

WORK, MIXTURE, DISTANCE PROBLEMS

WORK PROBLEMS

Work problems are solved using the concept that if a job can be completed in r units of time, then its rate (or fraction of the job completed) is $\frac{1}{r}$.

Examples of Work Problems

Example 1

John can completely wash and dry the dishes in 20 minutes. His brother can do it in 30 minutes. How long will it take them working together?

Solution: Let t = the time to complete the task so $\frac{1}{t}$ is their rate together. John's rate is $\frac{1}{20}$ and his brother's rate is $\frac{1}{30}$. Since they are working together we add the two rates together to get the combined rate.

$$\begin{aligned}\text{The equation is } \frac{1}{20} + \frac{1}{30} &= \frac{1}{t}. \text{ Solve using fraction busters: } 60t\left(\frac{1}{20} + \frac{1}{30}\right) = 60t\left(\frac{1}{t}\right) \\ 3t + 2t &= 60 \\ 5t &= 60 \\ t &= 12 \text{ minutes}\end{aligned}$$

Example 2

With two inflowing pipes open, a water tank can be filled in 5 hours. If the larger pipe can fill the tank alone in 7 hours, how long would the smaller pipe take to fill the tank?

Solution: Let t = the time for the smaller pipe so $\frac{1}{t}$ is its rate. The combined rate is $\frac{1}{5}$ and the larger pipe's rate is $\frac{1}{7}$. Since they are working together we add the two rates together to get the combined rate.

$$\begin{aligned}\text{The equation is } \frac{1}{7} + \frac{1}{t} &= \frac{1}{5}. \text{ Solve using fraction busters: } 35t\left(\frac{1}{7} + \frac{1}{t}\right) = 35t\left(\frac{1}{5}\right) \\ 5t + 35 &= 7t \\ 35 &= 2t \\ t &= 17.5 \text{ hours}\end{aligned}$$

Problems

Solve each work problem.

1. Susan can paint her living room in 2 hours. Her friend Jaime estimates it would take him 3 hours to paint the same room. If they work together, how long will it take them to paint Susan's living room?
2. Professor Minh can complete a set of experiments in 4 hours. Her assistant can do it in 6 hours. How long will it take them to complete the experiments working together?
3. With one hose a swimming pool can be filled in 12 hours. Another hose can fill it in 16 hours. How long will it take to fill the pool using both hoses?
4. Together, two machines can harvest a tomato crop in 6 hours. The larger machine can do it alone in 10 hours. How long does it take the smaller machine to harvest the crop working alone?
5. Steven can look up 20 words in a dictionary in an hour. His teammate Mary Lou can look up 30 words per hour. Working together, how long will it take them to look up 100 words?
6. A water tank is filled by one pump in 6 hours and is emptied by another pump in 12 hours. If both pumps are operating, how long will it take to fill the tank?
7. Two crews can service the space shuttle in 12 days. The faster crew can service the shuttle in 20 days alone. How long would the slower crew need to service the shuttle working alone?
8. Janelle and her assistant Ryan can carpet a house in 8 hours. If Janelle could complete the job alone in 12 hours, how long would it take Ryan to carpet the house working alone?
9. Able can harvest a strawberry crop in 4 days. Barney can do it in 5 days. Charlie would take 6 days. If they all work together, how long will it take them to complete the harvest?

MIXTURE PROBLEMS

Mixture problems are solved using the concept that the product (value or percentage) \times (quantity) must be consistent throughout the equation.

Examples of Mixture Problems

Example 1

Alicia has 10 liters of an 80% acid solution. How many liters of water should she add to form a 30% acid solution?

Solution: Use $(\%) \cdot (\text{liters}) = (\%) \cdot (\text{liters})$. Let x = liters of water added so $x + 10$ is new liters.

The equation is $(0.8) \cdot (10) = (0.3) \cdot (x + 10)$.

