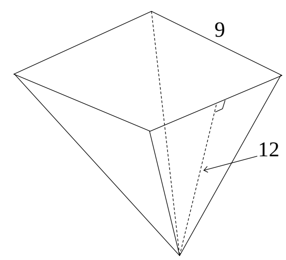
4. The base is a hexagon with an area of 120 square units, and the lateral faces are congruent.



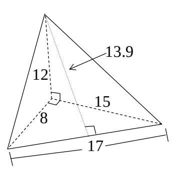
5. The base is a regular octagon with an area of 88 square units, and the lateral faces are congruent.



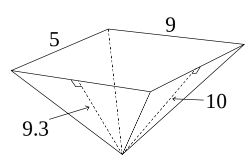
6. The base is a square and all four lateral faces are congruent.



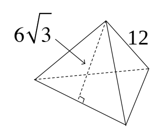
7.



8. The base is a rectangle and opposite lateral faces are congruent.

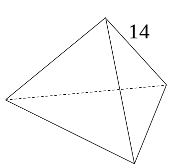


9. This is a regular tetrahedron.

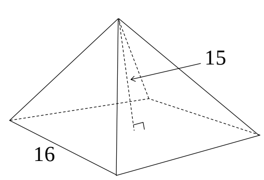


These last three problems require the use of the Pythagorean theorem.

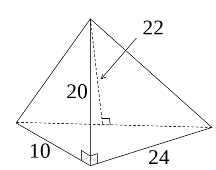
10. This is a regular tetrahedron.



11. The base is a square and the lateral faces are congruent.



12. The base is a right triangle.



Answers

1. 217 square units
2. 110.4 square units
3. 415 square units
4. 426 square units
5. 363.2 square units
6. 297 square units
7. 316.15 square units
8. 178.7 square units
9. $144\sqrt{3} \approx 249.42$ square units
10. $196\sqrt{3} \approx 339.48$ square units
11. 800 square units
12. 746 square units