Multiply by 10 to clear decimals and solve. $(8)(10) = (3)(x + 10)$

$$80 = 3x + 30$$

$$50 = 3x$$

$$x = 16 \frac{2}{3} \text{ liters}$$

Example 2

A store has candy worth \$0.90 a pound and candy worth \$1.20 a pound. If the owners want 60 pounds of candy worth \$1.00 a pound, how many pounds of each candy should they use?

Solution: Use $(\$)\cdot(\text{lbs}) = (\$)\cdot(\text{lbs})$.

Let x = pounds of \$0.90 candy so $60 - x$ = pounds of \$1.20 candy.

The equation is $0.90(x) + 1.20(60 - x) = 1.00(60)$.

Multiply by 100 to clear decimals and solve. $90(x) + 120(60 - x) = 100(60)$

$$90x + 7200 - 120x = 6000$$

$$-30x = -1200$$

$$x = 40$$

40 lbs. of \$0.90 candy and 20 lbs. of \$1.20 candy

Note: The second example above could also have been solved using two equations, where x = \$0.90 candy and y = \$1.20 candy:

$$x + y = 60 \text{ and } \$0.90x + \$1.20y = \$60.00$$

Problems

Solve each mixture problem.

10. How much coffee costing \$6 a pound should be mixed with 3 pounds of coffee costing \$4 a pound to create a mixture costing \$4.75 a pound?
11. Sam's favorite recipe for fruit punch requires 12% apple juice. How much pure apple juice should he add to 2 gallons of punch that already contains 8% apple juice to meet his standards?
12. Jane has 60 liters of 70% acid solution. How many liters of water must be added to form a solution that is 40% acid?
13. How many pound of nuts worth \$1.05 a pound must be mixed with nuts worth \$0.85 a pound to get a mixture of 200 pounds of nuts worth \$0.90 a pound?
14. A coffee shop mixes Kona coffee worth \$8 per pound with Brazilian coffee worth \$5 per pound. If 30 pounds of the mixture is to be sold for \$7 per pound, how many pounds of each coffee should be used?
15. How much tea costing \$8 per pound should be mixed with 2 pounds of tea costing \$5 per pound to get a mixture costing \$6 per pound?
16. How many liters of water must evaporate from 50 liters of an 8% salt solution to make a 25% salt solution?
17. How many gallons of pure lemon juice should be mixed with 4 gallons of 25% lemon juice to achieve a mixture which contains 40% lemon juice?
18. Brian has 20 ounces of a 15% alcohol solution. How many ounces of a 50% alcohol solution must he add to make a 25% alcohol solution?

DISTANCE PROBLEMS

Distance problems are solved using the concept that the distance equals the product of rate and time: $d = rt$. Drawing a diagram will very often help you to write the needed equation.

Examples of Distance Problems

Example 1

Carol and Jan leave from the same place and travel on the same road. Carol walks at a rate of two miles per hour. Carol left five hours earlier than Jan, but Jan bikes at a rate of six miles per hour. When will Jan catch up?

Solution: Draw a diagram like the one at right.

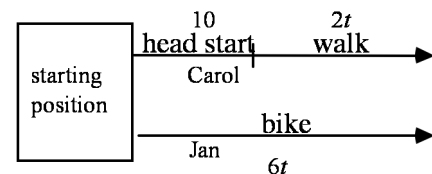
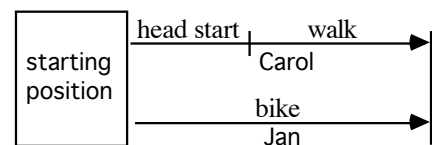
Carol's head start: $d = rt = 5 \cdot 2 = 10$. If $t =$ time for Jan to catch Carol, Jan's distance during that time is $d = rt = 6t$ and Carol's is $2t$.

Adding that information to the diagram:

Use the updated diagram to write and solve an equation showing Carol and Jan travel the same distance yields. The equation is: $10 + 2t = 6t$

$$10 = 4t$$

$$t = 2.5 \text{ hours}$$



Example 2

Two trucks leave a rest stop at the same time. One heads due east; the other heads due north and travels twice as fast as the first truck. The trucks lose radio contact when they are 47 miles apart. How far has each truck traveled when they lose contact?

Solution: Draw the top diagram. Since both trucks travel for the same time and the north truck is traveling twice as fast, its distance is twice as long. If $x =$ distance of the east truck, our diagram now becomes the right triangle in the bottom diagram. From the Pythagorean theorem we have: $(2x)^2 + x^2 = 47^2$

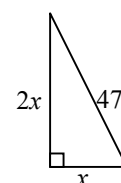
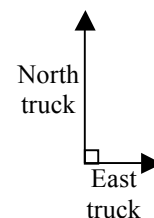
$$4x^2 + x^2 = 47^2$$

$$5x^2 = 2209$$

$$x^2 = 441.8$$

$$x \approx 21.0$$

The east truck traveled 21.0 miles and the north truck traveled 42.0 miles.



Problems

Solve each distance problem.

19. Matilda and Nancy are 60 miles apart, bicycling toward each other on the same road. Matilda rides 12 miles per hour and Nancy rides eight miles per hour. In how many hours will they meet?
20. Two cars start together and travel in the same direction. One car travels twice as fast as the other. After five hours they are 275 kilometers apart. How fast is each car traveling?
21. Two cars leave a parking lot at the same time. One car travels south at 55 mph and the other car travels west at 45 mph. How long will it take until the cars are 150 miles apart?
22. A cheetah spots a gazelle 132 meters away. The cheetah starts towards the gazelle at a speed of 18 meters per second. At the same instant, the gazelle starts moving away from the gazelle at 11 meters per second. How long will it take the cheetah to get dinner?
23. Two trucks leave the same rest stop at the same time traveling at the same speed. One heads south, and the other travels west. When the two trucks lose CB contact, they are 53 miles apart. How far has each truck traveled?
24. Cleopatra rode an elephant to the outskirts of Rome at two kilometers per hour and then took a chariot back to camp at 10 kilometers per hour. If the total traveling time was 18 hours, how far was it from camp to the outskirts of Rome?
25. Jethro and Mac were in a 24-hour bicycle race. Jethro biked at an average speed of 17 miles per hour and finished 60 miles ahead of Mac. What was Mac's average speed?
26. Cassie walked for a while at three miles per hour and then continued her journey on a bus at 15 miles per hour. Her time on the bus was twice as long as her time walking. How long did she ride on the bus if the total distance she covered was 66 miles?
27. A truck going 70 miles per hour passes a parked highway patrol car. When the truck is half a mile past the patrol car, the officer starts after it going 100 miles per hour. How long does it take the patrol car to overtake the truck?

Answers

- | | | | | | |
|-----|----------------------|-----|-----------------------------|-----|---|
| 1. | 1.2 hours | 2. | 2.4 hours | 3. | $6\frac{6}{7} \approx 6.86$ hours |
| 4. | 15 hours | 5. | 2 hours | 6. | 12 hours |
| 7. | 30 days | 8. | 24 hours | 9. | $\frac{60}{37} \approx 1.62$ days |
| 10. | 1.8 pounds | 11. | $\frac{1}{11}$ gallon | 12. | 45 liters |
| 13. | 50 pounds | 14. | 20 Kona, 10 Brazilian | 15. | 1 pound |
| 16. | 34 liters | 17. | 1 gallon | 18. | 8 ounces |
| 19. | 3 hours | 20. | slow 55 kph fast 110 kph | 21. | ≈ 2.1 hours |
| 22. | ≈ 19 seconds | 23. | 37.5 miles | 24. | 30 km |
| 25. | 14.5 mph | 26. | 4 hours | 27. | $\approx 0.0167 = \frac{1}{60}$ hour or 1 minute |